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Modelling and Forecasting of Crude Oil Price Return Volatility from 2006-2023: An Application of the Garch Models

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Abstract

The volatility dynamics of Nigerian crude oil prices from 2006 to 2023 were investigated in this work using the generalized autoregressive conditional heteroskedasticity (GARCH) modeling technique. Strong evidence of time-varying conditional heteroskedasticity in the dataset was found using diagnostic tests. The model information criteria showed that the dependencies were well represented by the parsimonious GARCH (1,1) formulation. Based on maximum log-likelihood and information criterion values, exponential GARCH (EGARCH) showed the best in-sample fit. The projection predicts prices beginning at a very high level in 2024 and then rapidly falling, with a steep downward slope in the expected price trajectory. The model predicts that by the end of 2025, crude oil prices will be significantly lower than they were at the start of the projection period. Overall, our empirical findings give good support for using EGARCH techniques to forecast volatility in Nigerian crude oil returns. As a result, practitioners now have access to efficient prediction tools that have been verified using rigorous statistical approaches to help them estimate future price risk.

1. Introduction

Natural crude oil, sometimes called fossil fuel or just "oil," is present in different concentrations across the globe. Processing it into useful products like petroleum motor spirit, jet fuel, diesel fuel, and petroleum gas transforms it from its unpleasant raw state. In 1956, Shell B.P. discovered crude oil at Oloibiri, Bayelsa State, Nigeria, which marked a watershed point in the country's political and economic history [1]. Crude oil is highly volatile, deflectional, and capital-intensive. Refined products are vital for poverty reduction because they encourage infrastructural development, which increases overall growth and development [2]. The significance of crude oil production to Nigeria is comparable to the indispensability of oxygen to life itself [3-4]. Despite ongoing governmental efforts, crude oil remains the driving force behind the country's economic policies [5]. Unfortunately, this heavy reliance has exposed every sector of the Nigerian economy to vulnerability, contributing to the prevailing hardships in the nation. Since the continuous deregulation of the downstream sector of Nigeria's oil industry in 2003, the collective consciousness of the average Nigerian has become considerably more cognizant of the profound impact of oil. Evaluating the impact of oil income on the Nigerian economy has become a crucial necessity. According to [6], Nigeria's oil-related fiscal income totaled $390 billion between 1971 and 2005. Nigeria had a population of around 173.6 million in 2014, making it Africa's most populated country. The country also claimed the title of Africa's greatest economy, with a GDP of $522.6 billion in 2013. In addition, Nigeria is the continent's largest oil producer. However, Nigeria's oil wealth has proven to be both a boon and a scourge. The petroleum sector has brought about significant changes in the Nigerian economy, particularly during the last five decades, supplanting agriculture as the country's economic foundation [7]. According to [8], oil and gas make up 83% of Nigeria's GDP and 90% of its foreign exchange profits. There have been several distortions and discouragements in diversifying the government's financial sources due to the overreliance on oil money. As an example, several nations' reliance on income taxes has diminished because of their abundant oil revenues. Inflationary tendencies are further reinforced by increased spending and low tax rates, especially on imported commodities. Therefore, infrastructural development, private sector investment stimulation, and advancements in the agricultural and manufacturing sectors have taken a backseat due to the reliance on petroleum revenue. However, it's worth noting that substantial proceeds from the domestic sale and export of petroleum products act as catalysts, triggering a multiplier effect across other sectors of the economy through government expenditures. The objective of this study is to provide a thorough comprehension of the volatility patterns displayed by petroleum prices in Nigeria. Through the utilization of advanced GARCH models, this research endeavors to reveal the fundamental elements influencing shifts in volatility. It also aims to pinpoint noteworthy volatility periods and offer precise forecasts regarding volatility. By accomplishing this, the research strives to offer a valuable understanding of the essence of volatility in petroleum prices. This will prove beneficial for decision-makers, stakeholders in the market, and policymakers, aiding them in adeptly managing the complexities presented by price fluctuations in the Nigerian petroleum industry.

