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Variable Sample Size Control Charts for Monitoring the Multivariate Coefficient of Variation Based on Median Run Length and Expected Median Run Length

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Abstract

The monitoring of a well-functioning process system has always held significant importance. In recent times, there has been notable attention towards employing control charts to oversee both univariate and multivariate coefficients of variation (MCV). This shift is in response to the concern of erroneous outcomes that can arise when traditional control charts are applied under the condition of dependent mean and standard deviation, as highlighted by prior research. To address this, the remedy lies in adopting the coefficient of variation. Furthermore, this study underscores the application of MCV in scenarios where multiple quality attributes are simultaneously under surveillance within an industrial process. This aspect has demonstrated considerable enhancement in chart performance, especially when incorporating the variable sample size (VSS) feature into the MCV chart. Adaptive VSS, evaluated through metrics like median run length (MRL) and expected median run length (EMRL), is also integrated for MCV monitoring. In contrast to earlier studies that predominantly focused on average run length (ARL), this research acknowledges the potential inaccuracies in ARL measurement. In this study, two optimal designs for VSS MCV charts are formulated by minimizing two criteria: firstly, MRL; and secondly, EMRL, both accounting for deterministic and unknown shift sizes. Additionally, to assess the distribution's variability in run lengths, the study provides the 5th and 95th percentiles. The research delves into two VSS schemes: one with a defined small sample size (n_S), and another with a predetermined large sample size (n_L) for the initial subgroup ($n(1)$). The approach taken involves the development of a Markov chain method for designing and deriving performance measures of the proposed chart. These measures include MRL and EMRL. Moreover, a comparative analysis between the proposed chart's performance and the standard MCV chart (STD) is presented in terms of MRL and EMRL criteria. The outcomes illustrate the superiority of the

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proposed chart over the STD MCV chart for all shift sizes, whether they are upward or downward, and when $n(1)$ equals n_S or n_L.

1. Introduction

In the world of quality control and industrial industries, Statistical Process Control (SPC) charts have developed into essential and effective instruments (Teng, 2021). They are pivotal in enhancing the quality attributes of the product. SPC, recognized as one of the most compelling statistical methods (Montgomery, 2019), guarantees processes' predictability and stability, thereby improving the quality of procedures and products (Evans & Lindsay, 2004).

Inherent production processes are unstable and unpredictable variations, despite meticulous design and maintenance efforts (Evans & Lindsay, 2004). These two primary sources of intrinsic variability are assignable causes and common causes of variation, as depicted in Fig. 1. According to Joglekar, 2003, common causes that are part of regular processes lead to minor effects and result in a statistically in-control (IC) process characterized by a consistent distribution, mean, and variance. On the other hand, assignable causes induce substantial variances that make the process uncontrollable. These factors encompass machinery problems, operator errors, and subpar raw materials (Montgomery, 2019). Various SPC tools have also been augmented to production processes and manufacturing procedures (Khatun et al., 2019). Control charts seem incredibly useful for tracking processes that stray from control due to assignable causes (Khaw et al., 2021). These charts play a pivotal role in identifying instances when a process veers out of control due to these identifiable reasons.

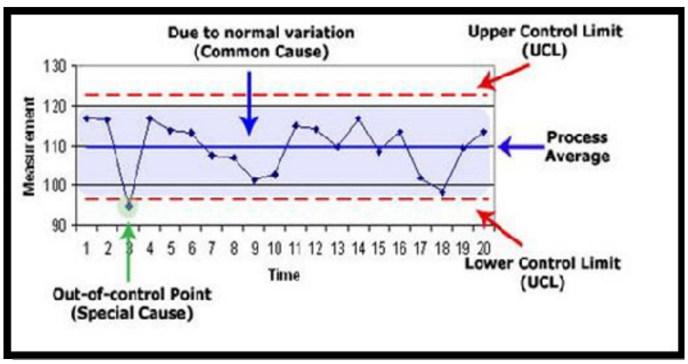

Fig. 1 *Components of statistical control charts (Baradaran & Dashtbani, 2014)*

Given a constant process mean, many control charts usually focus on overseeing both the process mean and variance, upholding the independence of the mean from the standard deviation (Khaw et al., 2019). Nonetheless, scenarios arise in which the mutual dependence of the mean and variability is observed within specific processes. In such instances, the mean is not steady and can impact (or be impacted by) the variance. This puts into question the traditional employment of X and S or R charts in such scenarios. Instead, a more suitable approach involves monitoring the coefficient of variation (CV) (Yeong et al., 2016). The application of CV is extensive, spanning various domains such as material engineering, manufacturing, science, finance, medicine, and biology (Khaw et al., 2018). A pivotal advancement occurred when (Kang et al., 2007) introduced the concept of CV control charts. Over recent decades, multiple versions of CV charts have been suggested to identify alterations in CV and enhance the established standard CV charts (Khaw et al., 2017; Khaw & Chew, 2019; Khaw et al., 2019; Yeong et al., 2018).

In addition to its statistical applications, the CV has demonstrated its utility across a range of diverse fields. For instance, Babu et al. (2016) utilized the CV to decrease speckle noise in ultrasound images, while Karthik & Manjunath (2019) applied it to rectify non-uniform grid line distances in noisy microarray images. Amelio (2019) and Bakowski et al. (2017) explored the role of CV in assessing sensory characteristics of virgin olive oil and compression ignition engine injection pressure, respectively. In a discrete uniform distribution context, Papatsouma et al. (2019) examined its potential as an estimator for population CV. The versatility of CV is evident in its deployment by Calif & Soubdhan (2016) for evaluating global solar radiation patterns and timeframe separation, as well as by Wachter et al. (2017) for investigating grip strength. Doring et al. (2018) adapted CV to detect worldwide trends in cereal yield stability. In the realm of image analysis, CV was employed by Singh & Singh (2019) to detect video frame and region duplication forgery, while in the context of robot path planning, it was harnessed for multiple objectives by Salmanpour et al. (2017). Scientific research, such as the analysis of neuronal spike trains, also integrated CV, recommending a G1-CV approach for optimal developmental face ventilation mode selection, as demonstrated by Lengler & Steger (2017) and Z. Zhou et al.

(2018). Furthermore, CV has found relevance in the study of home blood pressure (Ye et al., 2018) and in experiments concerning shear and tensile strength, as investigated by Centore (2015), Romano et al. (2005), and Ushigome et al. (2011). Univariate control charts are utilized when monitoring a single variable, while multivariate control charts are necessary when considering multiple variables (Khaw et al., 2021). (Castagliola et al., 2015) developed the Variable Sample Size Coefficient of Variation (VSS CV) chart, tracking the converted CV statistics. (Yeong et al., 2017) then adopted the VSS approach to monitor the CV as a result of this directly. Within the VSS scheme, the sample size, varying as a function of the previous sample's position, is categorized into three regions: the warning, central, and out-of-control (Khaw et al., 2019).

In numerous scenarios, monitoring multiple characteristics becomes necessary (Yeong et al., 2016). Within industrial contexts, monitoring multivariate processes is deemed an essential procedure (Khaw et al., 2021). Yeong used two one-sided Multivariate Coefficient of Variation (MCV) charts to fill the gaps in the multivariate process research. Synthetic MCV charts and adaptive MCV were elucidated (Khaw et al., 2018; Khaw & Chew, 2019). The latter serves as a statistical quality control tool for concurrent processes generating multiple correlated variables, boasting adaptability to detect process changes while remaining robust against gradual drifts or shifts. The statistical performance of the standard MCV chart is improved by this augmentation (Yeong et al., 2016). Run rules and variable parameter MCV charts were subsequently introduced (Chew et al., 2020). Recent contributions include the exponentially weighted moving average (EWMA) MCV chart and an adaptive EWMA MCV chart were proposed (Giner-Bosch et al., 2019; Haq & Khoo, 2019). An adaptive VSSID MCV control chart and a VSS MCV were created for MCV monitoring (Chew et al., 2020; Khaw et al., 2021). In VSS MCV, the adaptive sample size VSS scheme is incorporated into the standard Multivariate CV chart (MCV). Monitoring industrial processes frequently entails observing two or more interrelated quality characteristics (Khaw et al., 2021). The VSS MCV chart demonstrates superior accuracy through numerical comparisons. When it comes to spotting slight and moderate upward and downward MCV fluctuations, it outperforms the current standard MCV chart (Khaw et al., 2021).

Three adaptive charts for monitoring MCV were introduced (Khaw et al., 2018). These charts, employing the Markov chain approach, enhance sensitivity in detecting slight to moderate shifts in MCV compared to the conventional MCV chart. It was demonstrated through performance comparisons utilizing ATS, SDTS, and EATS criteria that the Variable Sample Size and Sampling Interval (VSSI) MCV chart outperformed the existing MCV charts. The VSSI MCV chart empowers engineers to promptly identify out-of-control signals by allowing variations in sample size and sampling intervals, thereby facilitating superior process control (Khaw et al., 2018).

The VSS chart is an adaptive control chart, relying on three charting parameters: sampling interval, sample size, and control limit coefficient. At least one of these parameters varies during implementation based on the statistical information from the preceding sample. This inherent flexibility renders the adaptive chart more robust and economically efficient for process monitoring than static control charts (Tagaras, 1998). The average run length (ARL) performance measure is frequently used in the literature to evaluate the VSS chart. Notable examples encompass the VSSI chart designed by (Khaw et al., 2018), the one-sided downward VSSI chart monitored by (Chew et al., 2020), and the VSS MCV chart illustrated by (Khaw et al., 2021). In the subsequent work, a Markov chain model was developed to derive performance measures for the chart, encompassing ARL, standard deviation of the run length (SDRL), average sample size (ASS), average number of observation (ANOS), and expected average run length (EARL). Numerical comparisons underscore that the proposed chart surpasses the standard MCV chart in effectively detecting slight to moderate upward and downward shifts in MCV.

Control chart analysis is a cornerstone of quality control, but recent studies have illuminated shortcomings in relying solely on the ARL measure. Teoh et al (2014) emphasis that this method misrepresents the performance across the full run length. Two primary concerns with ARL were highlighted: the skewed distribution of run lengths and the substantial Standard Deviation of Run Length (SDRL) (Montgomery, 2019). For right-skewed distributions, the average's prominence over the median can mislead practitioners (Khoo et al., 2012). Furthermore, the Variable Sample Size Coefficient of Variation (VSS CV) chart indicates that exclusive reliance on ARL can misinterpret its efficacy. The median run length (MRL) and expected MRL (EMRL)-based chart designs are introduced by VSS CV (Khaw et al., 2019). Due to ARL's practical constraints brought on by an inappropriate run-length distribution and high variability, a different performance measure and control chart must be considered. In light of these factors, the use of run-length percentiles emerges as a superior performance indicator compared to ARL (Khoo, 2004; Palm, 1990; Radson & Boyd, 2005; Zhou et al., 2012). Moreover, the significance of run-length percentiles in capturing run-length behaviour and augmenting control chart insights is evident (Zhou et al., 2012). Numerous articles demonstrate how skewness affects the MRL's dependability as a chart performance indicator. MRL's effectiveness is pronounced in right-skewed distributions due to its proximity to central tendency compared to ARL. Recognising these benefits, numerous researchers advocate for MRL as an alternative measure for control chart design and evaluation (Capizzi & Masarotto, 2008; Chong et al., 2022; Golosnoy & Schmid, 2007; Khoo et al., 2011, 2012; Lee & Khoo, 2017; Khaw et al., 2019; Lee & Khoo, 2006; Shmueli & Cohen, 2003; Teoh et al., 2014; 2015; 2016; 2017; Yeong et al., 2020). Therefore, harnessing the strengths of the MRL measure, this research advances optimal designs for the VSS MCV chart based on both MRL and EMRL. For the VSS MCV chart, two different optimal designs are created by minimising the out-of-control MRL (MRL_1) and out-of-control EMRL ($EMRL_1$) for known and unknown shift sizes, respectively. Notably, there is an unexplored application of MRL and EMRL as performance measures in investigating the VSS MCV chart under known and unknown shift size conditions, thus underscoring the importance of this research.

In addition, the standard (STD) MCV control chart, when based on MRL and EMRL, often fails to effectively optimize parameters. When process monitoring or parameter augmentation is necessary, its use can occasionally be ineffective. To address this, the need arises to optimize parameters for better process monitoring and identification of areas for improvement. The main goal is to minimize MRL_1 and $EMRL_1$. An optimisation algorithm for VSS MCV has been developed as a remedy, concentrating on MRL and EMRL. It's crucial to note that MRL-based designs work well when an exact shift size is specified. Adopting EMRL-based designs, on the other hand, becomes necessary when the precise shift magnitude cannot be determined. In numerous real-world situations, EMRL-based designs are indispensable. Due to a dearth of relevant historical data, quality practitioners often lack advanced knowledge about the magnitude of the upcoming shift size. Moreover, the shift size typically follows uncertain stochastic models, introducing variability.

In a nutshell, this endeavour is undertaken to accomplish objectives: (i) to develop two one-sided VSS MCV charts designed to monitor MCV, with a focus on MRL and EMRL criteria; (ii) to investigate the percentiles of the run-length distribution related to the VSS MCV chart; and, (iii) to create an optimization algorithm for the VSS MCV chart, specifically targeting scenarios where $n(1)=n_S$ and $n(1)=n_L$, while minimizing the values of MRL_1 and $EMRL_1$.

2. Methodology

In this section, the operation of the VSS MSV is discussed.

