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MATHEMATICAL MODEL TO PREDICT THE PERMEABILITY OF WATER TRANSPORT IN CONCRETE STRUCTURE

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Abstract

Coarse aggregate fine sand are component of concrete, the rate of macropores in concrete determine the rate porosity and void ratio in concrete structure, the influence of the permeability coefficient determine the rate of water transport in concrete. Mathematical model to predict the rate of permeability on water transport were mathematically developed, the model is to monitor the rate of water transport in concrete structure. Permeability established a relationship under the influence of macropores on the constituent that made of concrete, application of concrete placement determine the rate of permeability deposition in concrete structure, permeability establishment are under the influence of macropores between the mixture through the cement paste, considering the variables in the system, mathematical model were established to monitor the rate of water passing through concrete and also determine the rate of permeability coefficient on concrete structure.

Keywords: *Concrete Structures, Permeability and Mathematical model*

1.0 Introduction

The durability of concrete structure depends on the frequent rate of migration through the dissolved constituent. Such migration is influenced by permeability. This condition in terms of concrete mixture is through a continuous network of the micro pores that exist in the matrix of the concrete mix. Other influences are through the porosity that exists in the interfacial of the gradation structure of the aggregate. This study characterized the rapid and accuracy of measuring concrete permeability in a mix, this include establishment of theoretical model that describes the influence of permeability on concrete structure. Experiments are performed using transient permeability apparatus to monitor the measure coarse aggregate fine sand and water are the micropores between this material as a component of concrete known as porosity and void ratio in the concrete structure, the influence of the permeability coefficient determine the rate of water transport in concrete of water migration in concrete mixture, the apparatus allowed a rapid and accurate measure of water migration in concrete mix.

Concrete is a type of porous material and can be physically and chemically damaged due to its exposure to various environments from the placing of concrete to its to the service life. In particular, some external hazardous elements, such as sulfate, chloride ion, and carbon dioxide, permeate in concrete over a long-term period as a solution or a gaseous state and cause physical damage due to chemical reactions. These reactions affect the corrosion of steel bars applied in concrete and that decreases the durability life and strength of such steel bars. Thus, it is very important to insert

Corrosion inhibitors into steel bars in the case of a deterioration element that exceeds the critical amount of corrosion in the location of steel bars [1]. However, it is very difficult to guarantee corrosion resistance at the location of steel bars using conventional technology that applies corrosion inhibitors only on the surface of concrete [2-3]. This study attempts to develop a method that penetrates corrosion inhibitors up to the location of steel bars and investigate the penetration depth of corrosion inhibitors by verifying moisture migration in concrete under applied pressure. In the penetration of water in concrete, the penetration depth according to the passage of time can be estimated using the Darcy's law, which is also applicable to the penetration of sand stratums under low pressure conditions. Meanwhile, it is necessary to analyze the penetrative diffusion flow accompanied by internal deformation under high pressure conditions [4]. Under the circumstances, this study applied the experiment on the penetration depth of water for reinforced concrete structures using the water pressure applied to the holes in concrete as a variable. Based on the results of this experiment, this study also calculated coefficients of water penetration and diffusion and estimated the penetration depth for the concrete according to the water pressuring time and pressure. In addition, this study attempts to provide the basic data for the development of a diffusion method with high pressure penetration of corrosion inhibitors for penetrating the inhibitors to the location of steel bars through investigating the water penetration mechanism in concrete using a FEM analysis that reflects the interaction between solid and fluid [5-7].

To produce high quality long lasting concrete structures, cements of a high and consistent quality must be employed. Worldwide, the cement industry spends countless hours assuring the quality of its products, mainly based on laboratory tests. In the USA, most physical testing of cements is performed according to ASTM standards /1/; in Germany, testing is generally governed by the European Norm /2/. Technologies that could reduce the number of physical tests needed for cement production (and optimization) would clearly be a welcome addition. One such potential technology is the use of virtual testing. In virtual testing, starting materials are characterized and their performance predicted via the use of computer models. This should result in savings in both resources (materials, labor, etc.) and time, as 28 days of concrete performance can be simulated in just a few hours of computer time. An additional benefit of virtual testing is the capability to perform a large number of "what-if" type computations to explore new material systems and optimize existing ones, e.g., what is the optimum sulfate content and form for a particular cement or how will the performance of a cement change if its Blaine fineness is increased by 10 m2/kg.