2. Literature Review

Hansen & Lunde [9] have compared the various types of GARCH models in terms of their forecasting ability and concluded that the GARCH (1,1) model outperforms the other models in the case of financial time series data, including crude oil prices. Going by their findings, they recommended the GARCH (1,1) model for forecasting the volatility of Nigeria's crude oil prices because of the efficiency of this model. While working on the study, [10] major concern was on Volatility and Futures Price of Petroleum. The study's findings indicated that the proposed EGARCH model produced a highly accurate forecast, with a larger mean absolute error than the GARCH (1,1) model. As explained in the results and explanation sections, the asymmetrical response of volatility in the EGARCH model was helpful in the case of petroleum futures. [11] looked at the comparison between the simple GARCH (1,1) model and the Markov-Switching GARCH for the crude oil futures prices in his paper that was published in 2002. From the study, it was revealed that the Markov-Switching GARCH gave better forecasts by capturing regime-switching in volatility. This implies that structural changes and movement in the market forces have a considerable impact on the fluctuation in crude oil prices. [12] used the exponential generalized autoregressive conditional heteroskedasticity (EGARCH) model to examine daily fluctuations in oil prices. Also, from the estimated data, it is evident that the oil price series has long-run structural breakpoints, molded through asymmetric characteristics. According to [13] study, it was apparent that diverse forms of GARCH-type models offered reliable crude oil price volatility prediction. Among the tested models, IGARCH which set the conditional variance to include a unit root provided a good account for the high volatility. EGARCH also performed well because of its exigency for modeling the asymmetric effects. Similarly, [14] also examined the daily average of the spot price of crude oil for several crude oil prices, such as Nigerian Bonny Light and Forcados, the average of OPEC and non-OPEC countries, and US crude oil prices. The EGARCH (1, 1) model being used here helped the researchers observe that characteristics of the series such as the asymmetric effect and the clustering of volatility remained high in the oil price returns. Among the most sampled oil prices, the Nigerian Forcados were the most volatile, and at the same time, the most persistent. Moreover, in the context of all the assumed values of crude oil prices, the study ignored the impact of asymmetry and leverage.

Based on the characteristics of the dynamic and volatility of the crude oil prices, [15] applied the prediction models from the date between January 2, 1995, and March 11, 2010, which specifically include the daily spot price of both WTI and Brent. The selected GARCH family models were GARCH (1, 1), EGARCH (1, 1), GJR-GARCH (1, 1), APARCH (1, 1) and ARMA (1, 1). Therefore, this analysis establishes that the GJR-GARCH model performed better when it comes to the predictive ability than other models as hypothesized; this model is the most suitable to explain the return series of WTI and Brent. [16] collected daily prices in the range of June 23, 1998, to July 16, 2009. The following models of the Mixed ARCH family were used during the iterations: GARCH (1, 1); EGARCH (1, 2); GJR-GARCH (1, 2); and TGARCH (1, 2). These findings show that the proposed GJR-GARCH (1, 2) model has a higher predictive accuracy of the future market's oil price than other models. [17] used the bivariate GARCH models to test the volatility spillover effects of international oil prices on the European equity markets. In this study, it was noted that the bivariate GARCH model provided better forecasts since it incorporated the spillover effect of volatility between the markets. The results emphasized that other spillover effects are vital in managing risk and hedging mechanisms. In their study "Modeling and Predicting Crude Oil Prices in Nigeria", [18] have done a fairly good job of designing more accurate ARIMA and GARCH for precise prediction of crude oil prices. Subsequently, the data collected was utilized to evaluate which model was the most efficient. Given the diagnostic criteria, the empirical study identified the ARIMA (3, 1, 1) and GARCH (2, 1) models as the most suitable for forecasting the time series of crude oil prices. These chosen models were then used over a six-month forecast period. Lastly, their forecast exercises showed a steep rise in the prices of crude oil contrary to the overall historical trends. [19] linked four GARCH-related models: namely, GARCH (1,1), GJR-GARCH (1,1), EGARCH (1,1), and APARACH (1,1) models to examine the behavior of oil prices (the Brent and WTI crude oil markets) and their associated risk. Consequently, this research aimed to prove the hypothesis based on the arguments that there are opposite outcomes for the volatility of the two examined markets and that oil shocks are persistent.

