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Mixed Finite Element for Crack Analysis in Functionally Graded Material

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Abstract: This study presents a mixed finite element formulated using isoparametric techniques and extended to analyze quasi-static crack propagation in functionally graded materials. The mixed mode is analyzed for homogeneous and functionally graded beams subjected to three-point bending and for homogeneous and functionally graded plates under uniform axial tension. The mixed finite element results are compared with previous experimental and numerical results published in the literature.

Keywords: Crack, kinking, mixed finite element, functionally graded material

1. Introduction

Functionally graded materials (FGMs) are advanced composite materials with smoothly varying compositions and properties across their volume. In Japan, researchers proposed FGMs to eliminate sharp interfaces between two materials by developing a new composition graded in composition highly resistant to thermal barrier applications. The concept was to gradually change the material instead of using a traditional composite material, which would reduce stress concentration at the interface. As a result, the functionally graded material could handle extreme applied conditions without failing, as shown in Figure 1. Since then, FGMs have been utilized for various structural engineering applications in addition to thermal barriers.

Functionally graded materials (FGMs) possess significant qualities that make them popular in the manufacturing industry. As a result, they are widely utilized in various fields, and their potential applications are vast, indicating a promising future. Some industries where FGMs are currently applied are automobile, airspace, defence, and energy, which utilize compositional gradient FGM types. Additionally, defence and armoury make use of microstructural FGM types. Porosity gradient FGMs, on the other hand, are utilized in dental, autogenic, and filter industries, as depicted in Figure 2.

The development of functionally graded composite materials originated from the incapability of classical composite materials to withstand high thermal stress. Classical composite materials failed due to a clear and defined interface between the laminate composite materials, which caused an increase in stress concentration at the interface. The stress concentration resulted in the initiation of cracks and subsequent crack propagation.

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The researchers used the finite element method (FEM) to analyse cracks with arbitrary variables and structure reliably calculation (Zhao et al., 1997). FEM modelling based on a virtual crack extension to calculate directly stress intensity factors (SIFs) and energy release rate from existing cracks. The model was developed by determining the stiffness using energy across each element (Tilbrook et al., 2005). The dummy thermal loads technique was used to model Fgm by the finite element code ANSYS APDL(Hassan & Keleş, n.d.).k_2 min criterion was used to predicate crack growth direction, the finite element formulation based on the technique of isoperimetric transformation, where the continuity of material properties modelled as the continuity of displacement function to simulating the graded material, the numerical results confirmed the mathematical solution and proposed for a further study like 3D problem FEM computation (Li et al., 2000). A finite element formulation uses interpolation function and exponential material properties gradient. This element can solve the unformed displacement loading problem perpendicular to the gradient and gives critical importance to crack path analysis (Santare & Lambros, 2000). The FEM analysis of graded cellar structure proved that increasing the density flow increases the effective elastic modulus (Ajdari et al., 2009), there is a better performance of high-order elements than the conventional element in the same case when the elements have the same shape function (Kim & Paulino, 2002b). The material force proposed to evaluate J-integral in Fgm (Mahnken, 2007). a model capable of estimating the kinking angle crack in Fgm was studied (Abanto-Bueno & Lambros, 2006).

A computational simulation to solve linear quasi-static thermos elastic problem in plane strain state in Fgm with the graded finite element based on virtual crack closer technique (Burlayenko et al., 2016), this graded finite element used iso parametric quadrilateral for homogenous material in 2D plan ax symmetric problem, the weak path test for non-homogeneous material modelled and compared to the exact solution (Paulino & Kim, 2007).