Long-lasting structure is very important if it is to survive the harsh environment that it is often exposed to [8]. In Nigeria, where there is no maintenance culture, it is all the more imperative. Nigerian hot marine coastal waters, constitute an aggressive environment that has been found to be deleterious to concrete [9], leading to premature deterioration that affects the strength and durability characteristics of concrete structures. One of the major forms of chemical attack on concrete is the chloride ingress. This ingress leads to corrosion of reinforcement, reduction in strength, unserviceable structures, and structures that are aesthetically poor. According to Stanish [8], corrosion products put surrounding concrete in tension thereby causing tension cracking and spalling of the cover of concrete. The attendant adverse structural influences are: loss of bond between the reinforcement and concrete, loss of steel area, and loss of stiffness. The total effects of these are serious durability problems because of reductions in the strength, serviceability and aesthetics of concrete structures.

Result in early repair or premature replacement of the structures. Libby [10] and Gallegos & Quesada [11] listed many chloride-induced structural failures that required expensive rehabilitation work. Thus to increase the service life of coastal concrete structures, increasing the resistance of concrete to chloride penetration is very necessary. A common approach to prevent such deterioration is to prevent chloride penetration into the structure by using relatively impenetrable concrete. But impenetrability of chloride into concrete depends on its porosity which in itself is defined in terms of pore size, pore distribution and interconnectivity of the pore systems. And in increasing this resistance, it is necessary to take into cognizance the prevalence of elevated temperature curing conditions, lest it works against the durability. As previously observed by Detwiler [12], the effects of hot weather and/or accumulated heat of hydration can be mitigated by various measures, but only to a certain extent. Earlier works by Wee [13], Smith [14] and Kumar [15] with cement paste containing silica fume, fly ash, and granulated blast furnace slag suggested that supplementary materials could improve the performance of concrete cured at elevated temperatures against chloride intrusion. For this work, the author chose slag because it is available as by products from the steel rolling plants in Osogbo, Aladja, and Katsina in Nigeria.

And for the basis of comparison, degree of hydration, rather than the curing time was employed. Perenchio [16] had earlier pointed out that conclusions based on constant temperature curing would not necessarily apply to concrete cured under field conditions. And previous work by Detwiler [17] gave the number of days to reach 70% degree of hydration for Portland cement paste, cement paste containing silica fume and granulated blast furnace slag (at 30% replacement level). The numbers of days to reach this degree of hydration were employed for this work.

Presently natural resources are increasingly consumed due to rapid urbanization and thereafter human construction activities, so that various strategies are being investigated by engineers to protect and restore natural ecosystems all over the world. Permeable (porous/ pervious) pavement is termed as comprising materials that facilitate storm water infiltrate and transfer to the underlying subsoil [18-19].

In Australia, permeable pavement has been utilized as a potential tool of Water Sensitive Urban Design (WSUD) to manage natural water. From 1994 the University of New South Wales (UNSW) started to research into permeable concrete paving and more recently the University of South Australia (UniSA) is also involved. However, the previous studies conducted both in UNSW and UniSA mainly concentrated on water quality and pollution control through permeable pavements and, only the properties of base course materials in permeable pavement system and segmental paving have been studied. There is still a gap of optimizing the surface materials for permeable pavements pervious concrete. At ambient temperature conditions, a dosage of 5 fl oz/cwt of the HCA provides between 60 and 90 minutes of extra working time. Hydration controlling admixtures can eliminate inconsistencies and performance variability that may be brought on by the need to re-temper mixtures at the job sites [20]. Along with the HCA, VMA or viscosity modifying admixtures may be beneficial to the performance of pervious concrete. The use of VMAs results in better flow, quicker discharge time from a truck, and easier placement and compaction. Furthermore, VMAs prevent drain down, and may increase both compressive and flexural strength of pervious concrete. It should be noted that not all VMAs are made with pervious concrete in mind, and therefore, care should be taken when choosing the right VMA for pervious installation [20]. In California, Young's [21] reported that latex modifiers allowed harder surface finishing using Bunyan screeds, which in return produced "table-top" surface, and almost eradicated surface raveling. Latex modifiers assist in binding the cement paste to the aggregate. Mixtures with latex modifiers might allow utilization of pervious concrete in high speed pavement applications [21].

2.0 Theoretical Background

This model consider the cylinder sample depicted which produced total volume of $V = AL$ where A is the cross section area of the specimen and L is known to be the length, verification of this sample is through two pressurized reservoirs, this produced the upstream at ϕu and the downstream one at ϕd. These initial values can be expressed as upstream and downstream pressures are $\phi 1$ and $\phi 0$ respectively. In this condition application of partial differential expression which should govern pressure variation as a function of length and period $\phi(Z, t)$ within the sample which is expressed as but in this study concrete materials where considered, lack mix design application that has lead poor compressive strength in concrete, this has also developed to so many structural damage, the focus of this study are to determine the rate of permeability and there relation with the concrete constituent.

