Development of Analytical Models for RC Columns under Blast loading Considering Strain Rate Effects

Mohammad Esmaeilnia Omran¹, Somayeh Mollaei¹,*

¹Department of Civil Engineering
University of Kurdistan, Sanandaj, IRAN.

Received 4 October 2016; accepted 8 May 2017; available online 9 May 2017

Abstract: One of the most important examples of transverse excitation of beam-columns is structural reinforced concrete (RC) columns under blast loading. Under accidental or intentional explosions next to the buildings, external columns are the most critical and vulnerable structural elements. In this paper, two analytical approaches are used to predict the first maximum dynamic response of rectangular RC column under simultaneously effect of axial force and transverse blast loading. The first analytical model is based on continuous formulation of Euler-Bernoulli beam theory and the second model is a single degree of freedom (SDOF) approach. Both of the approaches consider strain rate effects on nonlinear behavior of materials (concrete and steel reinforcement) and secondary effects of P-δ. Results of proposed models for predicting the first maximum lateral response of column under impulsive, dynamic and quasi-static loading regimes are compared to the results of nonlinear finite element analysis. The outcomes indicate undesirable discrepancies under high levels of axial force and quasi-static loading conditions. Nevertheless, in the impulsive and dynamic regimes and moderate and low axial load ratio, the differences in the results are acceptable. Afterward, the analytical models are used to evaluate Pressure-Impulse (P-I) diagram for RC column under blast loading and effective factors on it.

Keywords: Continuous Model, SDOF Approach, RC column, Blast Load, Axial Force, Strain rate effects.

1. Introduction

The performance of axially loaded structural components under extreme dynamic loads, such as explosion and impact, is a common practical issue in a wide range of engineering problems especially in civil engineering. In the external explosions which may be occurred in the vicinity of buildings, external columns are often the most vulnerable structural components [1]. Despite the importance of this issue, in engineering applications the slight analytical studies have been done over the dynamic behavior of the axially loaded RC columns (beam-columns) under blast loading. Most studies in the field of evaluating RC columns under blast loading, has been done by means of numerical modeling by finite element software packages [1-8]. Nevertheless, analytical methods are essential tools to create a comprehensive insight to physical behavior of structural component [9]. Due to the high speed and acceptable accuracy of the analytical methods, they are especially suitable for initial designing and evaluation of residual strength of structural components after the explosion. Also, the results of analytical methods are applicable as fast tool for validation of numerical methods. Most of the studies in the field of nonlinear dynamic systems are related to lumped systems, and continuous structural components such as beams and columns are less investigated [10]. Details related to analytical models for free and force vibration of beams have in some research papers [11-18] and some books [19-24]. Most of these studies are related to elastic beam-columns, and solving such problems by considering non-elastic deformations and effects of strain rates under severe dynamic loadings are investigated in very few studies. For instance, a research has been done by Carta & Stochino (2013) on the continuous analytical models of RC beams under the explosion by considering high strain rate effects [25]. In their study, the effect of axial loads is not considered. Akbari et al (2014) introduced a new method for solving differential equation of nonlinear vibration of continuous beam-columns without considering the effects of high strain rate [26]. Single degree of freedom (SDOF) method is a simple analytical approach that is the basis of blast analysis and design references [27-31]. It has been shown that equivalent SDOF system can model the behavior of steel beams and columns [17, 32-34], slabs and walls [35-37] and RC beams [18, 25, 38, 39] under the blast loading with acceptable accuracy. But, studies on how to consider compressive axial load effects (P-δ) in SDOF models for RC columns under blast loading is very rare. As an example, Nassr et al (2013) have examined the axial load effect in SDOF models of slender steel columns under blast loading [33]. Some studies have also been conducted by US Army Corps of engineers that publishing the complete results of those studies has legal and military limitations [40, 41].