The fluctuation of Nigeria's crude oil markets was then estimated using the GARCH Model as provided in the study by [20]. Having considered the results depicted for the two models, EGARCH (1,1) was less efficient in modeling the data as compared to GARCH (1,1). [21] sought to establish the effectiveness of various GARCH-type models in the analysis of the Volatility of Nigerian crude oil returns based on the error distribution type. This one discovers that the GARCH(1,1) model with Student's t fitted better than the model under normal distribution indicating the effect of non-normality in the data. To ascertain the efficiency of the models, [22] employed structures of ARCH and GARCH for the volatility series of Nigeria's crude oil output. The two models compared by the researchers were GARCH (1,2) and GARCH (2,1); to assess the models' forecasting, the AIC, BIC, and SIC tests were used by the researchers. Therefore, comparing AIC and BIC values, it was observed that the GARCH (1,2) fitted the crude oil production volatility data better than the GARCH (2,1). According to the findings of [23] research, it is rather challenging to predict the Nigerian crude oil production and fluctuation of the prices. Their main purpose and findings focused on the stability of the positive definiteness characteristic in multivariate time series. As established in the prior results, it was evident that the multivariate time series was stable in offering the basis for further exploration. Due to the complexity of the structural relationship between crude oil price and fluctuating production, the researchers applied specialist MARCH and MGARCH models. MARCH [p (3,1)] demonstrates a high level of performance based on well-specified model criteria, indicating the model's potential to accurately replicate specific data patterns. Not only did the study show a good positive correlation between crude oil output and price, but more importantly they showed how these macro-economic variables interact and depend on each other. Oil price shocks were thus found to have a significant and enormous impact on output fluctuations and therefore economic growth given that causality was bilateral between crude oil output and price. The existing literature on the volatility of petroleum prices, particularly within the Nigerian market, has explored various econometric approaches to model and analyze this complex phenomenon. A common thread among these studies is the utilization of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models, which have proven to be a robust and widely accepted framework for capturing the dynamic volatility characteristics of commodity prices. This study builds on previous work by examining Nigeria's oil price changes from 2006-2023 more deeply. This study employs different types of GARCH models to gain clearer insight. In addition to the basic GARCH method, EGARCH and TGARCH applies to better capture how volatility responds to price rises and falls. This research also analyzes how volatility evolved over the years. It identifies any major shifts in patterns of change over time. This gives context to what influenced the observed behavior, the selected GARCH techniques are employed to forecast future volatility. Their performance is thoroughly tested to evaluate suitability for Nigeria's market conditions. The goal is to offer policymakers, industry, and researchers' useful perspectives on managing risks from oil price fluctuations in Nigeria. Insights could help in the navigation of challenges in this important sector.

3. Materials and Methods

3.1 Data

The data employed for this study were primarily obtained from secondary sources, which include the CBN's annual statistical bulletin and NBS's series of annual reports and country reports. The study period is defined by the years 2006 to 2023. The foundation of this analysis is based on the application of GARCH models as these models are efficient for identifying essential aspects of volatility. Particular attention is paid to choosing variants of GARCH models regarding the model check and criteria for model fitting. Estimation of model parameters is conducted with the help of historical data as well as focusing on the adequacy of the model and its fitness to provide good results through diagnostic checking.

3.2 Generalized Autoregressive Conditional Heteroskedasticity (GARCH) Model

The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model is widely used in financial econometrics to model and forecast time series volatility [25-26].

3.2.1 GARCH (p, q) Model Specification

The GARCH(p,q) model is simply an extension of the GARCH (1,1) model where p and q stand for the respective number of lagged conditional variance and squared residual terms included. The GARCH (1,1) model consists of two main equations: one for the return series, and the other for the conditional variance. The basic GARCH (1,1) model, introduced by [27], is specified as follows:

Mean Equation:
$r_t = \mu + \varepsilon_t$

Where:
r_t = Return at time t
μ = Mean return
ε_t = Residual (error term) at time t.