The finite element method used the displacement correlation technique to analyse cracks in orthotropic Fgm coating under thermal loads to evaluate SIFs. The influence of the parameter geometry, boundary condition and non-homogeneous parameter on SIFs were discussed (Yıldırım et al., 2008). The finite element was modelled to evaluate SIFs of the curved crack in anisotropic Fgm by introducing the clamped displacement discontinuous technique (Chang Jui-Hung & Liao Geo-Jih, 2014). Stress intensity parameters for functionally graded materials calculated using weight functions obtained from the virtual crack extension approach (Shi et al., 2014), evaluated quasi-static crack initiation in planar FGMs. A new finite element formulation with singularity proposed to analyse the crack problem in isotropic Fgm, Westergard stress function was employed to determine the particular element (Molavi Nojumi & Wang, 2017). A 2D virtual crack closer technique interface element proposed In graded finite element, subroutine ONAT and UEL were used through the finite element software Abaqus to analyse the dynamic crack problem by VCCT, the results accurate with the experiment result in the literature (Zhou et al., 2016). A basic formula for a quarter-point crack tip element was established to assess the elastic T-stress in finite element analysis of functionally graded materials (Sladek et al., 2016).

A recent study by (Derouiche et al., 2021) applied the virtual crack closure-integral technique, in combination with the derivative stiffness procedure, to analyse crack extension in anisotropic materials using a mixed finite element approach. Another study by (Sami et al., 2021) focused on crack propagation and the computation of energy release rates in an orthotropic medium, utilizing the virtual crack extension technique.

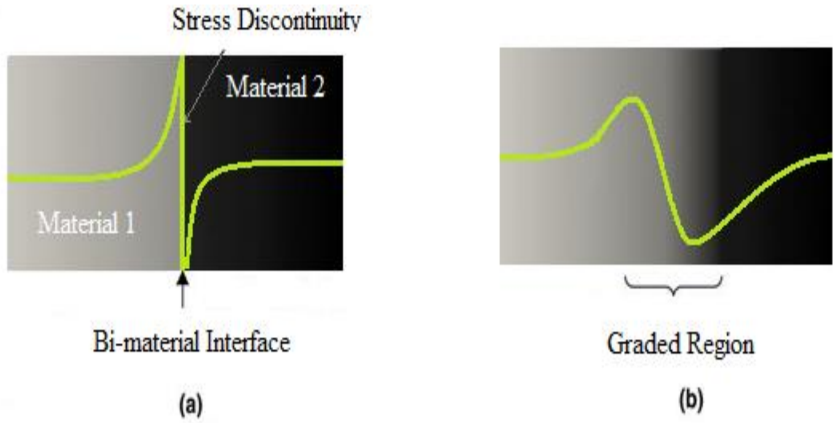

Fig. 1 - The high stress in the sharp interface in bi-material; (b) graded region to eliminate the interface (FGM's)

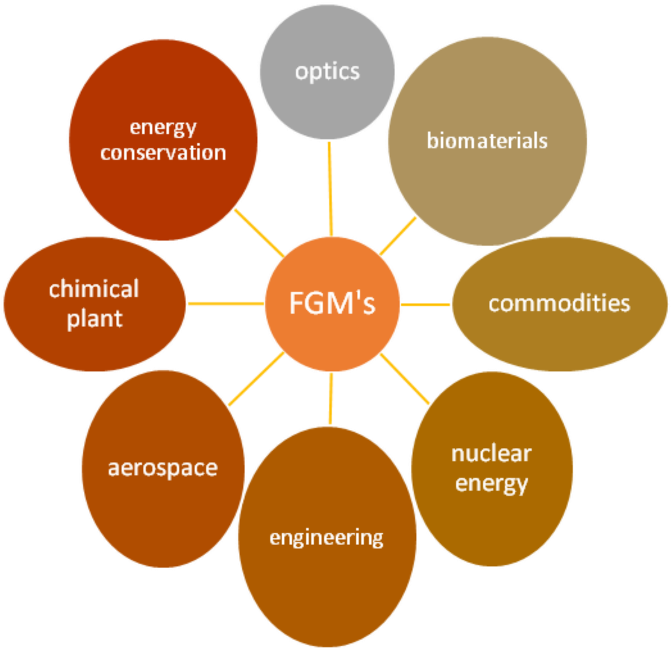

Fig. 2 - Field of FGM’s application

2. Formulation of RMQ-7 Element for Functionally Graded Materials

A mixed finite element is presented to analyse kinking cracks in functionally graded beams and plates. Modifications were introduced to the formulation of this mixed finite element to incorporate the gradual properties of functionally graded materials. The RMQ-7 (Reissner Modified Quadrilateral), which has 7 nodes with 2 degrees of freedom per node, was developed by (Bouzerd, 1992), (Bouziane et al., 2009) configured the RMQ-7 in the natural plane (ξ, η), as depicted in Figure 3.