For the express of the parameter in the system are

ϕ	=	porosity
k	=	permeability
μ	=	Macropore in pore fluid viscosity
K	=	Sample permeability
Z	=	Distance with the region i.e. measure from the upstream reservoir.
t	=	Time

β = Lumped compressibility

3.0 Governing Equation

The model designing water transport is presented by Roy et al 1993 the model was to monitor the behaviour of water that is passing through a concrete structure.

$$\frac{\partial^2 \phi}{\partial Z^2} + \beta\left(\frac{\partial \phi}{\partial Z}\right)^2 = \frac{\beta u}{k}\frac{\partial \phi}{\partial t} \qquad (1)$$

The mathematical expression of the parameter in the system where put into mathematical equation, the variables ϕ *denote* (Z, t) which is the function of Z distance and Time T, but for simplicity the equation where linearized so it will be easy to solve the problem under study. The concept of water forms the binding agent with cement paste fine sand coarse aggregate are mixed together to make up concrete structure, most time it reinforced either with mild steel or high tension steel depending on the imposed load, the concrete is placed on a form work, it is also vibrated for compaction, this compaction will only reduce the rate of void and decrease the rate permeability in the concrete structure, but it will not avoid permeability, the rate of permeability determine the rate of void ratio and porosity deposited in concrete structural component. Considering the equation by applying separation of variable; the variables in the system were represented, by the application of mathematical tools, a constant C_1 and C_2 where establish as the equation were derived where ϕ represent pressure, β and other parameters u K t where represented as λ^2 ,this condition express the entire variables.

$\frac{T^2}{T}\ \frac{\beta u}{K}$ Equal to λ^2. By splitting techniques this was applied to other variables. The parameters known as the variable in the system that produce the result of water transport in the concrete structure where all equal to λ^2 exception of pressure in the system denoted as ϕ. Permeability is the product of porosity, because of the pore fluid between the binding agents on the concrete structure, fine sand coarse aggregate reacting with the cement paste, are mixed together to produce concrete. The rate of permeability coefficient, are determine through structural characteristic from the concrete, where the permeability exist allows the fluid to pass through the concrete based on the rate of deposited micropores from permeability coefficient in the concrete structure. The model established a relationship within the variables in the system, where two constant were developed, this implies that the constant linearized the equation derived to solve water transport passing through the influence of permeability in concrete structure.
Substituting solution ϕ = Z, T into equation (1), we have

$$Z^{'}T + \beta\left(Z^{'}T\right)^2 = \frac{\beta u}{k} X'T \qquad (2)$$

$$\frac{T''}{T} + \beta\left(\frac{Z''T}{Z}\right)^2 = \frac{\beta u}{k}\ \frac{Z'T}{T} \qquad (3)$$

$$\frac{T''}{T} = \frac{\beta u}{k} + \beta - 2\frac{Z''}{Z} = \frac{\beta u}{k}\ \frac{Z''}{Z} - 2 \qquad (4)$$

$$\frac{T''}{T} = \beta - 2\frac{Z'}{Z} - \beta - \frac{\beta u}{k}\frac{Z'}{Z} - \frac{\beta u}{k} - \beta - 2 \qquad (5)$$

$$\frac{T''}{T} = \beta - 2 - \beta - \frac{\beta u}{k} - \frac{\beta u}{k} - \beta - 2 \qquad (6)$$

Considering when $Ln\ \Delta Z \Rightarrow 0$

$$\frac{T''}{T} = \beta - \frac{\beta u}{k} - \frac{\beta u}{k} - \beta = \lambda^2 \quad \ldots\ldots\ldots\ldots \quad (7)$$

$$\frac{T''}{T} = \lambda^2 \quad \ldots\ldots\ldots\ldots \quad (8)$$

$$\frac{\beta u}{k} = \lambda^2 \quad \ldots\ldots\ldots\ldots \quad (9)$$

$$\beta = \lambda^2 \quad \ldots\ldots\ldots\ldots \quad (10)$$

This implies that the equation can be expressed as:

$$\frac{\beta u}{k}\frac{Z'}{Z} = \lambda^2 \quad \ldots\ldots\ldots\ldots \quad (11)$$

$$\frac{\partial^2 y}{\partial Z^2} = \lambda^2 \quad \ldots\ldots\ldots\ldots \quad (12)$$