2.1 Operation of Two One-Sided VSS MCV Chart

The population MCV statistic, denoted as $\gamma = (\mu^T \Sigma^{-1} \mu)^{-\frac{1}{2}}$, was attained by Voinov & Nikulin (2011), where μ and σ refer to the mean vector and covariance matrix, respectively, γ can be estimated by the sample MCV $\hat{\gamma}$ when μ and σ are unspecified, Thus, $\hat{\gamma} = (\bar{x}^T S^{-1} \bar{x})^{-\frac{1}{2}}$ by replacing μ and σ to $\bar{x}$ and S, Here, $\bar{x}$ is the sample mean vector, whereas S is the sample covariance matrix. The computations of $\bar{x}$ and S are shown as follows:

$$\bar{X} = \frac{1}{n}\sum_{t=1}^{n} X_t \tag{1}$$

and

$$S = \frac{1}{n-1}\sum_{t=1}^{n}(X_t - \bar{x})(X_t - \bar{x})^T \tag{2}$$

Respectively, where $\bar{x}$ and S are independent of one another, note that n refers to the sample size $1<n<31$, and t is an index and refers to sample value $t=1$. A fixed sample size, n_0 used for the STD MCV chart (Yeong et al., 2016). The STD MCV chart and VSS MCV consist of two regions, safe and action regions, and three, safe, warning, and action regions, respectively. To enhance the performance of the STD MCV chart in detecting small to moderate MCV shifts, a sample size variation was carried out using the VSS scheme. In the proposed chart, the adopted sample size is adjusted between the lower limit of sample size, denoted as n_S, and the upper limit, n_L, satisfying the condition $n_S<ASS_0<n_L$. Notably, ASS_0 (also referred to as n_0) corresponds to the in-control Average Sample Size. It's worth mentioning that for the VSS MCV chart, ASS_0 matches the fixed sample size employed in the STD MCV chart. This study introduces two distinct one-sided VSS MCV charts: the upward and downward VSS MCV charts. These charts are formulated to ensure an unbiased comparison. The construction of the VSS MCV chart is as follows:

1. When the t^{th} sample MCV, $\hat{\gamma}$ falls in the safe region (below the warning limit for the upward chart, while $\hat{\gamma}$ falls above the warning limit for the downward chart), the process shows no indication of trouble. Hence, n_1 is taken to compute the $(t+1)^{th}$ sample MCV γ_i^2; the process still shows no trouble.
2. When $\hat{\gamma}$ falls in the warning region (above the warning limit and below the control limit for the upward chart, while $\hat{\gamma}$ falls below the warning limit and above the control limit for the downward chart). Still, there is a higher tendency for it to become out-of-control. Hence, n_2 is taken to compute $\hat{\gamma}_{t+1}$ and
3. When $\hat{\gamma}$ falls in the action region ($\hat{\gamma}$ falls above the control limit for the upward chart while $\hat{\gamma}$ falls below the control limit for the downward chart), the process indicates trouble at the t-sample due to the presence of

assignable causes; in this case, an immediate investigation should be taken, in Fig. 2 above condition is demonstrated.

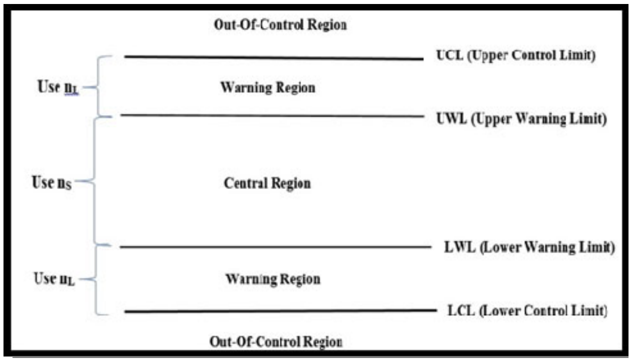

Fig. 2 *Two one-sided of the VSS chart (Teoh et al., 2016)*

The upper control and warning limits of the upward VSS MCV chart can be obtained as follows:

$$UCL = F_{\hat{\gamma}}^{-1}(1-\alpha|n_0,p,\delta_0) \quad (3)$$

and

$$LCL = F_{\hat{\gamma}}^{-1}(\alpha|n_0,p,\delta_0) \quad (4)$$

respectively, whereas the lower control and warning limits of the downward VSS MCV chart can be obtained as:

$$UWL = F_{\hat{\gamma}}^{-1}(1-\alpha'|n_0,p,\delta_0) \quad (5)$$

and

$$LWL = F_{\hat{\gamma}}^{-1}(\alpha'|n_0,p,\delta_0) \quad (6)$$

respectively, where $\delta_0 = \frac{n}{\gamma_0^2}$ and the determination of value α' is based on the desired (ARL_0) value when the process MCV is IC and $\alpha' > \alpha$, here, a = 1 when n(i) = n_S (where a = 1 corresponds to small sample size) and a = 2 when n(i) = n_L, (where a = 2 corresponds to large sample size). The n(i) stands for the selected sample size, Equations (3) – (6) follow an inverse cumulative distribution function (cdf) of $\hat{\gamma}$, i.e., $F_{\hat{\gamma}}^{-1} = (\alpha|n,p,\delta) = \sqrt{\left[\frac{(n(n-p))}{((n-1)p)}\times\frac{1}{F_F^{-1}}(1-\alpha|p,n-p,\delta)\right]}$. Here, $F_F^{-1}(\cdot)$ refers to an inverse cdf of a non-central F distribution while p stands for the number of quality characteristics, $1<n<31$ and $p<n$; in addition, because of the positive degree of freedom, this distribution is valid only when $p<n$ and δ are defined as $n/(\tau\gamma_0)^2$, where the shift size τ = 1 when the process shows no indication of trouble. At the same time, $\gamma_1 = \tau\gamma_0$ is an out-of-control MCV when $\tau \neq 1$, the values of $\tau > 1$ and $0 < \tau < 1$ refer to upward and downward MCV shifts, respectively.

Yeong et al. (2017)'s model of a three-state Markov-chain adopted for deriving the formulae for the MRL and EMRL of the VSS MCV chart, where 1st, 2^{nd,} and 3rd states indicate safe, warning which transient regions and action region which are absorbing state respectively. Subsequently, the resulting transition probability matrix **A** can be obtained as follows:

$$\mathbf{A} = \begin{pmatrix} A_{11} \\ A_{21} \\ 0 \end{pmatrix} \quad (7)$$

Where

$$A_{11} = \Pr(\hat{\gamma} \leq UWL|n_S,p,\delta_1) = F_{\hat{\gamma}}(UWL|n_S,p,\delta_1) \quad (8)$$

$$A_{12} = \Pr(UWL < \hat{\gamma} \leq UCL|n_S,p,\delta_1) = F_{\hat{\gamma}}(UCL|n_S,p,\delta_1) - F_{\hat{\gamma}}(UWL|n_S,p,\delta_1) \quad (9)$$

$$A_{21} = \Pr(\hat{\gamma} \leq UWL|n_L,p,\delta_1) = F_{\hat{\gamma}}(UWL|n_L,p,\delta_1) \quad (10)$$

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$$A_{22} = \Pr(\text{UWL} < \hat{\gamma} \leq \text{UCL}|n_L,p,\delta_1) = F_{\hat{\gamma}}(\text{UCL}|n_L,p,\delta_1) - F_{\hat{\gamma}}(\text{UWL}|n_L,p,\delta_1) \tag{11}$$

For the upward VSS MCV chart, while

$$A_{11} = \Pr(\hat{\gamma} \geq \text{LWL}|n_S,p,\delta_1) = 1 - F_{\hat{\gamma}}(\text{LWL}|n_S,p,\delta_1) \tag{12}$$

$$A_{12} = \Pr(\text{LCL} < \hat{\gamma} \leq \text{LWL}|n_S,p,\delta_1) = F_{\hat{\gamma}}(\text{LWL}|n_S,p,\delta_1) - F_{\hat{\gamma}}(\text{LCL}|n_S,p,\delta_1) \tag{13}$$

$$A_{21} = \Pr(\hat{\gamma} \geq \text{LWL}|n_L,p,\delta_1) = 1 - F_{\hat{\gamma}}(\text{LWL}|n_L,p,\delta_1) \tag{14}$$

$$A_{22} = \Pr(\text{LCL} < \hat{\gamma} \leq \text{LWL}|n_L,p,\delta_1) = F_{\hat{\gamma}}(\text{LWL}|n_L,p,\delta_1) - F_{\hat{\gamma}}(\text{LCL}|n_L,p,\delta_1) \tag{15}$$

For the downward VSS MCV chart, Equations (8) to (15) follow the cdf of $\hat{\gamma}$, i.e., $F_{\hat{\gamma}}(x|n,p,\delta) = 1 - F_F[(n(n - p))/((n - 1)px^2)|p,n - p,\delta]$, where $F_F(\cdot)$ denotes the cdf of a non-central F distribution, and $\alpha_a = n_a/\gamma_1^2$, where $a = 1$ (corresponds to small sample size) and $a = 2$ (corresponds to large sample size); subsequently, the ARL_1 and the out-of-control SDRL (SDRL_1) of the VSS MCV chart are obtained as :

$$\text{ARL}_1 = s^T(I - Q)^{-1}1 \tag{16}$$

and

$$\text{SDRL}_1 = \sqrt{2s^T(I - Q)^{-2}Q1 - \text{ARL}^2 + \text{ARL}} \tag{17}$$

respectively, where s = 2 × 1 vector of starting probabilities such that $s = (1,0)^T$ and $s = (0,1)^T$ with first sample size n_1 and n_2,respectively.

I = 2 × 2 identity matrix

Q = 2 × 2 transition probability matrix associated with the transient states

$1 = (1,1)^T$

The out-of-control ASS (ASS_1) stands for the ratio between the expected total number of observations taken in an infinite production horizon, which computation is done (Yeong et al., 2017):

$$\text{ASS}_1 = (n_S, n_L, n(1))\theta \tag{18}$$

where θ can be obtained from $\theta = B^{-1}(s,0)^T$; consequently, the matrix B shows

$$B = \begin{pmatrix} 1 & 1 & 1 \\ A_{12} & A_{22} - 1 & 0 \\ 1 - A_{11} - A_{12} & 1 - A_{21} - A_{22} & -1 \end{pmatrix} \tag{19}$$

for $n(1) = n_S$, when the first sample size is considered as n_S, while

$$B = \begin{pmatrix} A_{11} - 1 & A_{21} & 0 \\ 1 & 1 & 1 \\ 1 - A_{11} - A_{12} & 1 - A_{21} - A_{22} & -1 \end{pmatrix} \tag{20}$$

for $n(1) = n_L$, when the first sample size is considered as n_L, It must be noted that the performance evaluation of the VSS-type chart does not merely depend on ARL since there is no ignorance possibility of the actual number of observations taken in a sample, In this case, it is vital to use the out-of-control ANOS (ANOS_1) as one of the performance measures, where the equation shows as follows:

$$\text{ANOS}_1 = \text{ARL}_1 \times \text{ASS}_1 \tag{21}$$

When the VSS MCV chart model is a three-state Markov chain, the last state being the out-of-control state, the run length, i.e., the number of samples taken until an out-of-control signal produced, is represented as the number of transitions (since each shift represents a sample being taken) until the process reaches the absorbing state, i.e., the out-of-control state. Hence, the run length is a Discrete Phase Type (DPH) random variable of parameters (Q, q) (Kulkarni, 1999; Meuts, 1995). Thus, the probability mass function (p.m.f.) and c.d.f of the run length can be computed as follow:

$$f_{RL}(l) = P(RL = l) = q^T(Q^{l-1})r \tag{22}$$

$$F_{RL}(l) = P(RL \leq l) = 1 - q^T(Q^{l})1 \tag{23}$$

where $l \in \{1,2,3,\ldots\}$ and $q = (1,0)^T$ if the sample size of the first subgroup is $n(1) = n_S$, while $q = (0,1)^T$. If the first subgroup's sample size is $n(1) = n_L$, The (100θ) percentile of the Run-length distribution can be determined as the value I_θ such as Gan (1993).

$$P(RL \leq l_\theta - 1) \leq \theta \text{ and } P(RL \leq l_\theta) > \theta \tag{24}$$

where θ is in the range $0<\theta<1$, for example, the 30th percentile of the VSS MCV chart can be obtained from Equation (24) by setting θ=0.3, hence, to get the MRL, which is the 50th percentile, θ set as 0.5.

The out-of-control disease is caused by assignable cause(s), which results in a shift of a certain magnitude in the process; for a CV-type chart, the extent of the shift is denoted as the shift size τ, where τ is the ratio between the out-of-control CV (γ_1) over the in-control CV(γ_0),i.e., When $\tau = \frac{\gamma_1}{\gamma_0}$ The assignable cause(s) increases the CV.

The only way that $(100\theta)^{th}$ the percentile of the run-length distribution, I_θ from Equation (24), can be obtained is the specification of τ as an exact value. However, there is difficult the specification of τ as an accurate value in many real-life applications (Castagliola et al. 2011). Hence, this research also proposes the expected percentiles of the run-length distribution, $E(I_\theta)$, where τ only is specified as a range of possible values. There is no need to select an exact value. $E(I_\theta)$ can be computed as follows:

$$E(l_\theta) = \int_{\tau_{min}}^{\tau_{max}} f_\tau(\tau) l_\theta(\tau)\,d\tau \tag{25}$$

where $f_\tau(\tau)$, the probability density function (PDF) of the shift τ. is E (I_θ) can be computed as the expected value of I_θ over the density function $f_\tau(\tau)$; it is noted that E (I_θ) it does not quantify any statistical features of the chart for a specific shift size. Due to the fitting difficulty of the actual shape $f_\tau(\tau)$, previous literature regularly presumed that it follows a continuous uniform distribution over the interval (τ_{min},τ_{max}) (Castagliola et al. 2011), where the user sets τ_{min} and τ_{max}. Thus, Equation (25) reduces to:

$$E(l_\theta) = \int_{\tau_{min}}^{\tau_{max}} \frac{1}{\tau_{max} - \tau_{min}} l_\theta(\tau)\,d\tau \tag{26}$$

Due to inadequate evaluation of the integral in Equation (26), approximates it using the Gauss-Legendre quadrature method. A detailed explanation of the Gauss-Legendre quadrature method is given by (Kovvali, 2022).

2.2 Run-length Properties of the Multivariate Control Charts

The performances of the two optimal VSS MCV charts, one with $n(1) = n_S$ and the other with $n(1) = n_L$, are compared to that of the STD MCV. As such, this section outlines the run-length properties, including ARL, SDRL, MRL, EMRL, and percentiles of the run-length distribution for the VSS MCV charts.

2.3 Optimal Designs of the VSS MCV Chart

An optimization procedure is employed to determine the optimal parameters of the VSS MCV chart, aiming to minimize MRL_1. It is assumed that both MRL_0 and $EMRL_0$ are set to 250 for the computation of the optimal parameter combinations (n_S, n_L, α′). Three optimization criteria are examined to minimize the following objective function:

(i) Min (n_S, n_L, α') MRL_1 (τ), subject to MRL_0

(ii) Min (n_S, n_L, α') $EMRL_1$ (τ), subject to $EMRL_0$

The procedure aims to optimize the VSS MCV is conducted:

1. The process begins by establishing values for MRL_0 (or $EMRL_0$), p, and τ for minimizing MRL_1 (τ), or (τ_{min}, τ_{max}) for minimizing $EMRL_1$ (τ_{min}, τ_{max}).