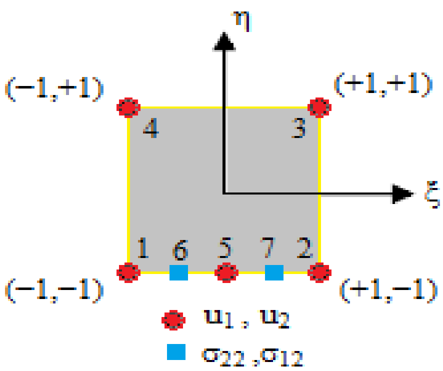

Fig. 3 - RMQ-7 mixed finite element

The displacements vector {u} is approximated by the interpolation functions matrix [N], as shown in equation (1).

$$\{u\} = [N]\{q\} \tag{1}$$

Where {q} is the nodal vector of displacements.
Ni interpolation functions approximate the displacements (Bouziane et al., 2009).

$$N_1 = -\frac{1}{4}(1-\xi)(1-\eta)\xi,\ N_2 = \frac{1}{4}(1+\xi)(1-\eta)\xi,\ N_3 = \frac{1}{4}(1+\xi)(1+\eta),$$
$$N_4 = \frac{1}{4}(1-\xi)(1+\eta), \quad N_5 = \frac{1}{2}(1-\xi^2)(1-\eta) \tag{2}$$

The stresses vector {σ} s approximated by the interpolation functions for stresses matrix [M], as shown in equation (3).

$$\{\sigma\} = [M]\{\tau\} \tag{3}$$

Where $\{\tau\}$ is the nodal stresses vector.

The interpolation functions Mi2 are utilized to evaluate σ12 and σ22. (Bouziane et al., 2009) For nodes 6 and 7 are given by:

$$M_{i2}^{6} = \frac{1}{6}(1-2\xi)(1-2\eta),$$
$$M_{i2}^{7} = \frac{1}{6}(1+2\xi)(1-2\eta) \quad , \quad i = 1,2 \tag{4}$$

The expression used to approximate the nodal displacement and stress was:

$$\begin{Bmatrix} \{\sigma\} \\ \{\varepsilon\} \end{Bmatrix} = \begin{bmatrix} [M] & [0] \\ [0] & [B] \end{bmatrix} \begin{Bmatrix} \{\tau\} \\ \{q\} \end{Bmatrix} \tag{5}$$

The transformation matrix gives the element matrix is denoted as $[B]$. The element matrix $[K_e]$ is defined as follows:

$$[K_e] = \begin{bmatrix} [K_{\sigma\sigma}] & [K_{\sigma u}] \\ [K_{\sigma u}] & [0] \end{bmatrix} \tag{6}$$

The sub-matrix $[K_{\sigma\sigma}]$ and $[K_{\sigma u}]$ are estimated as:

$$[K_{\sigma\sigma}] = -t\int_{A^e} [M]^T [S(z)][M] dA^e \tag{7}$$

$$[k_{\sigma u}] = t\int_{A^e} [M]^T [B] \partial dA^e \tag{8}$$

The compliance matrix is represented by [S(z)], Ae represents the element area , and t denotes the thickness represented by t. The matrix transpose is represented by T.

3. Crack Simulation in Fgm

Fracture analysis and crack propagation are conducted using a fixed finite element implanted in finite element code. The specific elements used consist of a two-dimensional mixed finite element with five displacement nodes and 2 stress nodes using the extension technique (Bouzerd et al., 2011) for energy release rate computation in homogenous materials (see Figure.4).