In this paper, a continuous formulation and a SDOF analytical model are used in order to estimate transverse...
displacement response of RC columns under simultaneous effects of compressive axial force and lateral blast loading. Both analytical approaches are based on Euler–Bernoulli beam theory and they consider the effects of secondary moments (P–\(\delta\)) and the effects of high strain rates on the nonlinear behavior of concrete and steel bars. In order to consider P–\(\delta\) effects for RC column modeling, the concept of reduced resistance function is used [42]. The response of column under blast loading with variable time-history and different levels of axial load is investigated. Then, using the introduced analytical models, P-I diagrams have been plotted for RC columns. P-I diagram is a graphical tool for assessment and initial design of structures and structural members under blast loading [43–47]. For a RC column (or beam-column) this failure level can be defined as maximum flexural deformation in mid height [27, 46, 47, 48], shear deformation at supports in the space of shear failure [49, 50], maximum rotation at supports or connections [27] or residual axial capacity of beam-column [2]. In this study, maximum deformation of the structural component is considered as failure criterion. The effects of axial load level and blast pressure time history on the column’s P-I diagram are investigated, as well.

2. Blast Loading

For engineering applications in order to simplify the blast loading, it has recommended different loading patterns on the structure faces [28–30]. In blast loading with medium to far standoff distance, pressure distribution on the structural face can be assumed to be uniform [2, 28, 30, 51]. Some of pressure time-histories, which are commonly used in analysis and design of structures under explosion, are triangular, rectangular, bilinear, exponential and rectangular with limited rise time [30]. According to the duration time of blast pressure \(t_0\), the structural maximum response is occurred in three different regimes: quasi-static, dynamic and impulsive [52]. In impulsive regime \(t_{\text{max}}\) (the time that maximum response occurs) is much longer than \(t_0\) and in this regime \(\frac{t_0}{T} < 0.1\) (\(T\) is natural period of vibration of the system). In dynamic regime \(t_{\text{max}}\) and \(t_0\) are almost close to each other (\(0.1 < \frac{t_0}{T} < 10\)) and in quasi-static regime \(t_{\text{max}}\) is much shorter than \(t_0\) (\(\frac{t_0}{T} > 10\)). In this paper, different explosion duration time is used to evaluate column’s behavior in all three regimens. The space distribution of lateral load is assumed to be uniform on one side of component (front face). Details and definitions of blast load parameters and their calculations can be found in various blast loading references [28, 30, 52]. It should be noted that in this paper, negative phase of explosion is neglected.

3. Analytical Models

3.1 Continuous Model

Consider a RC column (beam-column) with rectangular cross section under uniform distributed lateral dynamic loading and a fixed axial force. Governing differential equation of Euler-Bernoulli is defined by Eq. 1 [21–23, 53]:

\[
\frac{\partial^2 M}{\partial x^2} + N \frac{\partial^2 u(x,t)}{\partial x^2} + \mu \frac{\partial^2 u(x,t)}{\partial t^2} = q(x,t)
\]

Where, \(M\) is resistant bending moment of the section, \(N\) axial force, \(\mu\) mass of unit length, \(q\) lateral load, \(u\) lateral displacement, \(x\) initial column longitudinal axis and \(t\) is time variable. In this equation, shear deformation and rotational inertia of the section are ignored and displacements and rotations are supposed to be small. A force tending to return the structure to its position before loading is called structural resistant that the amount of this resistant is depended on the loading, structural geometry and its support condition [41]. Resistance equation is a force-displacement curve (or moment-curvature) which is depended on response history [24]. For a RC rectangular section under simultaneous effect of axial load and bending moment, a bilinear equation (Eq. 2) can be considered as an approximation of real moment-curvature diagram of the section; Such that the area under the bilinear function would be equal to the area under the real curve.

\[
M(\gamma) = \begin{cases} 
K_\gamma \psi_y + M_\gamma & ; 0 < \psi_y < \psi_y \leq \psi_u \\
K \psi_y + M_\gamma & ; \psi_y < \psi_u 
\end{cases}
\]

In Eq. 2, \(K\) and \(K\) are slope of resistance function in elastic and plastic range, respectively, \(\psi_y\) and \(M_y\), \(\psi_u\) and \(M_u\) are curvature and resistant moment of the section in yield and ultimate state, respectively. By substituting the resistance function in the equation of motion of the column (Eq. 1) and given that \(\psi = \frac{\delta u}{\delta x}\), differential equation of column will be as Eq. 3. The negative sign in this equation is considered since positive bending moment produces negative curvature in the section.

\[
\begin{cases} 
\frac{\partial^2 u(x,t)}{\partial x^2} + N \frac{\partial^2 u(x,t)}{\partial x^2} + \mu \frac{\partial^2 u(x,t)}{\partial t^2} = q(x,t) & ; \text{elastic} \\
\frac{\partial^2 u(x,t)}{\partial x^2} + N \frac{\partial^2 u(x,t)}{\partial x^2} + \mu \frac{\partial^2 u(x,t)}{\partial t^2} = q(x,t) & ; \text{plastic}
\end{cases}
\]

It must be noted that in the above equation, every arbitrary spatial and time distribution for lateral load \(q\) can be considered. By solving above equation, time history of the transverse displacement and curvature of the column can be calculated.