$$\partial y = \frac{\lambda^2}{\beta} Z \partial y \quad \ldots\ldots\ldots\ldots \quad (13)$$

$$\int \partial y = \int \frac{\lambda^2}{\beta} Z \, \partial y + C_1 \quad \ldots\ldots\ldots\ldots \quad (14)$$

$$y = \frac{\lambda^2}{\beta} Z^2 \, \partial y + C_1 + C_2 \quad \ldots\ldots\ldots\ldots \quad (15)$$

$$y = 0 \quad \ldots\ldots\ldots\ldots \quad (16)$$

$$\boxed{y = \frac{\lambda^2}{\beta} + C_1 Z + C_2} \quad \ldots\ldots\ldots\ldots \quad (17)$$

$$\Rightarrow \frac{\lambda^2}{\beta} + Z^2 + C_1 Z + C_2 = 0 \ldots\ldots\ldots \quad (18)$$

Applying quadratic expression, we have

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \ldots\ldots\ldots\ldots \quad (19)$$

$$X = \frac{-(C_1) \pm \sqrt{(C_1) - 4C_1\left(\frac{\lambda^2}{\beta}\right)C_2}}{2C_1} \quad \ldots\ldots\ldots\ldots \quad (20)$$

$$= \frac{C_1 \sqrt{C_1^{\,2} - 4C_2 \frac{\lambda^2}{\beta}}}{2C_1} \quad \ldots\ldots\ldots\ldots \quad (21)$$

$$X = \frac{-C_1 + \sqrt{C_1^{\,2} - 4C_2 \frac{\lambda^2}{\beta}}}{2C_1} \quad \ldots\ldots\ldots\ldots \quad (22)$$

$$\frac{X - C_1 + \sqrt{C_1^{\,2} - \dfrac{4C_2\lambda^2}{\beta}}}{2C_1} \qquad \text{……………………………………..} \qquad (23)$$

Subject equation (23) to the following boundary conditions and initial values conditions

$$t = 0, \quad \phi = Po \qquad \text{……………………………………………..} \qquad (24)$$

Therefore, $Z(z) = C_1\ell^{-M_1Z} + C_2\ell^{-M_2Z}$ …………………………… (25)

$$= \quad C_1\, Cos\, M_1 Z + C_2\, Sin\, M_2 Z \qquad \text{……………………………} \qquad (26)$$

Solving equation (17) gives

$$y = \frac{\lambda^2}{\beta} + C_1 Z + C_2$$

Deriving the relationship between the variables where constant were integrated to linearized the equation, application of quadratic expression were introduced, this concept were applied to express the integrated constant, denoted as C_1 and C_2, the relationship of the parameter denotes as λ^2 were established considering the boundary values of t = 0, $\phi = \phi 0$ the boundary were integrated to produce the model. This will be applied to monitor the rate of water transport in concrete structure, for further simplicity application of Suncidal expression were established, where the constant introduced, where expression in equation (25) as $Z(z) = C_1\,\ell^{-M_{1z}} + C_2{}^{-M_{2z}}$ deriving it produce the equation (27) and (28).

The final model equation for water transport was established as

$$\phi(z,t) = \left(C_1\, Cos\, M_1 \frac{\lambda^2}{\beta} Z + C_2\, Sin\, M_2 \frac{\lambda^2}{\beta} Z \right) \qquad \text{…………………….} \qquad (27)$$

But if $Z = \dfrac{V}{t}$

$$\phi(z,t) = \left(C_1\, Cos\, M_1 \frac{\lambda^2}{\beta}\frac{V}{t} + C_2\, Sin\, M_2 \frac{\lambda^2}{\beta}\frac{V}{t} \right) \qquad \text{…………………….} \qquad (28)$$

4.0 Conclusion

The model established consider all the variables in the system that influence the transport of water in concrete structure, initially, concrete attained strength by curing the structure component, but when it has attained the required strength, the fluid passing through the macropores established is through the influence of permeability, this will decrease the structural compressive strength, water transport on the structure are caused by the variables on the model developed. Applying the mathematical expression will definitely monitor the rate of water migrating on concrete structure, the variable that allows the water transport has been expressed. The permeability influence that relate with the macropores between the constituent of the concrete is the mixture of fine. Sand, cement, water and coarse aggregate, including the mixture and compressibility of concrete placement through compaction are expressed base on various rates. The model can be simulated to develop the theoretical values that can be compared with the experimental values for validation of permeability coefficient in concrete structure.

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