2. The n_S value is set as p + 1, followed by setting n_L as ASS_0 + 1, where n_S is greater than p.
3. The computation of α´ from Equation (27) takes place, considering MRL_0 (or $EMRL_0$) as 250 and adopting $ASS_0 = n_0$ from Khaw et al. (2018). The coefficient α' is computed using the equation, representing a warning limit coefficient.

$$\alpha' = 1 - \frac{F_\gamma(UCL|ASS_0, p, \gamma_0)(ASS_0 - n_L)}{n_S - n_L} \quad (27)$$

4. Subsequently, equations (16), (17), (18), (21), (24), and (26), respectively, based on the parameter combination (n_S, n_L, α´), are used to get $ARL_1(\tau)$, $SDRL_1(\tau)$, $ANOS_1(\tau)$, MRL1(τ), and $EMRL_1(\tau_{\min}, \tau_{\max})$.
5. The value of n_L is then unaltered, while n_S is incremented by one.
6. The process is iterated from step 3 to step 5 until n_S equals ASS_0-1.
7. The cycle then restarts by setting n_S=p+1 and increasing n_L by 1.
8. Given that industrial contexts typically do not involve large sample sizes, as Castagliola et al. (2015) stated, step 3 to step 7 are repeated until n_L exceeds 31. The maximum sample size of 31 is utilized as a reference in this paper, with the actual number of sample sizes adapted to the specific process.
9. The process is wrapped up by selecting the best set of parameters (n_S, n_L, α, p, MRL_1, $\theta_{0.05}$, $\theta_{0.95}$, MRL_0, ARL, ASS, SDRL, ANOS), which effectively minimizes $MRL_1(\tau)$ (for objective function (i)) and the $EMRL_1(\tau_{min}, \tau_{max})$ (for objective function (ii)).

With these optimal chart parameters, the 5th percentile (θ=0.05), MRL_1, and 95th percentile (θ=0.95) are computed from Equation (24) by letting θ=0.05, θ=0.50, and θ=0.95, respectively. This research examines scenarios involving both small and large initial sample sizes, specifically when $n(1)=n_s$ and $n(1)=n_L$. A large initial sample size is not required because there are no early signs that the process is out of control. It's crucial to remember that the initial sample size impacts the run duration, which indicates how many transitions must occur before the process reaches the out-of-control stage. Consequently, this has a direct influence on whether the first sample falls within the central region (state 1), the warning region (state 2), or the out-of-control region (state 3). In the context of the Markov chain, the state of the first sample influences the second sample's state, affecting the third sample's state and subsequent samples. Hence, the number of transitions needed for the Markov chain to reach the out-of-control state is directly influenced by the original sample size. It's important to remember that at the outset of process monitoring, there is no available information about the states of any of the samples.

3. Results

This section delves into the analysis and discussion of the performances of the two VSS schemes. This section presents the optimal parameter of the VSS MCV chart with a focus on minimizing MRL_1. Besides, insights into the computation of run-length distribution percentiles, MRL, and EMRL for both sides of the VSS MCV are also provided. The last part draws a comparison between the performances of the two optimal VSS MCV charts and the STD MCV charts, primarily based on MRL and EMRL metrics. Throughout this work, a meticulous presentation of all the results is computed when n(i) = n_S, n(i) = n_L, MRL_0=250, values of $\gamma_0 \in \{0.10,0.30,0.50\}$, $ASS_0 \in\{5,7\}$, $p \in(2,3,4)$, τ∈{0.30,0.40,0.50,0.60,0.7,0.80,0.9,1.10,1.20, 1.30,1.40,1.50,1.60,2} and (τ_{min}, τ_{max}) ∈ ([0.3,0.6), (0.5,0.8), (0.6,0.9), (0.5,1), (0.7,1), (1,1.5), (1,2), (1.2,1.8), (1.4,1.9), (1.5,2]). Due to the space constraints, the results of $ASS_0 \in\{10,15\}$ could be obtained from the first author upon request.

3.1 Optimal Parameters of the VSS MCV Chart for Minimizing MRL_1

Tables 1 to Table 4 present the optimal chart parameters (n_S, n_L, α', ARL, ASS, SDRL, ANOS) for the VSS MCV chart, focusing on minimizing MRL_1. Notably, the optimal parameter values for $n(1)=n_L$ are lower than those for $n(1)=n_s$, in line with the optimal parameter outcomes. As the shift size increases, ANOS values increase, and vice versa for upward shifts. As p and γ values increase, optimal parameter values rise correspondingly. The optimal parameters are achieved with fewer values, slight shift sizes, small γ value, and $n(1)=n_L$.

Furthermore, examining Table 1 in the context of $n(1)=n_L$, p=2, n_0=7, τ=0.3, it becomes evident that the optimal parameter values show a slight variation when considering various values of τ and γ_0. For instance, in the mentioned example, the ANOS value for $n(1)=n_L$, p=2, τ=0.3, γ_0=0.5 is 8.65, while for γ_0=0.1, γ_0=0.3 are 10.68, 10.22 respectively; and ANOS for p=2, $n(1)=n_L$, p=3, τ=0.3 equals 10.24. Additionally, when p is increased, optimal parameter values also increase. Similarly, an increase in γ leads to higher values, with a notable exception where a minor distinction occurs between small and large shift sizes. When shifting size increases, for downward optimal parameters values increase, and for upward is vice-versa.

Table 1 *Optimal n_S, n_L, α, ARL, ASS, SDRL, ANOS of the VSS MCV chart for minimizing MRL_1 when p=2, n(1)=n_S, ASS_0 ∈{5, 7}, γ_0 ∈{0.1, 0.3, 0.5}, τ ∈{0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.1,1.2,1.3,1.4,1.5,1.6, 2} and MRL_0=250*

τ	(n_S, n_L, α, ARL, ASS, SDRL, ANOS) VSS MCV chart (Minimizing MRL_1)		
	n_0 = 5		
	γ_0 = 0.1	**γ_0 = 0.3**	**γ_0=0.5**
0.3	3,9,0.3351,2.62,5.40,0.95,14.20	3,9,0.3351,2.59,5.35,0.91,13.87	3,9,0.3351,2.72,5.46,1.04,14.88
0.4	3,10,0.2876,4.01,6.43,2.14,25.80	3,10,0.2876,3.93,6.35,2.05,24.98	3,11,0.2520,3.91,6.47,1.98,25.36
0.5	3,16,0.1561,5.25,7.10,3.09,37.29	3,15,0.1689,5.23,7.14,3.02,37.38	3,13,0.2022,6.20,7.83,3.79,48.62
0.6	3,19,0.1274,8.85,8.81,6.01,78.06	3,18,0.1357,8.94,8.54,6.21,76.44	3,18,0.1357,10.51,8.79,7.77,92.45
0.7	3,31,0.0739,19.04,9.25,16.18,176.16	3,28,0.0825,19.45,8.45,16.91,164.43	3,30,0.0766,23.79,8.2,21.3,196.12
0.8	3,31,0.0739,100.36,7.18,98.75,721.43	3,31,0.0739,86.51,6.84,84.89,592.02	3,31,0.0739,102.49,6.5,101,671
0.9	3,10,0.2876,260.57,5.4,259.8,1422.9	3,27,0.0858,250.41,5.51,249.6,1379.8	3,31,0.0739,259.7,5.4,259,1417
1.1	4,25,0.0502,111.4 ,5.7 ,110.7,638.03	4,31,0.0397,118.72,5.74,118.08,681.5	137.48,5.56,136.87,765.4
1.2	3,30,0.0766,33.02,7.46,31.71,246.53	3,31,0.0739,38.84,7.35,37.62,285.82	3,30,0.0766,53.4,7.05,52.3,376.8
1.3	3,31,0.0739,12.92,8.27,11.32,106.95	3,30,0.0766,15.79,8.23,14.26,130.11	3,30,0.0766,23.01,8.07,21.5 185.9
1.4	3,18,0.1357,8.47,7.46,6.86,63.20	3,20,0.1200,9.87,7.76,8.29,76.63	3,27,0.0858,12.9,8.4,11.3,108.6
1.5	3,22,0.1077,5.65,7.37,4.08,41.71	3,16,0.1561,7.08,7.22,5.52,51.15	3,28,0.0825,8.51,8.47,6.8,72.1
1.6	4,20,0.0650,4.34,7.22,3.07,31.43	3,14,0.1840,5.55,6.73,4.02,37.42	3,19,0.1274,7.06,7.73,5.44,54.6
2.0	4,7,0.3351,2.92,5.10,1.93,14.92	4,13,0.1135,2.93,6.19,1.79,18.19	3,14,0.184,4.16,6.47,2.63,26.95
	n_0 = 7		
0.3	3,8,0.8005,2.93,5.46,1.37,16.03	3,8,0.8005,2.86,5.41,1.30,15.54	3,9,0.6675,2.5332,5.60,0.92,14.18
0.4	3,12,0.4459,2.78,6.86,1.12,19.15	3,12,0.4459,2.75,6.79,1.07,18.70	3,13,0.4016,2.77,7.02,1.07,19.51
0.5	3,15,0.3351,3.92,8.59,2.02,33.75	3,14,0.3653,4.03,8.47,2.16,34.21	3,16,0.3096,3.99,8.74,2.04,34.99
0.6	4,23,0.1602,5.17,11.75,2.94,60.85	4,21,0.1787,5.21,11.39,3.02,59.40	3,20,0.2374,6.43,11.28,4,72.59
0.7	3,27,0.1689,11.57,14.39,8.59,166.54	3,28,0.1623,10.45,13.10,7.70,137.12	3,31,0.1452,11.93,13.16,9.23,157
0.8	3,31,0.1452,54.38,12.6,52.25,685.75	3,31,0.1452,45.31,11.72,43.24,531.45	3,31,0.1452,55.99,11.04,54.1,618
0.9	3,13,0.4016,219.83,8.1,218.97,1782.6	3,27,0.1689,199.44,8.3,198.47,1668.3	3,29,0.1561,212.16,8.2,211,1742
1.1	4,28,0.1274,88.26,8.79,87.38,776.58	4,29,0.1224,96.95,8.70,96.11,844.23	4,28,0.1274,115.6,8.4,114.8,979.9
1.2	3,28,0.1623,22.87,10.84,21.31,248	3,29,0.1561,27.42,10.75,25.94,294.93	3,28,0.1623,38.91,10.3,37.5,401.1
1.3	3,23,0.2022,9.84,10.65,8.09,104.91	3,27,0.1689,11.21,11.27,9.48,126.51	3,30,0.1505,15.5,11.67,13.8,181.6
1.4	3,31,0.1452,5.54,10.32,3.81,57.18	3,22,0.2127,6.95,10.24,5.21,71.29	4,30,0.1178,8.48,12.01,6.87,101.9
1.5	4,19,0.2022,4.15,9.24,2.66,38.45	3,16,0.3096,5.48,8.75,3.84,48.03	3,20,0.2374,6.92,10.11,5.18,70.03
1.6	3,11,0.5013,4.21,6.90,2.71,29.08	3,19,0.2520,4.11,8.46,2.48,34.82	3,18,0.2686,5.51,9.30,3.80,51.34
2.0	3,8,0.8005,2.73,5.15,1.43,14.07	3,12,0.4459,2.81,6.14,1.40,17.31	5,18,0.1561,2.88,8.85,1.62,25.50

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Table 2 *Optimal n_S, n_L, α, ARL, ASS, SDRL, ANOS of the VSS MCV chart for minimizing MRL_1 when p=2, n(1)=n_L, $ASS_0 \in\{5, 7\}$, $\gamma_0 \in \{0.1, 0.3, 0.5\}$, $\tau \in\{0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.1,1.2,1.3,1.4,1.5,1.6,2\}$ and MRL_0=250*