3.2 Equivalent SDOF Model

SDOF analysis, based on Biggs (1964) method [19], is an essential part of blast engineering and the reason of its popularity is no need to specialized finite element software [17]. In this method the structural member is idealized as an equivalent mass-spring system with one degree of freedom. SDOF model is based on the assumption that structure experiences a deformation pattern that is described with only one parameter [35]. In order to consider plastic deformations, different
equivalent coefficients for elastic and plastic states are used. Consider a two ends hinged member under uniformly distributed loading. As an estimation of elastic deformation, static deformed shape of a simple beam under uniform loading can be applied [19]. In plastic range, it is assumed that a plastic hinge is formed in the middle length of the member, so the deformed shape is linear. To ensure that response parameters which are obtained from SDOF system have sufficient accuracy, this equivalent SDOF is selected in such a way that the maximum deformation correspond the deformation of a critical point in the actual structure [19].

In most practical applications, it is important to determine the maximum response of the system. In this study, the aim of providing SDOF model is to determine the first maximum transverse response of the column under blast loading. It has been shown that under the short and intense stimulations, such as impact and explosion, the amount of dissipated energy by structural damping is very low and ignorance of damping is a conservative assumption [30, 38, 41, 54, 55]. Therefore, in SDOF models in this article the system damping is ignored. Thus, the dynamic equation of motion for SDOF model of RC column without damping is expressed as Eq.4:

$$M_E(t)\ddot{u}_E(t)+K_E(t)u_E(t)=-P_E(t)$$

Where $P_E(t)$ is the equivalent loading, $M_E(t)$ equivalent mass, $K_E(t)$ equivalent stiffness, $u_E(t)$ equivalent displacement of SDOF system and $\ddot{u}_E(t)$ refers to the acceleration of the system. We have $M_E(t)=K_{LM} M_b$ that $K_{LM}$ is load-mass coefficient (for a simple beam with distributed uniform load $K_{LM} \approx 0.78$ in elastic range and 0.66 in plastic range) and $M_b$ is total mass of the column. In this paper, SDOF system behavior is assumed to be elasto-plastic that can be displayed by a bilinear load-displacement ($P-u$) diagram according to Fig. 1 in which $K_{E,pl}$ is equivalent plastic stiffness (the line slope in plastic range) and $K_{E,el}$ is equivalent elastic stiffness.

![Fig. 1 load-displacement diagram for SDOF model.](image)

Where $u_{E,y}$ and $P_y$ and $u_{E,u}$ and $P_u$ are displacement and load at ultimate and yield state, respectively. In a simple beam under uniformly distributed loading using equilibrium equations we have:

$$M_y = \frac{q_y l^2}{8} \quad \rightarrow \quad P_y = q_y l - \frac{3M_y}{l}, \quad P_u = \frac{3M_y}{l}$$

Where $q_y$ and $q_u$ is the uniform lateral load on the column at the yield and ultimate state, respectively. $M_y$ and $M_u$ are determined according to moment-curvature diagram of the section. If $\phi_y$ and $\phi_u$ are section curvature values in the ultimate and yield state respectively, yield displacement $u_{E,u}$ in the middle length of the member can be approximated as follows:

$$u_{E,u} = \frac{1}{48} f_y^2$$

To calculate $u_{E,u}$, assume that a plastic hinge is formed in the middle length then (with small displacements):

$$u_{E,u} = u_{E,y} + \frac{P_u}{2} \frac{l}{2}$$

Where, $\theta_{pu}$ is plastic rotation in the ultimate state. If plastic curvature $\phi_u (\theta_p = \theta_u - \phi_p)$ is constant through the length of the plastic hinge, then $\theta_{pu} = l_p \phi_p$ that $l_p$ is the length of plastic hinge. In order to estimate the plastic hinge length, several equations have been proposed [56, 57]. In this paper, suggested equation by Pauli and Priestley (1992) is used [58]:

$$l_p = 0.08 i + 0.022 d_y$$

Where $d_y$ and $P_y$ is diameter and yield stress of longitudinal reinforcement bars.