Variance Equation:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \quad (1)$$

Where:
σ_t^2 = conditional variance at time t.
α_0 = constant term.
α_i = coefficients for the lagged squared residuals (the ARCH terms).
β_j = coefficients for the lagged conditional variances (the GARCH terms).
ϵ_{t-i}^2 = squared residuals from past periods.
σ_{t-j}^2 = lagged conditional variances.

3.2.1 EGARCH Model Specification

The Exponential GARCH (EGARCH) model, proposed by [28], is designed to capture asymmetric effects in volatility, such as the phenomenon where negative shocks have a larger impact on volatility than positive shocks of the same magnitude. The EGARCH model represents the conditional variance in a logarithmic form, which ensures that the variance is always positive without needing to impose non-negativity constraints on the parameters. The model is specified as follows:

$$log(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{q} \alpha_i \left(\frac{\epsilon_{t-i}}{\sigma_{t-i}}\right) + \sum_{i=1}^{q} \gamma_i \left(\left|\frac{\epsilon_{t-i}}{\sigma_{t-i}}\right| - E\left[\left|\frac{\epsilon_{t-i}}{\sigma_{t-i}}\right|\right]\right) + \sum_{j=1}^{p} \beta_j \, log(\sigma_{t-j}^2) \quad (2)$$

Where:
$log(\sigma_t^2)$ = Natural logarithm of the conditional variance at time t.
α_0 = constant term.
α_i = coefficients for the standardized residuals.
γ_i = coefficients capturing the asymmetry (the leverage effect).
β_j = coefficients for the lagged log conditional variances.
ϵ_{t-i} = past residuals (errors).
σ_{t-i} = past conditional standard deviations.

3.2.2 TGARCH Model Specification

The Threshold GARCH (TGARCH) model is a statistical tool used to analyze volatility in time series data. It builds upon the standard GARCH model by incorporating a leverage effect. It provides a more realistic representation of volatility compared to the standard GARCH model and accurately captures the behavior of financial markets where negative shocks create higher levels of uncertainty [29].

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-1}^2 + \sum_{i=1}^{q} \gamma_i \, \epsilon_{t-1}^2 I(\epsilon_{t-1} < 0) + \sum_{j=1}^{p} \beta_j \, \sigma_{t-j}^2 \quad (3)$$

Where:
σ_t^2 = the conditional variance at time t.
α_0 = the constant term (Omega).
α_i = the coefficients for the lagged squared residuals (the ARCH terms).
γ_i = the coefficients capturing the asymmetry effect.
β_j = the coefficients for the lagged conditional variances (the GARCH terms).
$I(\epsilon_{t-1} < 0)$ = an indicator function that equals 1 if $\epsilon_{t-1} < 0$ and 0 otherwise.

The variance equation in the TGARCH (p,q) model extends the GARCH model by including several components. First, it incorporates a constant term (α_0). Additionally, it includes a sum of q lagged squared residuals (ϵ_{t-1}^2), which captures the impact of past shocks. To account for the asymmetry effect, it also includes a sum of q lagged squared residuals with an indicator function ($\epsilon_{t-1}^2 I(\epsilon_{t-1} < 0)$), reflecting the leverage effect where negative shocks have a different impact on volatility compared to positive shocks. Finally, the model includes a sum of p lagged conditional variances (σ_{t-j}^2), capturing the persistence of volatility over multiple periods.

3.3 Model Selection Criteria

The most commonly used types of information criterion include the Akaike information criteria (AIC) and the Bayesian information criteria (BIC) [30].

3.3.1 Akaike Information Criterion (AIC)

The AIC is a measure of the relative accuracy of a particular statistical model for a specified dataset. It allows for the comparison of models, paying special attention to the assets that underlie the trade-off between model goodness-of-fit and model parsimony. The AIC is calculated using the formula:

$$AIC=2k-2log(L) \tag{4}$$

Where:
k is the number of estimated parameters in the model.
L is the maximized value of the likelihood function for the model.
Higher values of AIC indicate a worse model while lower values are better. The AIC penalizes models with more parameters to control overfitting, thereby promoting models that provide an equally good fit but are less complex [31].