τ	(n_S, n_L, α, ARL, ASS, SDRL, ANOS) VSS MCV chart (Minimizing MRL_1)		
	n_0 = 5		
	γ_0 = 0.1	γ_0 = 0.3	γ_0=0.5
0.3	3,8,0.4016,1.95,5.40,1.36,10.57	3,8,0.4016,1.88,5.35,1.29,10.11	3,9,0.3351,1.54,5.46,0.91,8.44
0.4	3,11,0.2520,1.97,6.32,1.38,12.47	3,11,0.2520,1.89,6.24,1.30,11.85	3,12,0.2243,1.78,6.351.18,11.32
0.5	3,17,0.1452,1.81,6.94,1.23,12.60	3,16,0.1561,1.94,6.96,1.39,13.51	3,18,0.1357,1.83,6.98,1.31,12.78
0.6	3,27,0.0858,2.07,7.47,2.05,15.52	3,26,0.0894,2.13,7.28,2.38,15.55	3,29,0.0794,2.27,7.29,3.00,16.61
0.7	3,31,0.0739,9.94,9.25,13.61,92.01	3,29,0.0794,10.61,8.42,14.44,89.36	3,31,0.0739,14,8.19,18.8,114.76
0.8	3,31,0.0739,93.12,7.18,98.48,669.38	3,31,0.0739,78.5772,6.84,84.52,537.71	3,31,0.0739,95.1,6.5,100.76,623
0.9	3,10,0.2876,259.99,5.46,259.86,1419.72	3,27,0.0858, 247.9,5.51,249.6,1366.3	3,29,0.0794,258,5.45,259.6,1407
1.1	4 ,31,0.0397,108.31,5.79,110.08, 627.47	4,30,0.0411,116.71,5.73,118.13,668.9	4,30,0.0411,135.3,5.6,136.45,763
1.2	3,30,0.0766,28.42,7.46,31.36,212.22	3,29,0.0794,35.41,7.27,38.05,257.51	3,30,0.0766,49.61,7.0,52.2,349.8
1.3	3,31,0.0739,8,8.27,10.26,66.25	3,29,0.0794,11.49,8.18,13.75,94.06	3,30,0.0766,18.51,8.07,2.11,149
1.4	3,31,0.0739,3.15,8.09,4.28,25.58	3,26,0.0894,5.27,8.16,6.55,43.06	3,28,0.0825,8.56,8.46,10.40,72.4
1.5	3,21,0.1135,2.64,7.32,2.97,19.41	3,26,0.0894,2.83,7.87,3.45,22.31	3,25,0.0934,5.10,8.32,6.20,42.46
1.6	3,16,0.1561,2.34,6.63,2.29,15.51	3,20,0.1200,2.44,7.23,2.59,17.68	3,28,0.0825,2.82,8.25,3.50,23.35
2.0	3,8,0.4016,1.98,4.82,1.49,9.58	3,10,0.2876,2.02,5.37,1.60,10.87	3,14,0.1840,2.25,6.47,2.02,14.58
	n_0 = 7		
0.3	3,8,0.8005,1.95,5.46,1.36,10.68	3,8,0.8005,1.88,5.41,1.29,10.22	3,9,0.6675,1.54,5.60,0.91,8.65
0.4	3,11,0.5013,1.97,6.85,1.38,13.51	3,11,0.5013,1.89,6.77,1.30,12.86	3,12,0.4459,1.78,7.03,1.18,12.53
0.5	3,17,0.2876,1.80,8.39,1.20,15.17	3,16,0.3096,1.92,8.31,1.33,16.01	3,18,0.2686,1.80,8.56,1.20,15.44
0.6	3,27,0.1689,1.94,10,1.39,19.49	3,26,0.1762,1.92,9.72,1.39,18.70	3,29,0.1561,1.93,9.97,1.49,19.28
0.7	3,28,0.1623,6.97,14.35,7.28,100.14	3,30,0.1505,5.48,12.99,6.05,71.30	3,30,0.1505,7.64,13.26,8.6,101.4
0.8	3,31,0.1452,49.83,12.60,52.06,628.37	3,31,0.1452,40.53,11.72,43,475.39	3,31,0.1452,51.37,11,53.93,567.3
0.9	3,13,0.4016,219.12, 8.10,218.97,1776.89	3,27,0.16897,197.36,8.36,198.4,1650.9	3,31,0.1452,208.9,8.2,210.2,1718
1.1	4,29,0.1224,86.08,8.83,87.11,760.76	4,30,0.1178,94.93,8.74,95.89,829.90	4,31,0.1135,113.6,8.5,114.4,972
1.2	3,30,0.1505,18.98,11.03,20.45,209.42	3,29,0.1561,24.39,10.75,25.77,262.41	3,31,0.1452,34.7,10.5,36.2,366.8
1.3	3,28,0.1623,6.01,11.17,6.72,67.16	3,30,0.1505,7.52,11.56,8.46,87.11	3,31,0.1452,12,11.7,13.2,141.8
1.4	3,31,0.1452,2.44,10.32,2.60,25.21	3,25,0.1840,4.04,10.53,4.28,42.61	3,28,0.1623,5.94,11.50,6.55,68.3
1.5	3,21,0.2243,2.22,8.79,1.98,19.59	3,26,0.1762,2.30,9.72,2.21,22.41	3,24,0.1927,3.97,10.58,4.12,42
1.6	3,16,0.3096,2.07,7.63,1.66,15.84	3,20,0.2374,2.11,8.55,1.79,18.05	3,28,0.1623,2.29,10.20,2.22,23.3
2.0	3,8,0.8005,1.9,5.15,1.31,9.79	3,10,0.5726,1.91,5.83,1.35 11.16	3,14,0.3653,2.0533,7.2,1.57,14.8

Table 3 *Optimal n_S, n_L, α, ARL, ASS, SDRL, ANOS of the VSS MCV chart for minimizing MRL_1 when p=3, n(1)=n_S, ASS_0 ∈{5,7}, γ_0 ∈ {0.1, 0.3, 0.5}, τ ∈{0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.1,1.2,1.3,1.4,1.5,1.6,2} and MRL_0=250*

τ	(n_S, n_L, α, ARL, ASS, SDRL, ANOS) VSS MCV chart (Minimizing MRL_1)		
	n_0 = 5		
	γ_0 = 0.1	γ_0 = 0.3	γ_0=0.5
0.3	4,9,0.2022,3.61,6.13,1.76,22.18	4,9,0.2022,3.53,6.05,1.68,21.41	4,9,0.2022,3.78,6.15,1.88,23.30
0.4	4,12,0.1274,5.17,6.52,3.03,33.76	4,12,0.1274,5.14,6.43,3.04,33.13	4,10,0.1689,6.31,7,3.89,44.23
0.5	4,13,0.1135,8.74,7.52,5.88,65.77	4,13,0.1135,8.70,7.31,5.95,63.70	4,15,0.09342,9.22,7.11,6.62,65.68
0.6	4,19,0.0692,16.07,7.46,13.27,119.98	4,19,0.0692,16.25,7.09,13.69,115.39	4,23,0.0552,18.27,6.80,15.92,124.38
0.7	4,30,0.0411,41.43,6.88,39.13,285.43	4,30,0.0411,39.77,6.45,37.71,256.74	4,31,0.0397,49.26,6.28,47.36,309.51
0.8	4,29,0.0426,164.5,5.74,163.4,945.2	4,31,0.0397,145.59,5.65,144.4,823.6	4,31,0.0397,164.24,5.55,163.17,912.04
0.9	4,6,0.5013,202,291.6,5.14,291,1499	4,22,0.0581,287.4,5.18,286.8,1491.5	4,26,0.0480,293.31,5.16,292.69,1515.9
1.1	4,18,0.0739,130.21,5.4,129.54,712.5	4,6,0.5013,137.67,5.09,137.02,701.9	4,6,0.5013,154.85,5.09,154.22,788.24
1.2	4,28,0.0443,46.26,6.34,45.23,293.60	4,28,0.0443,53.66,6.24,52.71,335.13	4,31,0.0397,69.40,6.12,68.52,424.91
1.3	4,31,0.0397,18.85,7.07,17.51,133.42	4,30,0.0411,23.08,6.96,21.82,160.73	4,30,0.0411,33.32,6.73,32.17,224.62
1.4	4,20,0.0650,11.44,6.94,10.03,79.47	4,27,0.0461,12.98,7.21,11.60,93.74	4,29,0.0426,18.70,7.17,17.36,134.18
1.5	4,13,0.1135,8.49,6.46,7.11,54.93	4,15,0.0934,9.9,6.68,8.53,66.23	4,23,0.0552,12.92,7.18,11.52,92.93
1.6	4,9,0.2022,7.16,5.88,5.88,42.14	4,16,0.0858,7.08,6.73,5.68,47.74	4,18,0.0739,10.05,7,8.62,70.4
2.0	4,6,0.5013,4.22,4.96,3.15,20.95	4,9,0.2022,4.20,5.65,2.94,23.78	4,11,0.145,5.74,6.22,4.35,35.76
	n_0 = 7		
0.3	4,9,0.6011,2.98,6.48,1.42,19.36	4,9,0.6011,2.88,6.41,1.30,18.48	4,10,0.5013,2.57,6.58,0.95,16.96
0.4	5,13,0.2520,2.80,8.38,1.14,23.53	5,13,0.2520,2.77,8.30,1.11,23.06	5,15,0.2022,2.74,8.59,1.04,23.58
0.5	5,17,0.1689,3.92,9.87,1.98,38.76	5,16,0.1840,4,9.72,2.06,38.90	5,21,0.1274,4.08,9.72,2.23,39.69
0.6	4,27,0.1328,6.63,10.10,4.36,67.05	4,23,0.1602,6.46,10.35,4.04,66.95	4,20,0.1897,7.81,11.32,5.18,88.43
0.7	4,28,0.1274,14.38,13.28,11.36,191	4,28,0.1274,13.56,12.11,10.83,164.3	4,30,0.1178,16.34,11.97,13.71,195.74
0.8	4,31,0.1135,73.19,10.81,71.29,791.8	4,31,0.1135,61.29,10.21,59.42,625.8	4,31,0.1135,74.89,9.70,73.20,726.63
0.9	4,10,0.5013,241.83,7.62,241.05,1845	4,27,0.1328,224.51,7.9,223.64,1773	4,31,0.1135,235.09,7.8,234.26,1833.79
1.1	5,28,0.0894,98.37,8.29,97.59,815.58	5,31,0.0794,107.06,8.26,106.3,884.4	5, 18,0.1561,127.32,7.79,126.62,991.89
1.2	4,28,0.1274,27.39,10.10,25.95,276.95	4,28,0.1274,33.18,9.91 31.82,328.97	4,28,0.1274,46.04,9.56,44.82,440.33
1.3	4,25,0.1452,11.25,10.49,9.55,118.04	4,31,0.1135,12.74,11.01,11.06,140.4	4,31,0.1135,18.66,10.89,17.08,203.37
1.4	4,20,0.1897,6.96,9.65,5.25,67.18	4,20,0.1897,8.43,9.89,6.76,83.49	4,24,0.1523,11.32,10.59,9.63,119.94
1.5	4,14,0.3019,5.54,8.35,3.95,46.3	4,24,0.1523,5.55,9.75,3.87,54.14	5,28,0.0894,7.11,10.98,5.57,78.12
1.6	4,16,0.2520,4.12,8.12,2.54,33.51	5,21,0.1274,4.22,9.55,2.8,40.35	5,23,0.1135,5.66,10.33,4.15,58.56
2.0	4,9,0.6011,2.87,6.02,1.55,17.28	5,10,0.4016,2.96,6.97,1.83,20.70	4,12,0.3767,4.09,7.48,2.58,30.67

Table 4 *Optimal nS, nL, α, ARL, ASS, SDRL, ANOS of the VSS MCV chart for minimizing MRL1 when p=3, n(1)=nL, ASS0 ∈{5, 7}, 0 ∈ {0.1, 0.3, 0.5}, τ ∈{0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.1,1.2,1.3,1.4,1.5,1.6,2} and MRL0=250*

τ	(n_S, n_L, α, ARL, ASS, SDRL, ANOS) VSS MCV chart (Minimizing MRL_1)		
	n_0 = 5		
	γ_0 = 0.1	γ_0 = 0.3	γ_0=0.5
0.3	4,10,0.1689,1.46,5.98,0.82,8.76	4,9,0.2022,1.89,6.05,1.3 11.46	4,10,0.1689,1.55,6.02,0.92,9.37
0.4	4,12,0.1274,2,6.52,1.42,13.07	4,12,0.1274,1.92,6.43,1.35,12.39	4,13,0.1135,1.8,6.4,1.25,11.55
0.5	4,18,0.0739,1.94,6.57,1.76,12.75	4,17,0.0794,2.12,6.55 2.18,13.91	4,19,0.0692,2.07,6.46,2.5,13.44
0.6	4,28,0.0443,2.82,6.52,5.37,18.42	4,27,0.0461,3.2,6.35,6.45,20.38	4,29,0.0426,4.18,6.35,8.75,26.58
0.7	4,31,0.0397,24.24,6.87,34.47,166.71	4,31,0.0397,22.93,6.44,33.43,147.9	4,31,0.0397,33.44,6.28,44.7,210.16
0.8	4,31,0.0397,154.03,5.77,161.91,889.0	4,31,0.0397,134.8,5.65,144.01,762.5	4,31,0.0397,154.46,5.55,162.87,857.7
0.9	4,6,0.5013,291.49,5.14,291.07,1498.5	4,27,0.0461,284,5.19,285.9,1474.3	4,31,0.0397,290.06,5.16,292.11,1499.44
1.1	4,27,0.0461,126.98,5.57,128.79,707.6	4,27,0.0461,135.55,5.53,137.1,750.3	4,28,0.0443,154.21,5.47,155.52,844.63
1.2	4,31,0.0397,39.527,6.41,43.9, 253.4	4,31,0.0397,47.48,6.30,51.53,299.4	4,29,0.0426,65.64,6.08,68.92,399.72
1.3	4,31,0.0397,12.29,7.07,16.23,87.05	4,30,0.0411,16.99,6.96,20.94,118.34	4,29,0.0426,28.02,6.71,31.95,188.07
1.4	4,23,0.0552,6.39,7.07,8.46,45.24	4,29,0.0426,7.08,7.27,9.9,51.5	4,31,0.0397,12.11,7.23,15.84,87.66
1.5	4,22,0.0581,3.4,7.01,4.59,23.85	4,27,0.0461,3.8,7.26,5.51,27.62	4,28,0.0443,6.86,7.36,9.48,50.55
1.6	4,17,0.0794,2.82,6.66,3.36,18.8	4,21,0.0614,3.07,7,3.98,21.49	4,29,0.0426,3.88,7.43,5.77,28.9
2.0	4,9,0.2022,2.14,5.49,1.84,11.78	4,11,0.1452,2.25,5.89,2.09,13.28	4,15,0.0934,2.68,6.61,2.95,17.75
	n_0 = 7		
0.3	4,10,0.5013,1.46,6.5,0.82,9.53	4,9,0.6011,1.89,6.41,1.3,12.15	4,10,0.5013,1.55,6.58,0.92,10.24
0.4	4,12,0.3767,1.99,7.71,1.41,15.39	4,12,0.3767,1.91,7.62,1.32,14.6	4,13,0.3351,1.78,7.78,1.18,13.92
0.5	4,18,0.2164,1.84,8.88,1.25,16.41	4,17,0.2328,1.93,8.78,1.35,17.02	4,19,0.2022,1.81,8.91,1.23,16.21
0.6	4,28,0.1274,1.98,9.93,1.52,19.68	4,27,0.1328,1.95,9.63,1.59,18.87	4,29,0.1224,2.13,9.91,1.98,21.2
0.7	4,31,0.1135,7.46,13.2,8.79,98.66	4,31,0.1135,6.78,11.96,8.41,81.23	4,30,0.1178,10.45,11.97,12.55,125.15
0.8	4,31,0.1135,67.8,10.81,71.08,733.63	4,31,0.1135,55.55,10.21,59.15,567.2	4,31,0.1135,69.3893,9.7015,73,673.18
0.9	4,10,0.5013,241.36,7.62,241.05,1841	4,27,0.1328,222.34,7.9,223.63,1756	4,31,0.1135,232.76,7.8,234.25,1815.56
1.1	5,28,0.0894,96.44,8.29,97.56,799.54	5,31,0.0794,105.1,8.26,106.29, 868.1	5,27,0.0934,124.97,8.02,125.7,1002.37
1.2	4,29,0.1224,23.49,10.18,25.36,239.28	4,30,0.1178,28.96,10.05,30.9,291.29	4,30,0.1178,28,42.02,9.68,43.9,407
1.3	4,30,0.1178,6.77,10.87,8.04,73.64	4,29,0.1224,9.47,10.86,10.86,102.95	4,31,0.1135,14.87,10.89,16.67,162.03
1.4	4,21,0.1787,4.27,9.75,4.56,41.66	4,26,0.1387,4.57,10.45,5.21,47.86	4,31,0.1135,6.52,11.17,7.74,72.9
1.5	4,22,0.1689,2.37,9.17,2.33,21.75	4,27,0.1328,2.49,9.91,2.68,24.74	4,25,0.1452,4.5,10.49,5.03,47.29
1.6	4,17,0.2328,2.16,8.22,1.87,17.79	4,21,0.1787,2.23,8.99,2.09,20.08	4,29,0.1224,2.5,10.26,2.74,25.67
2.0	4,9,0.6011,1.92,6.02,1.37,11.61	4,11,0.4301,1.96,6.66,1.45,13.08	4,15,0.2747,2.15,7.91,1.76, 17.01

3.2 Optimal Parameters of the VSS MCV Chart for Minimizing $EMRL_1$

Tables 5 to 8 show the optimal parameters values (n_S, n_L, α) for minimizing $EMRL_1$. These tables consider scenarios where $n(1) = n_S$, $n(i) = n_L$, MRL_0 =250, γ_0 ∈ {0.10, 0.30, 0.50}, ASS_0 ∈{5, 7}, p ∈(2, 3) and (τ_{min}, τ_{max}) ∈ ([0.3,0.6), (0.5,0.8), (0.6,0.9), (0.5,1), (0.7,1), (1,1.5), (1,2), (1.2,1.8), (1.4,1.9), (1.5,2]). These tables provide a comprehensive insight into the relationship between various parameter combinations and their impact on $EMRL_1$ minimization under the specified conditions.