3.3 Secondary moments (P-δ) effects

P-δ effects is considered explicitly in the calculations of both continuous and SDOF equation of motions. In each time step of calculations, a dynamic uniform equivalent lateral load (ELL) is applied to the column; The amount of ELL is calculated in a way that maximum bending moment resulted by that is equal to maximum bending moment caused by axial load with the eccentricity equal to calculated transverse displacement ($u$) at that time step. This assumption is based on the concept of reduced resistance function [42]. Equivalent lateral load $\delta(t)$ corresponded to P-δ effects for a simple one-way member is calculated by Eq.9 [36, 42]:

$$\delta(t) = \frac{N}{l^2} u(t)$$

Where $N$ is compressive axial load and $u(t)$ is displacement in the mid-height.

3.4 Moment-Curvature (M-φ) Analysis

Finite element software open system for earthquake engineering simulation (OpenSees), which has been created by the PEER center, can simulate the behavior of various structural systems [59, 60]. This software is an open code and free package which has some modules that simplify structural modeling and analysis process. For solving the governing equations of continuous and SDOF models, M-φ diagram of the section must be prepared at each time step of the calculations. In this paper, for preparing $M-\phi$ diagram, OpenSees software is used to due to its simplicity and quick calculations. In current paper, a subprogram which is written for RC section moment-curvature analysis has been used [61]. In this subprogram, the classical theory of Euler-Bernoulli is used to calculate $M-\phi$ for RC sections. For concrete materials model Concrete01 Material-Zero Tensile Strength type is
selected [62] which is based on the model of Kent & Park (1971), neglecting the tensile strength of concrete. In this material model, stress in the confined core concrete is calculated by the following equation [63]:

$$\sigma_c = \begin{cases} K f_c \left[ \frac{2\varepsilon_c}{0.002K} - \frac{\varepsilon_c}{0.002} \right]^2 & ; \varepsilon_c \leq 0.002K \\ K f_c (1 - Z_m \varepsilon_c) & ; \varepsilon_c > 0.002K \end{cases}$$

where \( f_c \) is characteristic compressive strength of concrete (MPa) and \( \varepsilon_c \) is strain in concrete fiber. Parameters in above equation are \( K = 1 + \frac{f_{sh} f_c}{f_y} \) and \( Z_m = \frac{0.5}{3.2 \sqrt{f_{sh} f_c} \times 0.002} \) in which \( f_{sh} \) is stirrup yield stress and \( f_y \) is stirrup volume relative to the volume of core concrete, \( h \) and \( s \) is width and height of concrete core and stirrup intervals (mm), respectively. Maximum concrete stress is \( \sigma_{cm} = 0.002K \) which occurs at strain \( \varepsilon_{cm} = 0.002K \) and the ultimate stress is assumed to be \( \sigma_u = 0.2Kf_c \) that occurs at the ultimate strain \( \varepsilon_u = \frac{0.8}{Z_m} > 0.002K > 0.004 + 0.08f_y \). For concrete cover (non-confined) it is assumed \( f_{sh} = f_y = 0 \) in the above equations.

For steel rebars, Steel01 Material model is used which is linear elastic-perfect plastic behavior. At each time step of calculations of the analytical models, \( M-\phi \) curve is drawn and a bi-linear function is fitted to the curve that \( K \) and \( \bar{K} \) are modified in each step due to the effects of strain rates. Curvature of the section is \( \theta(x) = -\frac{h_s}{A_s} \), so \( M_\phi \) bending moment corresponding with \( \phi \) is determined using \( M-\phi \) diagram. Afterward, depth of neutral axis \( x \) of the section is calculated by writing rotational equilibrium for the section under \( M_\phi \) (Fig. 3).

Fig. 3 RC section: 1) distribution of stress in the section; 2) distribution of strain in the section.