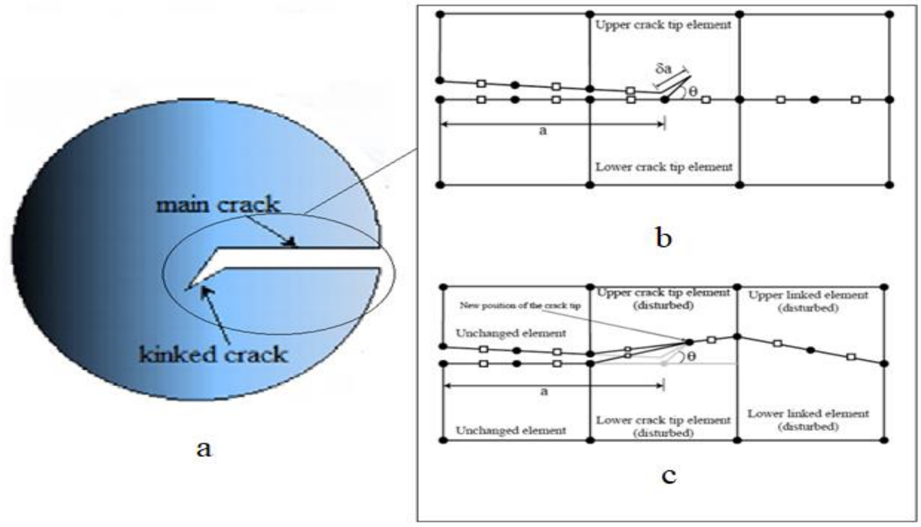

Fig. 4 - (a) Crack Tip in Functionally graded materials; (b) Crack geometry after kinking; (c) Reorganization of the mesh around the crack tip

Parks (Parks, 1974) proposed the virtual crack extension method to calculate the energy release rate(G) as:

$$G = -\frac{dU}{da} = -\frac{1}{2}\sum_{i=1}^{ne}\{u\}_i^t\left\{\frac{\Delta ki}{\Delta a}\right\}\{u\}i \tag{10}$$

The system's potential energy (U) is determined by various factors, such as the crack length (a), virtual crack extension length (Δa), and differences in stiffness matrices (ΔKi) and nodal displacement vectors (u) of the elements i at the virtual extension of the crack tip.

This study uses a series of layers consisting of homogeneous materials to model the functionally graded material. Each layer comprises multiple homogeneous elements with constant material properties, and their characteristics correspond to the centre of each element. This technique was used recently by (Benmalek et al., 2021) in the analysis Fgm beam bending (see Figure.5) and coupled to the virtual crack extension technique (Bouzerd et al., 2011) to compute the energy release rate of crack extension in functionally graded material.

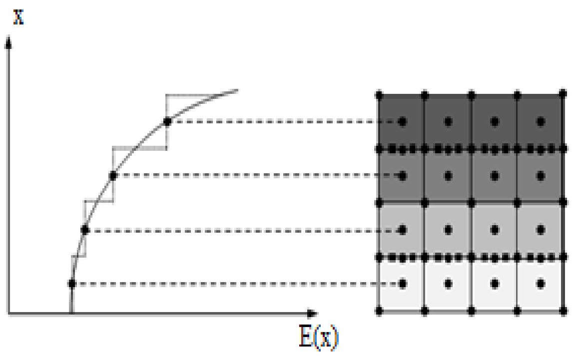

Fig. 5 - Numerical representation of an Fgm

4. Cracked Beam Subjected to Three-Point Bending

4.1 Example 1

(Marur & Tippur, 1998) utilized a gravity-assisted casting technique to produce Fgm specimens using two-part slow-curing epoxy and uncoated solid glass sphere fillers. The specimen geometry and boundary conditions are illustrated in Figure 6. The Young's modulus and Poisson's ratio in the material gradient region linearly vary from the epoxy side to the glass-rich side. . (Marur & Tippur, 2000) calculated the stress intensity factors from the experimental strain data, while (Kim & Paulino, 2002a) employed various methods such as the displacement correlation technique

(DCT), the modified crack closure (MCC) method, and the J-integral. The results are compared with the present results in Table 1.