Table 5 *Optimal n_S, n_L, α of the VSS MCV chart for minimizing $EMRL_1$ when p=2, $n(1)=n_S$, $ASS_0 \in \{5, 7\}$, $\gamma_0 \in \{0.1,0.3,0.5\}$, $(\tau_{min}, \tau_{max}) \in$([0.3,0.6), (0.5,0.8), (0.6,0.9), (0.5,1), (0.7,1), (1,1.5), (1,2), (1.2,1.8), (1.4,1.9), (1.5,2]) and MRL_0=250*

(τ_{min}, τ_{max})	(n_S, n_L, α) VSS MCV chart (Minimizing $EMRL_1$) n_0 = 5		
	γ_0 = 0.1	γ_0 = 0.3	γ_0=0.5
(0.3,0.6)	4,17,0.0794	4,17,0.0794	3,17,0.1452
(0.5,0.8)	3,31,0.0739	3,31,0.0739	3,31,0.0739
(0.6,0.9)	3,31,0.0739	3,31,0.0739	3,31,0.0739
(0.5,1.0)	3,29,0.0794	3,31,0.0739	3,31,0.0739
(0.7,1.0)	3,29,0.0794	3,31,0.0739	3,31,0.0739
(1.0,1.5)	4,31,0.0397	4,31,0.0397	3,31,0.0739
(1.0,2.0)	4,27,0.0461	4,31,0.0397	3,31,0.0739
(1.2,1.8)	3,30,0.0766	3,31,0.0739	3,31,0.0739
(1.4,1.9)	4,20,0.0650	4,27,0.0461	4,30,0.0411
(1.5,2.0)	4,18,0.0739	4,20,0.0650	4,28,0.0443
	n_0 = 7		
(0.3,0.6)	5,23,0.1135	5,22,0.1200	4,24,0.1523
(0.5,0.8)	3,31,0.1452	3,31,0.1452	3,31,0.1452
(0.6,0.9)	3,29,0.1561	3,31,0.1452	3,31,0.1452
(0.5,1.0)	3,29,0.1561	3,28,0.1623	3,31,0.1452
(0.7,1.0)	3,29,0.1561	3,31,0.1452	3,31,0.1452
(1.0,1.5)	4,31,0.1135	5,31,0.0794	4,29,0.1224
(1.0,2.0)	4,31,0.1135	4,30,0.1178	4,30,0.1178
(1.2,1.8)	4,30,0.1178	5,31,0.0794	4,31,0.1135
(1.4,1.9)	5,23,0.1135	5,30,0.0825	4,29,0.1224
(1.5,2.0)	5,19,0.145	5,26,0.0977	6,31,0.042

Table 6 *Optimal n_S, n_L, α of the VSS MCV chart for minimizing $EMRL_1$ when p=2, $n(1)=n_L$, $ASS_0 \in \{5, 7\}$, $\gamma_0 \in \{0.1, 0.3,0.5\}$, $(\tau_{min}, \tau_{max}) \in$ ([0.3,0.6), (0.5,0.8), (0.6,0.9), (0.5,1), (0.7,1), (1,1.5), (1,2), (1.2,1.8), (1.4,1.9), (1.5,2]) and MRL_0=250*

(τ_{min}, τ_{max})	(n_S, n_L, α) VSS MCV chart (Minimizing $EMRL_1$) n_0 = 5		
	γ_0 = 0.1	γ_0 = 0.3	γ_0=0.5
(0.3,0.6)	3,27,0.0858	3,25,0.0934	3,28,0.0825
(0.5,0.8)	3,31,0.0739	3,31,0.0739	3,31,0.0739
(0.6,0.9)	3,31,0.0739	3,31,0.0739	3,31,0.0739
(0.5,1.0)	3,31,0.0739	3,31,0.0739	3,31,0.0739
(0.7,1.0)	3,29,0.0794	3,31,0.0739	3,31,0.0739

(1.0,1.5)	3,31,0.0739	4,31,0.0397	3,31,0.0739
(1.0,2.0)	4,31,0.0397	3,31,0.0739	4,31,0.0397
(1.2,1.8)	3,31,0.0739	3,31,0.0739	3,31,0.0739
(1.4,1.9)	3,30,0.0766	3,29,0.0794	3,31,0.0739
(1.5,2.0)	3,21,0.1135	3,26,0.0894	3,30,0.0766
	n_0 = 7		
(0.3,0.6)	3,27,0.1689	3,25,0.1840	3,28,0.1623
(0.5,0.8)	3,31,0.1452	3,31,0.1452	3,31,0.1452
(0.6,0.9)	3,31,0.1452	3,31,0.1452	3,31,0.1452
(0.5,1.0)	3,29,0.1561	3,31,0.1452	3,31,0.1452
(0.7,1.0)	3,29,0.1561	3,31,0.1452	3,31,0.1452
(1.0,1.5)	4,31,0.1135	4,31,0.1135	4,31,0.1135
(1.0,2.0)	4,31,0.1135	4,31,0.1135	4,30,0.1178
(1.2,1.8)	3,31,0.1452	3,31,0.1452	3,31,0.1452
(1.4,1.9)	3,30,0.1505	3,29,0.1561	3,31,0.1452
(1.5,2.0)	3,21,0.2243	3,26,0.1762	3,30,0.1505

Table 7 *Optimal n_S, n_L, α of the VSS MCV chart for minimizing $EMRL_1$ when p=3, n(1)=n_S, ASS_0 ∈{5, 7}, γ_0 ∈ {0.1, 0.3,0.5}, (τ_{min}, τ_{max}) ∈ ([0.3,0.6), (0.5,0.8), (0.6,0.9), (0.5,1), (0.7,1), (1,1.5), (1,2), (1.2,1.8), (1.4,1.9), (1.5,2]) and MRL_0=250*

(τ_{min}, τ_{max})	(n_S, n_L, α) VSS MCV chart (Minimizing $EMRL_1$) n_0 = 5		
	γ_0 = 0.1	γ_0 = 0.3	γ_0=0.5
(0.3,0.6)	4,16,0.0858	4,15,0.0934	4,18,0.0739
(0.5,0.8)	4,31,0.0397	4,31,0.0397	4,31,0.0397
(0.6,0.9)	4,31,0.0397	4,31,0.0397	4,31,0.0397
(0.5,1.0)	4,31,0.0397	4,31,0.0397	4,31,0.0397
(0.7,1.0)	4,31,0.0397	4,31,0.0397	4,31,0.0397
(1.0,1.5)	4,31,0.0397	4,30,0.0411	4,31,0.0397
(1.0,2.0)	4,31,0.0397	4,31,0.0397	4,30,0.0411
(1.2,1.8)	4,31,0.0397	4,31,0.0397	4,31,0.0397
(1.4,1.9)	4,19,0.0692	4,25,0.0502	4,29,0.0426
(1.5,2.0)	4,21,0.0614	4,20,0.0650	4,26,0.0480
	n_0 = 7		
(0.3,0.6)	5,22,0.1200	5,21,0.1274	5,20,0.1357
(0.5,0.8)	4,31,0.1135	4,31,0.1135	4,31,0.1135
(0.6,0.9)	4,31,0.1135	4,31,0.1135	4,31,0.1135
(0.5,1.0)	4,29,0.1224	4,31,0.1135	4,31,0.1135
(0.7,1.0)	4,29,0.1224	4,31,0.1135	4,31,0.1135
(1.0,1.5)	5,31,0.0794	5,31,0.0794	5,31,0.0794

(1.0,2.0)	5,31,0.0794	5,30,0.0825	5,30,0.0825
(1.2,1.8)	5,28,0.0894	5,31,0.0794	5,31,0.0794
(1.4,1.9)	5,22,0.1200	5,28,0.0894	5,28,0.0894
(1.5,2.0)	5,27,0.0934	6,21,0.0692	5,27,0.0934

Table 8 *Optimal n_S, n_L, α of the VSS MCV chart for minimizing $EMRL_1$ when p=3, n(1)=n_L, ASS_0 ∈{5, 7}, γ_0 ∈ {0.1, 0.3, 0.5}, (τ_{min}, τ_{max}) ∈([0.3,0.6), (0.5,0.8), (0.6,0.9), (0.5,1), (0.7,1), (1,1.5), (1,2), (1.2,1.8), (1.4,1.9), (1.5,2]) and MRL_0=250*

(τ_{min}, τ_{max})	(n_S, n_L, α) VSS MCV chart (Minimizing $EMRL_1$) n_0 = 5		
	γ_0 = 0.1	γ_0 = 0.3	γ_0=0.5
(0.3,0.6)	4,28,0.0443	4,26,0.0480	4,29,0.0426
(0.5,0.8)	4,31,0.0397	4,31,0.0397	4,31,0.0397
(0.6,0.9)	4,31,0.0397	4,31,0.0397	4,31,0.0397
(0.5,1.0)	4,31,0.0397	4,31,0.0397	4,31,0.0397
(0.7,1.0)	4,31,0.0397	4,31,0.0397	4,31,0.0397
(1.0,1.5)	4,31,0.0397	4,31,0.0397	4,31,0.0397
(1.0,2.0)	4,30,0.0411	4,31,0.0397	4,31,0.0397
(1.2,1.8)	4,31,0.0397	4,31,0.0397	4,31,0.0397
(1.4,1.9)	4,31,0.0397	4,30,0.0411	4,31,0.0397
(1.5,2.0)	4,22,0.0581	4,27,0.0461	4,31,0.0397
	n_0 = 7		
(0.3,0.6)	4,28,0.1274	4,26,0.1387	4,29,0.1224
(0.5,0.8)	4,31,0.1135	4,31,0.1135	4,31,0.1135
(0.6,0.9)	4,31,0.1135	4,31,0.1135	4,31,0.1135
(0.5,1.0)	4,31,0.1135	4,31,0.1135	4,31,0.1135
(0.7,1.0)	4,29,0.1224	4,31,0.1135	4,31,0.1135
(1.0,1.5)	5,31,0.0794	5,31,0.0794	5,31,0.0794
(1.0,2.0)	5,31,0.0794	5,31,0.0794	5,29,0.0858
(1.2,1.8)	4,31,0.1135	4,31,0.1135	4,31,0.1135
(1.4,1.9)	4,31,0.1135	4,30,0.1178	4,30,0.1178
(1.5,2.0)	4,22,0.1689	4,27,0.1328	4,31,0.1135

3.3 VSS MCV MRL Chart Percentiles

A VSS MCV chart designed based on MRL and an initial sample of either $n(1)=n_S$ or $n(1)=n_L$ is adopted to facilitate this analysis. Tables 9 - 12 display downward and upward optimal chart parameters ($\theta_{0.05}$, MRL_1,$\theta_{0.95}$) and corresponding values for the STD MCV when considering MRL_0 = 250, γ_0 ∈ {0.10, 0.30, 0.50}, ASS_0 ∈ {5,7}, and τ ∈ {0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.1, 1.2,1.3,1.4,1.5,1.6,2}. These tables are a valuable resource for practitioners seeking to implement an MRL and EMRL-based design for the VSS MCV chart as well as the STD MCV.