In Fig. 3, strain distribution in the section height is supposed to be linear and tensile strength of concrete is ignored. \( \sigma_c \) and \( \varepsilon_c \) is stress and strain in compressive concrete, \( \sigma_s \), \( \varepsilon_s \) and \( \sigma_{sh} \), \( \varepsilon_{sh} \) are stress and strain in bars behind and front facing blast wave, respectively. Also, \( x \) is depth of neutral axis, \( b \) width, \( h \) height, \( d \) effective depth of section, \( A_s \) and \( A_{sh} \) are reinforcement areas in behind and front face of the section, respectively. By writing equilibrium equation of moments around \( A_s \) and substitute \( \sigma_c \) from Eq.10, considering linear strain at the section height Eq.15 is obtained:

$$M_\phi = \begin{cases} b.K.f_c \int \left[ \frac{\varepsilon_s}{0.002K} - \frac{\varepsilon_c}{0.002K} \right] (d - y) dy & ; \varepsilon_c \leq 0.002K \\ b.K.f_c \int (1 - Z_m \varepsilon_c)(d - y) dy & ; \varepsilon_c > 0.002K \end{cases}$$

Yielding of steel \( A_{sh,s} \) should be controlled i.e. if
\[ \{E_x|q|\varphi(x\dot{\cdot})\> f_{35} \]
then stress in \( A_{35} \) is replaced by \( f_{35} \) and
the equation would be solved again. After finding \( x \),
strain in concrete and rebars can be calculated using
\( e_x = \varphi(x\dot{\cdot}) \) and \( e_{35} = \varphi(x\dot{\cdot}) \).

By comparing the values of strains with previous step
of calculations, strain rate in concrete and steel is
determined and DIF factors are calculated using
\( m_\phi \) subprogram to determine \( \mathbf{R} \) and \( \mathbf{K} \) for the next step
\((j=j+1)\). Specified criterion for ending this cycle is
declared as reaching the ultimate strain in compressive
concrete.

4.2. SDOF Model

SDOF equation of motion in the elastic and plastic
range, for a tow end hinged beam- column, is expressed
as Eq.16 which is FD form of Eq.4 :
\[
\begin{align*}
0.75M_0 \left( 1 - 2\frac{\rho^2 + \rho'_{0}}{\rho^2 + \rho'_{0}} \right) + K_{\rho_{0}} u_{\rho_{0}} &= \varsigma_{\rho_{0}}^{-1} \dot{u}_{\rho_{0}} \quad ; 0 \leq \rho_{0} \leq \rho_{0} \\
0.66M_0 \left( 1 - 2\frac{\rho^2 + \rho'_{0}}{\rho^2 + \rho'_{0}} \right) + K_{\rho_{0}} u_{\rho_{0}} &= \varsigma_{\rho_{0}}^{-1} \dot{u}_{\rho_{0}} \quad ; \ \rho_{0} \leq u_{\rho_{0}} \leq u_{\rho_{0}}
\end{align*}
\]
Where \( l \) is length of the member and other parameters
are defined before. All the steps for solving above
equation and calculating displacement response \( u(t) \) is the
same with continuous model. However, Eq.16 is simpler
and shorter to solve rather than Eq.14, because it doesn't
have spatial derivatives and order of the equation is 2time
lesser.

5. Results and Discussion

In this section, the process of analytical solutions
discussed before are used to estimate dynamic response
of considered RC column under different blast pressure
time-histories. Results obtained by continuous and SDOF
models are compared to implicit finite element analysis
by OpenSees Version 2.3.1 [59] and explicit finite
element analysis by ABAQUS Version 6.14-2 [73]. The
considered column has square section with symmetric
reinforcement and other specifications of the model and
loading conditions are shown in Fig. 4 and table 1. RC
column is designed as a conventional structural column in
2-3 story building in accordance with requirements of
ACI 318-14 [74].

\[ N_{\text{max}} = \frac{1}{2} f \left( A_2 - A_2 \right) \]

Where, \( A_2 \) is the area of longitudinal bars and \( A_2 \) is gross area of the section. In practical
applications, usually the ratio of axial load in columns to
its axial capacity is about 0.1-0.4 [1, 4]. For evaluating
column's response in quasi-static, impulsive and dynamic
regimes the amount of time duration of explosion is
changed (table 2).