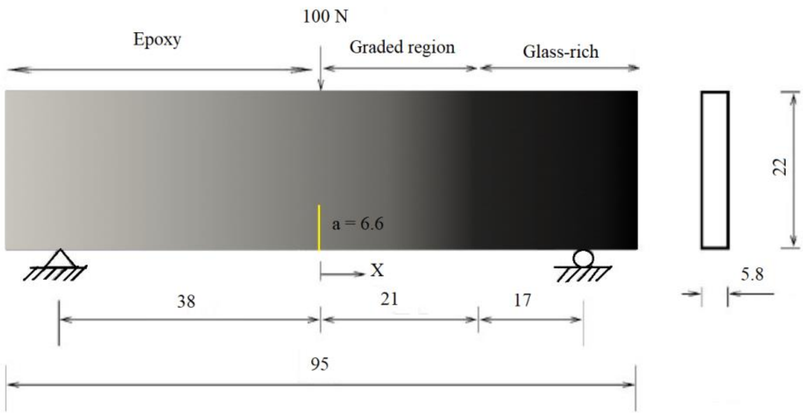

Fig. 6 - Three-point bending test geometry and boundary conditions specimens

Table 1 - Comparison of kink angle obtained by the present MFE, the FEM simulation and analytical results

	(Kim & Paulino, 2002a)			(Marur & Tippur, 2000)	Present MFE
Parameter	MMC	J integral	DCT	FEM	G
Angle	2.87	2.67	2.64	3.24	2.87

4.2 Example 2

Fracture analysis was conducted on homogenous and graded beams under three-point bending in mode 1 and mixed mode, as depicted in Figures 7 (a) and 7 (b). The homogeneous beam had material properties of E=2890MPa, ν=0.4, and K_{Ic} =1.09MPa√m. Linear material properties for the graded beam are given in Table 2.

Table - The material proprieties of the graded beam

X_2	E(MPa)	ν	K_{ic}(MPa√m)
0	1780	0,41	0,99
60	4000	0,39	1,19

(Kim & Paulino, 2004) utilized 992 elements with 12 different elements around the crack tip and the final mesh with 2875 nodes. (Khazal & Saleh, 2019) used extended element-free Galerkin method with 1856 non uniformly distribution nodes. This study treated the problem with 702 mixed finite elements and 1849 nodes.

The numerical results compared with numerical and experimental results for both homogenous beam and graded beam in the two cases of loading and dressed in the tables below:

Table 4 - Crack direction angle in homogenous and Fgm (case a)

	Case a	
	(Kim & Paulino, 2002a), (Khazal & Saleh, 2019)	Present MFE
PMMA	0	-0.02
FGM	0	-0.01

Table 5 - Crack direction angle in homogenous and Fgm (case b)

		Case b		
		Numerical Δa=1.5 (Galvez et al., 1996)	***Experimental*** Δa=1.5 (Galvez et al., 1996)	Present MFE Δa=1.5
1 step	PMMA	18.44	26.57	25.84
	FGM	24.44	26.57	24.37