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Table 9 *Comparison $\theta_{0.05}$, MRL, and $\theta_{0.95}$ of STD MCV with VSS MCV when p=2, n(1)=n_S, $ASS_0 \in \{5, 7\}$, $\gamma_0 \in \{0.1, 0.3, 0.5\}$, $\tau \in \{0.3,0.4,0.5,0.6,0.7, 0.8,0.9,1.1,1.2,1.3,1.4,1.5,1.6,2\}$ and MRL_0=250*

τ	($\theta_{0.05}$, MRL,$\theta_{0.95}$) STD MCV MRL			($\theta_{0.05}$, MRL,$\theta_{0.95}$) VSS MCV MRL		
			n_0=5			
	γ_0=0.1	γ_0=0.3	γ_0=0.5	γ_0=0.1	γ_0=0.3	γ_0=0.5
0.3	1,9,39	1,9,36	1,9,39	2,2,4	2,2,4	2,2,5
0.4	2,21,88	2,19,80	2,21,87	2,3,8	2,3,8	,2,3,8
0.5	3,38,163	3,35,151	3,38,164	2,4,11	2,4,11	2,5,14
0.6	5,63,273	5,59,255	5,63,272	2,7,21	2,7,21	2,8,26
0.7	8,97,417	7,92,394	8,97,416	3,14,51	3,14,53	3,17,66
0.8	11,140,602	10,134,577	11,138,597	7,70,297	6,60,256	7,71,304
0.9	15,190,820	14,187,808	15,191,825	14,181,779	14,174,749	14,180,777
1.1	6,81,347	7,85,368	8,97,418	6,77,332	7,82,354	8,95,411,
1.2	3,35,148	3,38,163	4,48,204	3,23,96.	3,27,114	4,37,158
1.3	2,18,76	2,21,87	2,27,117	2,9,35	2,11,44	2,16,66
1.4	1,11,45	1,13,53	2,18,75	2,6,22	2,7,26	2,9,36
1.5	1,7,30	1,9,36	1,13,53	2,4,14	2,5,18	,2,6,22
1.6	1,5,21	1,6,26	1,9,39	1,3,10	1,4,13	2,5,18
2	1,2,9	1,3,11	1,5,18	1,2,7	1,2,6	1,3,9
			n_0=7			
0.3	1,2,8	1,2,8	1,2,9	2,2,6	2,2,5	2,2,47
0.4	1,6,23	1,5,22	1,6,25	2,2,5	2,2,5	2,2,5
0.5	1,14,58	1,13,53	2,14,60	2,3,8	2,3,8	2,3,8
0.6	3,29,126	2,27,116	3,30,128	2,4,11	2,4,11	2,5,14
0.7	5,57,243	4,53,227	5,57,245	3,9,28	2,8,26	3,9,30
0.8	8,101,435	7,95,408	8,100,430	5,38,159	4,32,132	5,39,164
0.9	12,162,700	12,160,688	13,163,704	12,153,657	11,139,596	12,147,634
1.1	5,68,290	6,73,313	7,84,362	5,61,263	6,67,289	7,80,345
1.2	2,26,110	3,29,125	3,37,160	3,16,65	3,19,79	3,27,114
1.3	1,13,53	2,15,62	2,20,86	2,7,26	2,8,30	2,11,43
1.4	1,7,30	1,9,36	1,13,53	2,4,13	2,5,17	2,6,22
1.5	1,5,19	1,6,24	1,9,36	1,3,9	2,4,13	2,5,17
1.6	1,3,13	1,4,17	1,6,26	1,3,10	1,3,9	2,4,13
2	1,2,5	1,2,7	1,3,11	1,2,5	1,2,5	1,2,6

Table 10 *Comparison $\theta_{0.05}$, MRL, and $\theta_{0.95}$ of STD MCV with VSS MCV when p=2, n(1)=n_L, $ASS_0 \in \{5, 7\}$, $\gamma_0 \in \{0.1, 0.3, 0.5\}$, $\tau \in \{0.3,0.4,0.5,0.6,0.7, 0.8,0.9,1.1,1.2,1.3,1.4,1.5,1.6,2\}$ and MRL_0=250*

τ	($\theta_{0.05}$, MRL,$\theta_{0.95}$) STD MCV MRL			($\theta_{0.05}$, MRL,$\theta_{0.95}$) VSS MCV MRL		
	n_0=5					
	γ_0=0.1	γ_0=0.3	γ_0=0.5	γ_0=0.1	γ_0=0.3	γ_0=0.5
0.3	1,9,39	1,9,36	1,9,39	1,1,5	1,1,4	1,1,3
0.4	2,21,88	2,19,80	2,21,87	1,1,5	1,1,5	1,1,4
0.5	3,38,163	3,35,151	3,38,164	1,1,4	1,1,5	1,1,4
0.6	5,63,273	5,59,255	5,63,272	1,1,5	1,1,5	1,1,6
0.7	8,97,417	7,92,394	8,97,416	4,1,38	4,1,41	6,1,53
0.8	11,140,602	10,134,577	11,138,597	63,2,290,	52,1,248,	64,1,296,
0.9	15,190,820	14,187,808	15,191,825	180,13,779	171,11,746	178,12,776
1.1	6,81,347	7,85,368	8,97,418	74,4,328	80,5,353	93,6,408
1.2	3,35,148	3,38,163	4,48,204	18,1,91	23,1,112	33,1,154
1.3	2,18,76	2,21,87	2,27,117	3,1,29	6,1,39	11,1,61
1.4	1,11,45	1,13,53	2,18,75	1,1,12	2,1,19	4,1,30
1.5	1,7,30	1,9,36	1,13,53	1,1,9	1,1,10	2,1,18
1.6	1,5,21	1,6,26	1,9,39	1,1,7	1,1,8	1,1,10
2	1,2,9	1,3,11	1,5,18	1,1,5	1,1,5	1,1,6
	n_0=7					
0.3	1,2,8	1,2,8	1,2,9	1,1,5	1,1,4	1,1,3
0.4	1,6,23	1,5,22	1,6,25	1,1,5	1,1,5	1,1,4
0.5	1,14,58	1,13,53	2,14,60	1,1,4	1,1,5	1,1,4
0.6	3,29,126	2,27,116	3,30,128	1,1,5	1,1,5	1,1,5
0.7	5,57,243	4,53,227	5,57,245	4,1,22	3,1,18	4,1,25
0.8	8,101,435	7,95,408	8,100,430	34,2,154	27,1,127	35,1,159
0.9	12,162,700	12,160,688	13,163,704	152,11,656	136,9,593	144,9,629
1.1	5,68,290	6,73,313	7,84,362	59,3,260	65,4,286	78,5,342
1.2	2,26,110	3,29,125	3,37,160	12,1,60	16,1,76	23,1,107
1.3	1,13,53	2,15,62	2,20,86	3,1,20	4,1,25	7,1,39
1.4	1,7,30	1,9,36	1,13,53	1,1,8	2,1,13	3,1,19
1.5	1,5,19	1,6,24	1,9,36	1,1,6	1,1,7	2,1,12
1.6	1,3,13	1,4,17	1,6,26	1,1,5	1,1,6	1,1,7
2	1,2,5	1,2,7	1,3,11	1,1,5	1,1,5	1,1,5

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Table 11 *Comparison $\theta_{0.05}$, MRL, and $\theta_{0.95}$ of STD MCV with VSS MCV when p=3, n(1)=n_S, $ASS_0 \in \{5, 7\}$, $\gamma_0 \in \{0.1, 0.3, 0.5\}$, $\tau \in \{0.3,0.4,0.5,0.6,0.7, 0.8,0.9,1.1,1.2,1.3,1.4,1.5,1.6,2\}$ and MRL_0=250*

τ	($\theta_{0.05}$, MRL, $\theta_{0.95}$) STD MCV MRL			($\theta_{0.05}$, MRL, $\theta_{0.95}$) VSS MCV MRL		
			n_0=5			
	γ_0=0.1	γ_0=0.3	γ_0=0.5	γ_0=0.1	γ_0=0.3	γ_0=0.5
0.3	2,26,112	2,24,104	2,26,111	2,3,7	2,3,7	2,3,7
0.4	4,45,194	4,43,183	4,45,195	2,4,11	2,4,11	2,5,14
0.5	6,69,297	5,66,283	6,70,301	2,7,20	2,7,20	2,7,22
0.6	8,98,422	7,94,405	8,99,426	3,12,42	3,12,43	3,13,50
0.7	10,131,563	10,126,543	10,132,567	4,29,119	4,28 115	4,35,144
0.8	13,168,724	13,164,706	13,168,723	9,114,491	9,101,434	9,114,490
0.9	16,207,891	16,206,889	16,209,903	16,202,873	15,199,860	16,203,877
1.1	7,91,391	8,96,411	8,107,462	7,90,389	8,96,411	9,108,463
1.2	4,42,180	4,46,197	5,56,242	3,32,137	4,37,159	4,48,206
1.3	2,23,98	2,26,111	3,34,146	2,13,54	2,16,67	3,23,98
1.4	2,14,60	2,17,70	2,23,98	2,8,31	2,9,36	2,13,53
1.5	1,10,41	1,12,48	2,17,71	2,6,23	2,7,27	2,9,36
1.6	1,7,30	1,9,36	1,13,54	1,5,19	1,5,18	2,7,27
2	1,3,12	1,4,16	1,7,27	1,3,10	1,3,10	1,4,14
			n_0=7			
0.3	1,4,17	1,4,15	1,4,17	2,2,6	2,2,5	2,2,4
0.4	1,11,44	1,10,40	1,11,45	2,2,5	2,2,5	2,2,5
0.5	2,23,96	2,21,88	2,23,97	2,3,8	2,3,8	2,3,8
0.6	4,43,185	3,40,171	4,43,185	2,5,15	2,5,14	2,6,18
0.7	6,74,318	6,69,299	6,74,319	3,11,37	3,10,35	3,12,44
0.8	9,119,513	9,113,486	9,118,508	6,51,215	5,43,180	5,52,221
0.9	13,175,754	13,173,747	14,177,762	13,168,723	12,156,671	13,163,703
1.1	6,73,316	6,79,339	7,91,390	6,68,293	6,74,319	6,88,380
1.2	3,30,126	3,33,142	4,42,181	3,19,79	3,23,97	4,32,135
1.3	2,15,63	2,17,73	2,24,100	2,8,30	2,9,35	2,13,53
1.4	1,9,36	1,10,44	2,15,63	2,5,17	2,6,22	2,8,31
1.5	1,6,23	1,7,29	1,10,43	2,4,13	2,4,13	2,5,18
1.6	1,4,16	1,5,21	1,8,32	1,3,9	1,3,10	1,4,14
2	1,2,7	1,2,9	1,4,15	1,2,6	1,2,7	1,3,9

Table 12 *Comparison $\theta_{0.05}$, MRL, and $\theta_{0.95}$ of STD MCV with VSS MCV when p=3, n(1)=n_L, ASS_0 ∈{5, 7}, γ_0 ∈{0.1, 0.3, 0.5}, τ ∈{0.3,0.4,0.5,0.6,0.7, 0.8,0.9,1.1,1.2,1.3,1.4,1.5,1.6,2} and MRL_0=250*

τ	**($\theta_{0.05}$, MRL,$\theta_{0.95}$) STD MCV MRL**			**($\theta_{0.05}$, MRL,$\theta_{0.95}$) VSS MCV MRL**		
				n_0=5		
	γ_0=0.1	**γ_0=0.3**	**γ_0=0.5**	**γ_0=0.1**	**γ_0=0.3**	**γ_0=0.5**
0.3	2,26,112	2,24,104	2,26,111	1,1,3	1,1,4	1,1,3
0.4	4,45,194	4,43,183	4,45,195	1,1,5	1,1,5	1,1,4
0.5	6,69,297	5,66,283	6,70,301	1,1,5	1,1,5	1,1,5
0.6	8,98,422	7,94,405	8,99,426	1,1,8	1,1,13	1,1,20
0.7	10,131,563	10,126,543	10,132,567	8,1,95	7,1,92	16,1,125
0.8	13,168,724	13,164,706	13,168,723	104,2,477	90,1,423	104,2,480
0.9	16,207,891	16,206,889	16,209,903	202,15,872	196,13,855	200,13,873
1.1	7,91,391	8,96,411	8,107,462	87,5,384	93,5,409	106,7,465
1.2	4,42,180	4,46,197	5,56,242	25,1,128	31,1,151	44,1,203
1.3	2,23,98	2,26,111	3,34,146	5,1,46	9,1,59	17,1,92
1.4	2,14,60	2,17,70	2,23,98	2,1,24	2,1,28	5,1,45
1.5	1,10,41	1,12,48	2,17,71	1,1,13	1,1,15	2,1,27
1.6	1,7,30	1,9,36	1,13,54	1,1,10	1,1,11	1,1,16
2	1,3,12	1,4,16	1,7,27	1,1,6	1,1,7	1,1,9
			n_0=7			
0.3	1,4,17	1,4,15	1,4,17	1,1,3	1,1,4	1,1,3
0.4	1,11,44	1,10,40	1,11,45	1,1,5	1,1,5	1,1,4
0.5	2,23,96	2,21,88	2,23,97	1,1,4	1,1,5	1,1,4
0.6	4,43,185	3,40,171	4,43,185	1,1,5	1,1,5	1,1,5
0.7	6,74,318	6,69,299	6,74,319	4,1,26	3,1,24	5,1,36
0.8	9,119,513	9,113,486	9,118,508	46,2,210	37,1,174	47,2,215
0.9	13,175,754	13,173,747	14,177,762	167,13,722	154,10,669	161,10,700
1.1	6,73,316	6,79,339	7,91,390	66,4,291	72,4,317	86,6,376
1.2	3,30,126	3,33,142	4,42,181	15,1,74	19,1,91	28,1,130
1.3	2,15,63	2,17,73	2,24,100	3,1,23	5,1,32	9,1,48
1.4	1,9,36	1,10,44	2,15,63	2,1,14	2,1,15	3,1,22
1.5	1,6,23	1,7,29	1,10,43	1,1,7	1,1,8	2,1,15
1.6	1,4,16	1,5,21	1,8,32	1,1,6	1,1,6	1,1,8
2	1,2,7	1,2,9	1,4,15	1,1,5	1,1,5	1,1,6

3.4 VSS MCV EMRL Chart Percentiles

VSS MCV chart with the EMRL-based design and an initial sample $n(1)=n_S$ or $n(1)=n_L$ is adopted. Tables 13 to 16 provide a comprehensive representation to the expected 5th,50th and 95th percentiles $E\theta_{0.05}$, $EMRL_1$, and $E\theta_{0.95}$ of the VSS MCV chart and STD MCV when MRL_0 = 250, values of γ_0 ∈ {0.1, 0.3, 0.5}, ASS_0 ∈{5, 7}, p∈(2, 3), and (τ_{min}, τ_{max}) ∈ ([0.3,0.6), (0.5,0.8), (0.6,0.9), (0.5,1), (0.7,1), (1,1.5), (1,2), (1.2,1.8), (1.4,1.9), (1.5,2]).