<table>
<thead>
<tr>
<th>axial load ratio</th>
<th>natural vibration period (sec)</th>
<th>duration of explosion ( t_d ) (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0331</td>
<td>0.000331</td>
</tr>
<tr>
<td>0.21N_{\text{max}}</td>
<td>0.0263</td>
<td>0.000263</td>
</tr>
<tr>
<td>0.4N_{\text{max}}</td>
<td>0.0248</td>
<td>0.000248</td>
</tr>
</tbody>
</table>

In 3-D ABAQUS modeling, concrete is meshed with
17 mm C3D8R-SOLID elements, longitudinal bars with
20 mm B31-BEAM element and stirrups with 20 mm
truss elements. This type of elements is suitable for
stirrups since confinement effect of stirrups is considered
only once with definition of confined concrete.
specification. Material behavior for concrete is introduced with point data and using recommended Eq.10. Some of modeling assumptions are Poisson's ratio for Steel 0.3, steel material model is elastic-perfectly plastic, Poisson ratio for concrete 0.2 and concrete Damage Plasticity is used for crack modeling in concrete. In ABAQUS/Explicit dependency of concrete properties to high strain rates is considered by calculating the equivalent plastic strain rate [73] and strain rate dependency of steel material is also enabled for its material model. The analysis has been done in two phases; In the first phase, a quasi-static linear analysis is done in which initial axial load is applied on the rolling end of the member trough 1 sec time duration. In the second phase, non-linear dynamic analysis has been done which trough it the axial force is constant. Therefore, in this phase the results of first phase are as predefined field introduced to the model. RC member is assumed to be horizontal with free air blast loading above it. Blat load is applied with triangular time history to upper face of the member. Supports condition is ideal hinge in one end and simple roller at the other end. In Fig. 5 an image of concrete and reinforcement meshing of the column in ABAQUS is shown.

![Meshing of concrete and bars in ABAQUS.](image)

The amount of pressure at the first moment is \( P_0 = 1000 \) kPa and uniformly distributed. Since in this paper only the first maximum response of the structure under blast loading is considered, for saving in time and volume of output data, the analysis continues until cover the first maximum response.

In Fig. 6 transverse displacement histories in the middle of the length, for different blast loading regimes and axial load ratios, are shown. In this figure, FE curves refer to the results from explicit FE analysis by ABAQUS and CM curves show the results from continuous model (CM). Final points in the CM curves, show obtaining the failure criterion in analytical model.

![Fig. 6 Results of FE and continuous model for transverse displacement in the middle of length.](image)

According to Fig. 6, in impulsive loading regime the amount of displacement in the analytical model has an appropriate matching with FE results. In dynamic regime, estimated displacement in analytical model is appropriate but reaching to the failure criterion of calculation is a hinder to precise estimation of maximum response value. In quasi-static regime, due to the large deformations, analytical model has not given acceptable results. In the high axial load levels, results discrepancies have increased. Also it can be concluded that increasing the axial load ratio in the member has caused to reduce of the displacement response which implies increasing blast resistance capacity of the column.

For another validation of analytical process introduced here, considered RC column with axial load ratio \( 0.21 N_{\text{max}} \) and explosion of 40 kg TNT at 4 m standoff distance is analyzed. Reflected over-pressure on the face subjected to blast wave is assumed uniformly
distributed. Presented SDOF analytical process is used to study dynamic behavior of RC column under the blast loading. The results are compared to a 2-D finite element analysis of dynamic response of the member using OpenSees software. Material Models are the same with described models in M-α subprogram. 10 cm dispBeamColumn elements (with 10 points of integration) are selected in length of the member and for longitudinal reinforcement, Straight type element is used. To define column section, fiber model has been used that divides the section into 30 mm concrete fibers and steel bars. In the OpenSees commands and algorithms no prediction or recommendation exist on how consider the effects of strain rate on material properties. Therefore, in this study DIF coefficients of the last step of the calculation in SDOF model are used. Analysis of the considered column with OpenSees, consists of three distinct phases; First, the column is taken under a static analysis under axial load. During this phase, displacement of the nodes in the roller end of the member along the longitudinal axis of the column is recorded (phase I). Then, dynamic model with applying DIF coefficients to the material models is done. At the beginning of this stage, during another static analysis, determined displacements from the first phase is applied to the end nodes (phase II) and then dynamic analysis under lateral blast loading is implemented (phase III). Thus, for dynamic analysis under blast loading, column model can be updated for considering the effects of strain rate.