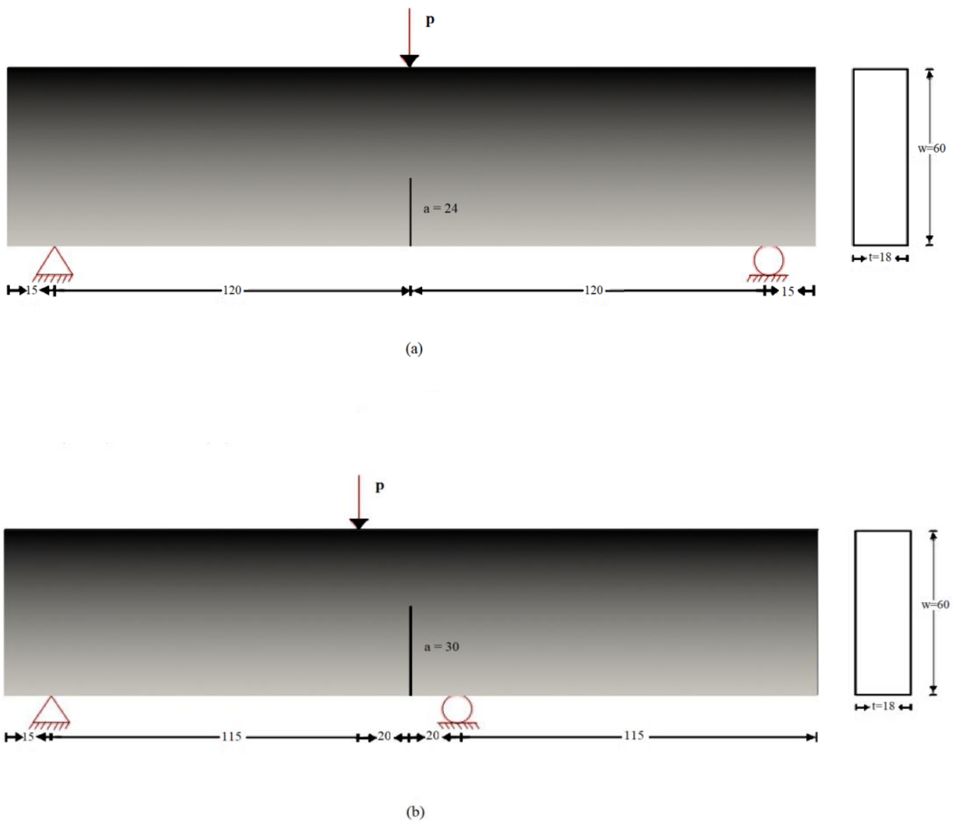

Fig. 7 - Geometry and boundary conditions of tree-point bending test (a) mode 1 and; (b) mixed mode

Table 6 presents the influence of crack length extension for homogenous, and Fgm crack direction in the Figure. 7 case a, in this case, the present element computes the new kink angle, where coefficient of crack extension (cf) variation d does not affect the results values in both homogenous and Fgm.

Table 6 - Crack direction angle computed by the present MFE for different cf

Parameter	**cf=10**	**cf=100**	**cf=1000**
PMMA(HOM)	-0.02	0.05	0.07
FGM	-0.01	0.05	0.07

Table 7 - Crack direction angle computed by the present MFE for different Δa.

Parameter	**Δa=1.5**	**Δa=0.1**	**Δa=0.01**	**Δa=0.001**
PMMA	-0.02	0.06	0.07	0.07
FGM	-0.02	0.06	0.07	0.07

Table 7 presents the influence of crack extension in homogenous and Fgm crack direction in Figure. 7 case b, the present element computes the kink angle for different crack propagation (Δa), where the results are stable for different crack extensions in both homogenous and Fgm.

5. Kinking Crack

Crack propagation in homogenous and Fgm plates under uniaxial tension is studied in this paper. (Hirshikesh et al., 2019) utilized 191735 linear triangular elements. While in this study, the problem was treated with 2380 mixed elements and 6145 nodes.

5.1 Kinking Crack in Homogenous:

A sheet of irradiated ECO (polyethene carbon monoxide) is studied as a homogeneous material. With the elastic properties E= 280MPa, ν = 0.45.

An inclined edge cracked specimen, dimensions and geometry are shown in table 8, and boundary conditions are shown in Figure 8. The Two specimens with π/3 and π/6 crack inclination angles were studied.

Table 8 - Edge cracked specimen, dimensions and geometry

	H	W	h	a	Φ(rad)
Homogenous 1	90	70	45	33	π/3
Homogenous 2	90	70	45	40	π/6

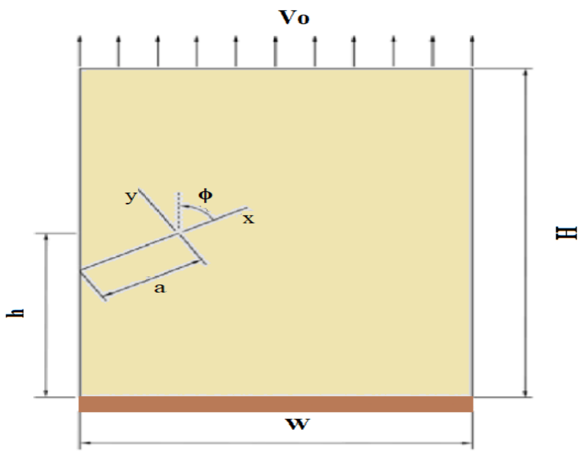

Fig. 8 - Geometry and boundary conditions of a homogeneous plate under uniaxial tension