Table 13 *Comparison $E\theta_{0.05}$, EMRL, and $E\theta_{0.95}$ of STD MCV with VSS MCV when p=2, $ASS_0 \in\{5, 7\}$, n(1)=n_S $\gamma_0 \in \{0.1, 0.3, 0.5\}$, $(\tau_{min}, \tau_{max}) \in$([0.3,0.6), (0.5,0.8), (0.6,0.9), (0.5,1), (0.7,1), (1,1.5), (1,2), (1.2,1.8), (1.4,1.9),(1.5,2] and MRL_0=250*

(τ_{min}, τ_{max})	($\theta_{0.05}$, EMRL, $\theta_{0.95}$) STD MCV MRL			($\theta_{0.05}$, EMRL, $\theta_{0.95}$) VSS MCV MRL		
	n_0=5					
	γ_0=0.1	γ_0=0.3	γ_0=0.5	γ_0=0.1	γ_0=0.3	γ_0=0.5
(0.3,0.6)	2.73,31.12,132.93	2.58,29,123.32	2.73,31.21,133.08	2,4.0708,11.0460	2,4.07,11.13	2,4.3279,11.8505
(0.5,0.8)	6.51,82.08,353	6.26,77.91,334.75	6.51,82.27,353.9	2.94,16.82,65.12	2.82,15.44,60.21	3.04,18.53,73.06
(0.6,0.9)	9.32,119.78,516.6	9.12,115.39,496.6	9.32,120.27,518.3	5.66,58.23,244.77	5.4,52.33,219.68	5.75,58.30,245.42
(0.5,1.0)	9.84,125.68,541.07	9.45,121.83,525.3	9.84,125.98,542.6	7.33,80.21,341.32	6.95,76.55,325.71	7.15,80.05,340.61
(0.7,1.0)	12.95,167.4,722.03	12.48,163,702.5	12.82,166.73,719.3	10.61,128.37,550.4	10.03,121.7,522.3	10.55,127.7,547.9
(1.0,1.5)	4.03,49.38,214.52	4.21,52.37,226.37	4.83,59.97,257.5	4.20,44.14,187.47	4.330,46.86,200.09	4.93,53.75,228.4
(1.0,2.0)	2.59,26.61,119.28	2.59,28.66,126.90	2.93,33.80,146.74	2.626,23.487,98.76	2.69,25.13,105.06	3.20,29.06,121.10
(1.2,1.8)	1.30,10.46,43.29	1.36,11.91,50.16	1.67,16.53,69.73	1.60,6.24,22.27	1.753,7.275,26.32	2.17,9.72,37.20
(1.4,1.9)	1,5.24,21.13	1,6.25,25.49	1.10,9.25,38.73	1.015,3.47,10.64	1.104,3.962,12.65	1.35,5.22,18.04
(1.5,2.0)	1,4.07,15.61	1,4.83,19.12,	1,7.32,30.10	1,2.823,8.41	1,3.350,9.868,	1.10,4.28,13.62
	n_0=7					
(0.3,0.6)	1.40,11.31,47.26	1.2570,10.51,43.7	1.40,11.66,48.86	1.82,2.45,5.24	1.82,2.45,5.24	2,2.63,6.19
(0.5,0.8)	4.040,46.49,199.62	3.67,43.53,186.53	4.05,46.83,200.9	2.46,9.50,32.59	2.36,8.75,29.51	2.46,10.10,36.19
(0.6,0.9)	6.46,83.22,358	6.41,79.06,339.29	6.41,82.94,356.78	4.51,39.24,161.76	4.22,33.39,136.87	4.328,37.92,156.37
(0.5,1.0)	7.62,95.00,411.43	7.24,91.90,398.00	7.55,94.99,410.86	6.42,65.99,278.77	5.97,61.44,259.47	6.17,64.10,271.25
(0.7,1.0)	10.72,137.66,593.2	10.28,133.8,576.5	10.67,137.12,590.7	9.25,106.48,454.61	8.56,98.76,421.32	9.05,103.53,442.33
(1.0,1.5)	3.49,42.14,185.14	3.84,45.07,196.71	4.12,51.86,223.96	3.82,37.38,156.99	3.84,39.97,169.09	4.43,45.89,194.41
(1.0,2.0)	2.24,22.44,101.65	2.37,24.07,108.58	2.59,28.36,125.53	2.48,19.72,81.52	2.54,21.35,87.75	2.83,24.54,101.24
(1.2,1.8)	1.17,7.22,29.43	1.18,8.51,35.26	1.36,11.84,49.89	1.35,4.45,14.18	1.25,5.20,17.81	1.60,6.93,24.58
(1.4,1.9)	1,3.41,13.18	1,4.29,16.66	1,6.38,25.82	1,2.553,6.89	1,2.874,7.88	1.35,3.81,10.77
(1.5,2.0)	1,2.67,9.72	1,3.2,12.40	1,5.02,19.60	1,2.257,5.510	1,2.46,6.41	1,3.133,9.501

Table 14 *Comparison $E\theta_{0.05}$, EMRL, and $E\theta_{0.95}$ of STD MCV with VSS MCV when p=2, n(1)=n_L ,$ASS_0 \in\{5, 7\}$,$\gamma_0 \in \{0.1, 0.3, 0.5\}$, $(\tau_{min}, \tau_{max}) \in$([0.3,0.6), (0.5,0.8), (0.6,0.9), (0.5,1), (0.7,1), (1,1.5), (1,2), (1.2,1.8), (1.4,1.9), (1.5,2])and MRL_0=250*

(τ_{min}, τ_{max})	($\theta_{0.05}$, EMRL, $\theta_{0.95}$) STD MCV MRL			($\theta_{0.05}$, EMRL, $\theta_{0.95}$) VSS MCV MRL		
	n_0=5					
	γ_0=0.1	γ_0=0.3	γ_0=0.5	γ_0=0.1	γ_0=0.3	γ_0=0.5
(0.3,0.6)	2.73,31.12,132.93	2.58,29.00,123.32	2.73,31.21,133.08	1,1,1.643,	1,1,1.876,	1,1,1.793,
(0.5,0.8)	6.51,82.08,353.00	6.26,77.91,334.75	6.51,82.27,353.90	1.015,9.13,48.62	1,7.68,43.77,	1,10.49,57.25
(0.6,0.9)	9.32,119.78,516.6	9.12,115.39,496.6	9.32,120.27,518.3	2.79,50.82,233.79	2.20,44.48,207.97	2.50,50.55,235
(0.5,1.0)	9.84,125.68,541.0	9.45,121.83,525.3	9.84,125.98,542.6	5.18,74.85,331.30	4.66,70.69,314.47	4.84,74.01,330.03
(0.7,1.0)	12.95,167.40,722	12.48,163.01,702	12.82,166.73,719	7.87,123.50,545	7.18,116.29,516.6	7.38,122.47,542.7
(1.0,1.5)	4.03,49.38,214.52	4.21,52.37,226.37	4.83,59.97,257.57	2.96,40.17,181.70	3.11,43.13,195.65	3.41,49.91,224.43
(1.0,2.0)	2.59,26.61,119.28	2.59,28.66,126.90	2.93,33.80,146.74	1.95,20.51,92.25	2.07,22.01,98.53	2.23,25.53,116.40
(1.2,1.8)	1.30,10.46,43.29	1.36,11.91,50.16	1.67,16.53,69.73	1,2.55,14.52	1,3.37,18.94	1,5.30,30.537

(1.4,1.9)	1,5.24,21.13	1,6.25,25.49	1.10,9.25,38.73	1,1,3.462	1,1.104,5.309	1,1.361,9.22
(1.5,2.0)	1,4.07,15.61	1,4.83,19.12,	1,7.32,30.10	1,1,3.501	1,1,3.675	1,1.104,5.86
			n_0=7			
(0.3,0.6)	1.40,11.31,47.26	1.25,10.51,43.77	1.40,11.6,48.86	1,1,1.64	1,1,1.82	1,1,1.74
(0.5,0.8)	4.04,46.49,199.62	3.67,43.53,186.53	4.05,46.83,200.96	1.01,5.32,24.54	1,4.45,21.22	1,6.01,28.28
(0.6,0.9)	6.46,83.22,358	6.4,79.06,339.2	6.41,82.94,356.78	2.45,34.87,155.50	1.92,28.88,130.65	2.22,33.27,150.4
(0.5,1.0)	7.62,95.00,411.43	7.24,91.90,398.00	7.55,94.99,410.86	4.80,62.71,273.74	4.28,57.85,253.58	4.46,60.64,265.5
(0.7,1.0)	10.72,137.66,593	10.28,133.88,576.	10.67,137.12,590	7.22,103.66,451.8	6.41,95.38,417.52	6.78,100.22,439.1
(1.0,1.5)	3.49,42.14,185.14	3.84,45.07,196.71	4.12,51.86,223.96	2.8,34.73,153.75	2.87,37.34,165.19	3.20,43.08,190.70
(1.0,2.0)	2.24,22.44,101.65	2.37,24.07,108.58	2.59,28.36,125.53	1.84,17.86,77.66	1.98,19.13,83.87	2.14,22.17,97.79
(1.2,1.8)	1.17,7.22,29.43	1.18,8.51,35.26	1.36,11.84,49.89	1,2.08,9.73	1,2.61,12.63	1,4.01,20.23
(1.4,1.9)	1,3.41,13.18	1,4.29,16.66	1,6.38,25.82	1,1,2.77	1,1.10,3.86	1,1.30,6.25
(1.5,2.0)	1,2.67,9.72	1,3.22,12.40	1,5.02,19.60	1,1,2.86	1,1,2.94	1,1.10,4.14

Table 15 *1 Comparison $E\theta_{0.05}$, EMRL, and $E\theta_{0.95}$ of STD MCV with VSS MCV when p=3, n(1)=n_S , ASS_0 ∈{5, 7}, γ_0 ∈{0.1, 0.3, 0.5}, (τ_{min}, τ_{max}) ∈([0.3,0.6), (0.5,0.8), (0.6,0.9), (0.5,1), (0.7,1), (1,1.5), (1,2), (1.2,1.8), (1.4,1.9), (1.5,2]) and MRL_0=250*

(τ_{min}, τ_{max})	($\theta_{0.05}$, EMRL, $\theta_{0.95}$) STD MCV MRL			($\theta_{0.05}$, EMRL, $\theta_{0.95}$) VSS MCV MRL		
			n_0=5			
	γ_0=0.1	γ_0=0.3	γ_0=0.5	γ_0=0.1	γ_0=0.3	γ_0=0.5
(0.3,0.6)	4.81,58.37.41	4.57,55.51,238.7	4.85,58.9,253.03	2.17,6.274,19.67	2.17,6.328,20.00	2.173,7.001,23.57
(0.5,0.8)	8.95,114.68,494.2	8.72,111.46,479	9.05,116.28,501.3	3.95,31.02,127.64	3.58,28.92,119.45	4.128,33.886,141.3
(0.6,0.9)	11.46,149.1,642.6	11.28,146.15,629	11.67,151.29,651	7.559,83.158,353.	7.01,76.29,324.55	7.4,84.096,358.4
(0.5,1.0)	11.72,151.33,652	11.62,148.94,641	11.85,153.24,659	8.39,98.35,420.6	8.24,93.55,399.16	8.30,98.53,421.77
(0.7,1.0)	14.25,188.35,812	14.13,185.40,799	14.35,188.89,814	12.51,155.51,669.	11.742,145.869,62	12.32,153.84,661.2
(1.0,1.5)	4.63,55.36,239.1	4.77,58.46,252.3	5.57,67.08,288.1	4.78,49.932,212.	4.880,53.295,227	5.330,60.9,260.48
(1.0,2.0)	2.78,30.42,134.4	2.93,32.82,143.4	3.25,38.72,166.8	2.88,26.99,112.7	2.95,28.73,120.6	3.56,33.38,140.4
(1.2,1.8)	1.45,13.54,56.74	1.63,15.57,65.3	2.02,21.18,90.1	1.65,8.514,32.2	1.769,9.92,38.4	2.292,13.50,54.1
(1.4,1.9)	1.01,7.14,29.22	1.104,8.55,35.1	1.35,12.62,53.36	1.257,4.6,15.22	1.350,5.22,17.9	1.742,7.050,25.4
(1.5,2.0)	1,5.54,21.91	1,6.60,26.91	1.10,10.13,42.1	1.05,3.73,11.86	1.173,4.28,13.8	1.550,5.60,19.7
			n_0=7			
(0.3,0.6)	1.82,18.39,77.70	1.72,16.84,71.5	1.82,18.62,78.7	2,2.911,7.05	2,2.911,7.05	2,3.224,7.67
(0.5,0.8)	4.92,61.49,264.5	4.72,57.85,248	5.02,61.59,264	2.72,12.18,44.45	2.574,11.05,40	2.673,13.24,49.5
(0.6,0.9)	7.90,99.82,429.5	7.39,95.23,410	7.91,99.57,428	5.22,47.75,198.8	4.609,41.13,17.2	5.055,46.4,193.5
(0.5,1.0)	8.45,108.57,467	8.30,105.44,454	8.45,108.7,467.9	6.92,73.21,310.2	6.42,67.51,285.8	6.61,71.20,301.5
(0.7,1.0)	11.59,151.27,652	11.53,147.79,637	11.72,151.03,650	10.0,118.98,509	9.34,108.29,46.9	9.76,114.5,489.9
(1.0,1.5)	3.84,45.27,197.8	4.03,48.36,210.3	4.63,55.65,239.8	4.05,40.24,169.2	4.27,42.99,181.9	4.78,49.44,209.8
(1.0,2.0)	2.42,24.23,109	2.59,26.09,116.9	2.74,30.81,135.5	2.54,21.40,88.1	2.62,22.98,95.13	,2.88,26.55,110.4
(1.2,1.8)	1.18,8.51,35.09	1.30,10.19,41.7	1.46,13.97,59.2	1.36,5.21,17.73	1.499,6.150,21	1.7539,8.32,30.8
(1.4,1.9)	1,4.22,16.53	1,5.22,20.53	1.05,7.86,31.79	1.01,2.957,8.22	1.10,3.365,9.72	1.350,4.293,13.6
(1.5,2.0)	1. 3.18,12.18	1,3.98,15.34	1,6.12,24.35	1,2.464,6.67	1,2.823,8.12	1.104,3.623,10.5

Table 16 *1 Comparison E$\theta_{0.05}$, EMRL, and E$\theta_{0.95}$ of STD MCV with VSS MCV when p=3, n(1)=n_L,ASS_0 ∈{5, 7}, γ_0 ∈{0.10, 0.30, 0.50}, (τ_{min}, τ_{max}) ∈([0.3,0.6), (0.5,0.8), (0.6,0.9), (0.5,1), (0.7,1), (1,1.5), (1,2), (1.2,1.8), (1.4,1.9), (1.5, 2]) and MRL_0=250*