In Fig. 7, the amount of calculated first maximum lateral displacement in SDOF model is compared with the results of FE analysis by OpenSees. Note that the results of FE analysis are recorded from applying time of the blast load to the structure. It can be seen that displacement increasing with time obtained by SDOF method is very close to the results of FE analysis but reaching failure criterion of the section has caused lower maximum displacement in compare to the amount calculated by OpenSees analysis.

In this step, the effects of the blast pressure time-history and axial load ratio on the P-I diagram of the RC columns is studied. Considered failure criterion is maximum displacement in the middle of the length $L_{max}$ to be the same with deformation corresponding 2 degrees of rotation at the column support which usually causes compressive concrete to crush [30]. For preparing P-I diagram, in the considered level of axial load and the shape of blast load time history (include rectangular, triangular and exponential forms), the sufficient number of corresponding points is calculated which resulting maximum displacement is equal to considered failure criterion. For considered column defined before, SDOF model is used to prepare P-I diagrams. P-I curves obtained in each case, for zero axial load case, is shown in Fig. 8. In this figure can be seen that changes in the form of blast loading, have more influence through the dynamic range of response.

In the next step, the effect of the axial load on the P-I diagram of the concrete columns have been studied. P-I curves obtained under triangular blast loading, is shown in Fig. 9. It can be seen that the amount of axial load has impact on the position and shape of the diagram P-I. Such that, with increasing axial load ratio, P-I diagrams are shifted closer to the center of the coordinate which implies the reducing blast pressure and impulse required to achieve target damage level. This is likely because of the fact that increasing in the axial load level, leads to increase flexural capacity of the column, but decrease the maximum rotation of the support (at the failure state), which subsequently the absorbed strain energy is also reduced. Therefore, maximum resisting blast load by

![Fig. 7 Time history of maximum transverse displacement in RC column obtained by SDOF method.](image)

![Fig. 8 P-I diagrams of beam-column (with zero axial load) under lateral uniform blast load with different time-history shapes.](image)

![Fig. 9 P-I curve of concrete columns.](image)
column is reduced compared with the case of without axial load.

![Graph showing P-I diagrams for column with different ratio of axial load under uniform blast load with triangular shape.]

It should be noted that increasing in axial load ratio, cause to increase the bending and shear capacity and resulted lateral displacement reduces. On the other hand, according to Fig. 9 it can be concluded that increasing axial load causes reduction of column blast resistance. So, considering the appropriate level of axial load in RC column is essential to evaluate of existing columns or design of the column under blast loading.

6. Summary

In this article transverse deformation of reinforced concrete columns under constant axial load and lateral blast loading, is calculated with tow analytical methods. In the introduced analytical models, material non-linearity, strain rate effects on considered material properties and also P-δ effects are included. One of the introduced models is based on continuous Euler-Bernoulli beam theory and the other one is based on equivalent single degree of freedom method. Obtained results from the analytical models for estimating the first maximum response of the column has compared to results obtained by nonlinear finite element analysis (ABAQUS and OpenSees software). Also, Using introduced SDOF model, the effects of blast loading form and axial load ratio on the RC column's P-I curve is investigated.

According to obtained results, in the moderate and low axial load levels, the continuous model estimates the maximum amount of deformation of the column with acceptable accuracy, in impulsive and dynamic loading regimes. When the axial load level is high or in the quasi-static regime, the accuracy of the results is lower. Also, presented SDOF method despite the simplicity and low calculation time has sufficient accuracy and reliable results. It has been shown that axial load level and the form of the blast pulse are effective on the P-I diagram. According to the results, by increasing axial load P-I diagram approaches to center of the coordination which indicates reduction in the necessary pressure and impulse values for reaching target failure level. This implies the importance of considering axial load in assessment and designing of the RC columns under blast loading. In SDOF approach, by using equivalent coefficients or equivalent length of plastic hinge, some approximations take into account. Also, useful information such as displacement, curvature and rotation profile can't be achieved through the column length. On the other hand, uncertainties in the nature of the blast loads cause the accuracy of the idealized systems can't be guaranteed. However, in SDOF method a good understanding of the parameters affecting the structural behavior and its dynamic response can be achieved.

References


[74] ACI Committee 318. Building code requirements for structural concrete (ACI 318-14) and commentary. American Concrete Institute, (2014), 519 pp.