Table 9 - Crack extension with angle π/6 in homogenous

HOMOGENEOUS π/6										
(Abanto-Bueno & Lambros, 2006)	(Becker Jr et al., 2001)	(Abanto-Bueno & Lambros, 2006)	NUM (Oral et al., 2008) Rc/a			Present MFE cf			Present MFE Δa	
			0.01	0.05	0.1	1000	100	0.1	0.01	0.001
-52±1.5	-49	-47.9	-52.5	-54.3	-54.5	-53.13	-50.24	-52.83	-53.31	-53.37

Table 10 Crack extension with angle π/3 in homogenous

HOMOGENEOUS π/3											
(Abanto-Bueno & Lambros, 2006)	(Becker Jr et al., 2001)	(Abanto-Bueno & Lambros, 2006)	(Hirshikesh et al., 2019)	(Oral et al., 2008) Rc/a			Present MFE cf			Present MFE Δa	
				0.01	0.05	0.1	1000	100	0.1	0.01	0.001
-28±1.5	-27	-26.9	-30.4	-32.4	-31.3	-30.8	-27.53	-24.55	-26.87	-27.75	-27.85

5.2 Kinking Crack in Fgm:

A sheet of Fgm (graded eco) specimen fabricated by (Abanto-Bueno & Lambros, 2006). The geometry and boundary condition is shown in Figure 9, and the material properties are given as follows:

$$E(x) = -9.2x10^{-7} X^4 + 4.1x10^{-4} X^3 - 5.0X^2 - 0.2X + 437.2 \qquad (11)$$

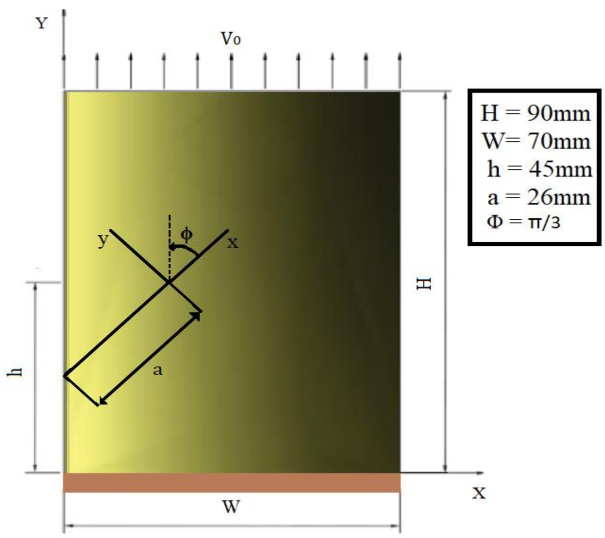

Fig. 9 - Geometry and boundary conditions of Fgm plate under uniaxial tension

Table 11 - Crack extension with angle π/3 in Fgm

FGM π/3												
(Abanto-Bueno & Lambros, 2006)	(Becker Jr et al., 2001)	(Abanto-Bueno & Lambros, 2006)	(Hirshikesh et al., 2019)	(Oral et al., 2008) Rc/a			Present MFE cf			Present MFE Δa		
				0.01	0.05	0.1	10	100	1000	0.1	0.01	0.001
-28±1.5	-24.00	-24.20	-29.90	-30.4	-30.1	-29.8	-28.70	-26.53	-30.00	-27.98	-28.85	-28.93

6. Conclusion

This study proposes a mixed finite element for analysing kinking cracks in functionally graded materials. The present element yields excellent numerical simulation results compared to experimental and numerical results with fewer computational efforts and elements. Assessing the energy release rate within the crack tip field of functionally graded materials is typically carried out using a formulation requiring fewer nodes. The graded material is modelled as a series of homogeneous layers, and the properties are located at the centre of each homogeneous element. The mixed finite element predicts the direction of kinking crack and provides accurate results for numerical examples and experiments in the literature.

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