(τ_{min}, τ_{max})	($\theta_{0.05}$, EMRL,$\theta_{0.95}$) STD MCV MRL			($\theta_{0.05}$, EMRL,$\theta_{0.95}$) VSS MCV MRL		
	n_0=5					
	γ_0=0.1	γ_0=0.3	γ_0=0.5	γ_0=0.1	γ_0=0.3	γ_0=0.5
(0.3,0.6)	4.81,58.37. 41	4.57,55.51,238	4.85,58.90,253.03	1,1,1.709	1,1,2.352	1,1,2.712
(0.5,0.8)	8.95,114.6,494.2	8.72,111.4,479	9.05,116.28,501.35	1.01,17.6,99.	1,15.04,90.3	1,19.8,113.9
(0.6,0.9)	11.4,149.1,642.6	11.28,146.1,629	11.67,151.29,651.91	3.34,71.36,336.33	2.67,63.45,306.83	3.02,71.58,342.94
(0.5,1.0)	11.7,151.3,652.6	11.6,148.9,641	11.85,153.24,659.9	5.58,89.07,402.22	5.08,83.31,380.29	5.28,88.58,404.02
(0.7,1.0)	14.2,188.3,812.9	14.13,185.4,799	14.35,188.89,814.78	8.88,148.5,661.91	7.8,138.04,619.27	8.19,146.45,653.78
(1.0,1.5)	4.6,55.36,239.1	4.77,58.46,252	5.57,67.08,288.13	3.11,44.74,205.7	3.11,47.99,220.67	3.49,56.09,255.28
(1.0,2.0)	2.78,30.4,134.4	2.93,32.82,143	3.25,38.72,166.84	2.074,22.90,104.44	2.128,24.47,112.39	2.28,28.44,132.14
(1.2,1.8)	1.45,13.54,56.7	1.63,15.57,65.3	,2.02,21.18,90.17	1,3.34,22.03	1,4.50,28.63	1,7.54,45.66
(1.4,1.9)	1.01,7.14,29.22	1.104,8.5,35.12	1.35,12.62,53.36	1,1,4.763	1,1.104,7.874	1,1.520,15.243
(1.5,2.0)	1,5.5455,21.91	1,6.60,26.9189	1.104,10.13,42.19	1,1,4.43	1,1,5.08	1,1.104,9.090
	n_0=7					
(0.3,0.6)	1.82,18.39,77.7	1.72,16.84,71.5	1.82,18.62,78.70	1,1,1.643	1,1,1.86	1,1,1.77
(0.5,0.8)	4.9,61.49,264.5	4.72,57.85,248	5.02,61.59,264.66	1.01,7.01,34.05	1,5.91,29.62	1,7.795,39.4
(0.6,0.9)	7.9,99.82,429.5	7.39,95.23,410	7.9198,99.57,428.5	2.71,42.63,191.31	2.10,35.47,162.56	2.40,40.96,186.29
(0.5,1.0)	8.4,108.57,467.8	8.30,105.44,454	8.45,108.75,467.93	5.01,69.50,304.54	4.36,63.17,278.19	4.766,66.89,294.53
(0.7,1.0)	11.5,151.27,652	11.53,147.7,637	11.72,151.03,650.9	7.81,115.54,505.5	6.8,104.29,458.8	7.13,110.56,485.93
(1.0,1.5)	3.84,45.27,197.8	4.03,48.3,210	4.63,55.65,239.88	2.8,37.139,165.7	3.04,39.98,178.62	3.292,46.53,206.54
(1.0,2.0)	2.42,24.23,109	2.59,26.09,116	2.74,30.81,135.54	1.91,19.01,83.91	2.07,20.42,90.72	2.19,23.89,106.81
(1.2,1.8)	1.18,8.51,35.09	1.3,10.19,41.7	1.46,13.97,59.26	1,2.25,11.66	1,2.943,15.490	1,4.806,24.912
(1.4,1.9)	1,4.22,16.5	1,5.2294,20.53	1.0505,7.86,31.79	1,1,2.881	1,1.104,4.462	1,1.454,8.073
(1.5,2.0)	,1,3.18,12.18	1,3.9816,15.34	1,6.1252,24.3513	1,1,2.999	1,1,3.2	1,1.1,4.8

3.5 Performance Comparison for MCV Shift of Known Size

Tables 9 to 12 compare the performance of two optimal VSS schemes: the STD MCV chart when the shift size τ is known a priori. It is crucial to note that in the scope of this study, the parameters are set as follows: MRL_0 = 250, γ_0={0.1, 0.3, 0.5}, p={2, 3}, n_0 = {5, 7} and τ ∈{0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 2}. In these tables, $\theta_{0.05}$ and $\theta_{0.95}$ denote the 5th and 95th percentiles of the run-length distribution, serving as indicators of the distribution's variation and spread. The values of $\theta_{0.05}$ and $\theta_{0.95}$ are provided in these tables to measure the variation and spread of the run-length distribution. The charting parameters for both the STD MCV and optimal VSS MCV charts are determined when MRL_0 =250 and n_0={5, 7}. As shown in Table 9, with the parameters set at p=2, n(1)=n_S, n_0=7, τ = 0.50, γ_0=0.3, the values of ($\theta_{0.05}$, MRL, $\theta_{0.95}$) for downward of STD MCV and VSS MCV are (1,13,53) and (2,3,8) respectively and for upward with τ = 1.4, the values are (1,9,36), (2,5,17) respectively.

Across all tables, a consistent observation emerges: as the γ_0 increases, the chart parameters' values also increase for all shift sizes. It is important to highlight that for small shift sizes τ ∈{0.30,0.40}, and within the range τ ∈ ([0.3,0.6),(0.5 0.8)), there is minimal or no discernible difference in optimal chart parameter values. When τ increases, MRL_1 and $EMRL_1$ increase for the downward chart and vice-versa for the upward chart. With an increase in n_0, optimal chart parameter values decreased for all shift sizes. Furthermore, when p increases, optimal chart parameters values increase, MRL and EMRL values and the disparity between $\theta_{0.05}$ and $\theta_{0.95}$ of VSS MCV becomes smaller for n(1)=n_L compared to VSS MCV charts with n(1)=ns and STD MCV.

3.6 Performance Comparison for MCV Shift of Unknown Size

Tables 13 to 16 show the comparison of STD MCV with VSS MCV based on EMRL for a range of shift size (τ_{min}, τ_{max}) ∈ ([0.3,0.6), (0.5,0.8), (0.6,0.9), (0.5,1), (0.7,1), (1,1.5), (1,2), (1.2,1.8), (1.4,1.9), (1.5,2]) for both side of control chart considered. The values of (E$\theta_{0.05}$, EMRL$_1$, E$\theta_{0.95}$) for VSS MCV and STD MCV control charts compared for different shift sizes, n_0={5, 7}, γ_0 = {0.1, 0.3, 0.5}, p={2, 3} and MRL$_0$=250. For example, in Table 14 for p=2, $n(1)$=n_L, γ_0 = 0.5, (τ_{min}, τ_{max}) = (0.6,0.9) and n_0=5, the values of (E$\theta_{0.05}$, EMRL$_1$, E$\theta_{0.95}$) for downward of VSS MCV and STD MCV control charts are (9.32,120.27,518.3) and (2.50,50.55,235) respectively and for upward with (τ_{min},τ_{max}) = (1.2,1.8) the values are (1.67,16.53,69.73) and (1,5.30,30.537), respectively.

In Table 13, it is observed that the values of (E$\theta_{0.05}$, EMRL$_1$, E$\theta_{0.95}$) decrease as the range for (τ_{min}, τ_{max}) becomes more focused on larger values of τ. For example, for γ_0 = 0.10 and n_0 = 5,(E$\theta_{0.05}$, EMRL$_1$, E$\theta_{0.95}$) = (7.331, 80.219, 341.32) for (τ_{min}, τ_{max}) = (0.50,1.00), while (E$\theta_{0.05}$,EMRL$_1$,E$\theta_{0.95}$) = (2.626, 23.487, 98.766) when (τ_{min}, τ_{max})= (1.00, 2.00). The disparity between E$\theta_{0.05}$ and E$\theta_{0.95}$ diminishes as the shift size range becomes more concentrated towards larger τ values.

4. Discussion

The STD MCV chart has a fixed sample size of n_0. To ensure a fair comparison with the STD MCV chart in terms of the MRL and EMRL criteria, the ASS$_0$ of the VSS MCV chart is set as n_0. In this research, (ASS$_0$= n_0) = (5, 7) are considered. According to (Castagliola et al., 2015), an extremely large n_2 value is not practical in the industry. Thus, the maximum n_2 value is set as 31 to minimize the MRL$_1$, and EMRL$_1$ values, to detect the increase and decrease of MCV shifts.

In addition, as previously mentioned in the earlier sections, the same values for τ, (τ_{min}, τ_{max}), p, γ_0 and ASS$_0$, with $n(1)$ = n_S, $n(1)$ = n_L, are considered, subsequently, MRL$_0$ assumes to be 250. Tables 9 to Table 16 are helpful for practitioners who want to implement an MRL and EMRL-based design for the VSS MCV chart. For example, if the user would like to have an MRL$_0$ of 250, n_0=7 and p=2, γ_0=0.10, τ = 0.60, and $n(1)$=n_L then by referring to Table 2, the practitioner should adopt the optimal chart parameters (n_S, n_L, α, ARL, ASS, SDRL, ANOS) = (3,27,0.1689,1.94,10,1.39,19.49), with these optimal chart parameters, by referring to Table 10, the practitioner would obtain the 5th, 50th and 95th percentiles as 1, 1 and 4, respectively. For the VSS MCV chart, and by referring to Table 10, the values of ($\theta_{0.05}$, MRL, $\theta_{0.95}$) for the STD MCV chart are 1, 14 and 58, respectively. STD MCV could not optimize parameters to minimize the MRL and EMRL. This finding has significant implications, as it provides a more efficient allocation of resources in future research endeavours, leading to potential savings in both time and money.

When ASS$_0$ is larger ($\theta_{0.05}$, MRL$_1$,$\theta_{0.95}$) generally becomes smaller, this shows that fewer samples are needed to detect an out-of-control condition when a larger ASS$_0$ is taken; the difference between the 5th and 95th percentiles also generally decreases with larger ASS$_0$ for both sides of the chart in Tables 9 to 16.

The ANOS$_1$ value is presented in Table 1. The results show a decreasing trend from small to large MCV shifts; for example, from Table 1, when p= 2, n_0 = 5, γ_0=0.10, $n(1)$=n_S, from τ = 0.3 to τ= 0.9, the ANOS$_1$ values of the VSS MCV chart increase from 14.20 to 1422.9 (for minimizing MRL$_1$) and it's vice versa for upward, note that the τ which is closer to one indicates a smaller MCV shift; thus τ= 0.5 is a larger MCV shift than τ= 0.9. Based on Tables 9 to Table 16 for $n(1)$=n_S, $n(1)$=n_L, and p=2 and 3, the VSS MCV chart shows smaller values of MRL$_1$ and $\theta_{0.95}$ in comparison with the STD MCV chart for all shift sizes. This shows that varying the sample size improves the detection ability of the MCV chart; this is especially true for small values of τ and n_0. The difference between $\theta_{0.05}$ and $\theta_{0.95}$ for the VSS MCV chart is also smaller than that of the STD MCV chart, which means that the VSS MCV chart shows less variation in the run-length distribution compared with the STD MCV chart, the VSS MCV chart outperforms the STD MCV chart for all ranges of τ considered.

According to Tables 9 to Table 16, overall, for all magnitudes of shifts, the VSS MCV chart outperforms the STD MCV chart, VSS MCV has a smaller difference between 5 and 95 per cent run-length. The optimal VSS MCV chart with $n(1) = n_L$, generally has the lowest variation in the run-length distribution compared to the other competitive control charts (VSS MCV for $n(1) = n_S$ and STD MCV). In continuing for more information, asses the comparison VSS MCV chart with p=2, $n(1)$=n_L, with p=3 and $n(1)$= n_L. For downward VSS MCV chart (Table 10) with $n(1)$=n_L, p=2, n_0= 7, γ_0=0.1 τ=0.5, the MRL$_1$ and difference between 5 and 95 per cent are 1 and 3, respectively for downward with p=3, the values are 1 and 4, for upward with p=2, τ=1.4 the values are 1 and 7 for upward with p=3, τ=1.4 the values are 2 and 13. EMRL$_1$ value for downward with p=2 n_0=7 and τϵ (0.5, 1) the values are 62.718 and 268.938. The values of EMRL$_1$ for upward with p=2 and τϵ (1.4, 1.9) are 1 and 1.777. EMRL$_1$ values and the difference between 5 and 95 per cent run length for downward with p=3 are 73.214, 303.355, for upward with p=3 the values are 2.957 and 7.209, the VSS MCV showed that p=2, $n(1)$=n_L, has smaller values for upward and both side of EMRL based design.

Then downward of VSS MCV-based MRL for p=2 in comparison with p=3, the value of MRL and the difference between 5 per cent and 95 per cent of run-length are not significant while for upward, p=2 has a smaller value than p=3. For the upward and downward of the VSS MCV chart based on EMRL, p=2 has a smaller value than p=3 and overall, as a result, data obtained from our study indicate that: VSS MCV for $n(1)=n_S$ and $n(1)=n_L$ exhibits superior performance compared to the STD MCV for all shift size based on MRL_1 and $EMRL_1$. The result shows that 95 per cent of run-length distribution has a higher value than 50 per cent in all the results, saving timing and money for future research when the control chart assesses based on MRL and EMRL, with optimization parameters, and finds optimal parameters to minimize MRL_1 and $EMRL_1$.

5. Conclusion

In the existing literature, no attempt has been made to propose a VSS MCV chart based on MRL and EMRL. The culmination of this study underscores a significant departure from the prevailing literature on the VSS MCV chart. Typically, efforts to reduce the excessive ARL have been the focus of optimisation efforts for chart parameters. However, this study highlights the risks of adopting these characteristics blindly, demonstrating how doing so might result in inaccurate interpretations and jeopardize the reliability of charts. In response, an innovative design is proposed, advocating for minimising the out-of-control MRL_1 to determine optimal chart parameters. This alternative approach decreases the possibility of misunderstanding, bolstering the accuracy of the chart's interpretation. The offered tables of optimal chart parameters for the VSS MCV chart, customized to MRL and EMRL designs, equip the practitioners with valuable tools to direct their implementation. This study expands on existing work by comprehensively comparing VSS MCV and STD MCV for MRL and EMRL designs. The VSS MCV chart consistently outperforms the STD MCV chart, displaying improved sensitivity and efficiency across a range of shift magnitudes. This superiority is particularly pronounced for $n(1)=n_L$ compared to $n(1)=n_S$ regarding MRL, EMRL values, and the span between $\theta_{0.05}$ and $\theta_{0.95}$. The result has substantial cost and resource ramifications because fewer samples are needed to detect out-of-control signals. The proposed chart can be further extended for the investigation of measurement errors in future studies.

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Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

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