3.3.2 Bayesian Information Criterion (BIC)

The BIC, also known as the Schwarz Criterion, is similar to the AIC but introduces a stronger penalty for models with more parameters. This criterion inclines to select lower-order models than the AIC since it increases the penalty for each additional parameter more severely. It is calculated using the formula:

$$BIC=klog(n)-2log(L) \tag{5}$$

This criterion penalizes each extra parameter more severely, tending to pick lower-order models than the AIC [32].
k is the number of estimated parameters in the model.
n is the number of observations.
L is the maximized value of the likelihood function for the model.

Similar to the case of AIC, a lower BIC value would be preferred. Nonetheless, the BIC penalty for the number of parameters grows in proportion with the log of the sample size; therefore, it is even less tolerable to overfitting than the AIC [32].

3.4 Correlogram

A correlogram chart is another special chart used in time series analysis that displays the ACF and PACF of a time series. These plots are also used for detecting the presence of autocorrelation in the data and for choosing the right model for time series forecasting.

3.5 Model Selection Criteria

Before doing any type of estimation, it is paramount to test the variables for order integration to establish whether they are stationary or have a unit root. This is conducted using the Augmented Dickey-Fuller (ADF) test.
The ADF test regulates a simple autoregressive model of order p for a variable Yt:

$$\Delta Y_t = \delta Y_t - 1 + \sum \Phi i \Delta Y_{t-i} + \varepsilon_t \tag{6}$$

Where:
Δ = the difference operator ($\Delta Y_t = Y_t - Y_{t-1}$),
δ = the coefficient on the lagged level, and
Φi = coefficients on lagged differences.
The final term of the equation εt stands for white noise.

ADF test assumes the null hypothesis to be δ = 0, in other words, the series has unit root and is non-stationary. The alternative is δ < 0 which implies that the series is nonlinear (around a deterministic trend or mean). To check the null, the t-statistic of the coefficient Y is obtained and then compared with the critical values given by Dickey and Fuller. When the value of the computed t-stat is less than the critical value then the null is rejected as stated by [33]. In other words, the ADF test greatly enables one to analyze the time series characteristics and the stationarity of individual variables as a precondition to employing multivariate time series models such as GARCH, which assume stationarity. Ascertaining the validity of this assumption is critical to make meaningful interpretations of results.

3.6 Heteroskedascity Test

The heteroscedasticity test is conducted to test the hypothesis of Time-varying or Constant variance in the return data. To this end, the least squares technique is used to estimate the form of the moving average equation. Then, the heteroscedasticity test is conducted using the ARCH-LM test.
The null and alternative hypotheses for the test are as follows:
H_0: The volatility is homoscedastic (constant over time)
H_1: The volatility is heteroscedastic (varies over time)
The ARCH-LM test is widely used in identifying the existence of ARCH effects in a time series [34-35].

3.7 Forecasting Performance of GARCH Models

Volatility prediction for the future is one of the key uses of GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models. Thus, a proper assessment of the accuracy of the forecast of volatility by a GARCH model is essential before using the model for this purpose. The following are the main components and procedures to test the quality of the GARCH type of models for forecasting.
Mean Squared Error (MSE):

$$MSE = \frac{1}{n}\sum_{t=1}^{n}(\sigma_t^2 - \hat{\sigma}_t^2)^2 \tag{7}$$

Calculates the average of the squares of the differences between the actual and estimated variances. This metric is applicable in determining the accuracy of the forecasts as highlighted by [36].
Root Mean Squared Error (RMSE):

$$\text{RMSE} = \sqrt{MSE} \tag{8}$$

The square root of MSE provides a scale that is interpretable in the context of the original data.
Mean Absolute Error (MAE):

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|\sigma_t^2 - \hat{\sigma}_t^2| \tag{9}$$

Calculates the mean of the absolute deviations of the observed from the predicted standard deviations, and provides information on the overall errors in terms of their magnitude.
Mean Absolute Percentage Error (MAPE):

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{\sigma_t^2 - \hat{\sigma}_t^2}{\sigma_t^2}\right| \tag{10}$$

Provides a percentage error measure, making it easier to interpret the forecast accuracy in relative terms.

4. Empirical Results

Figure 1 and Figure 2 presents the CRUDE_OIL_PRICE trend and the descriptive statistics of the series, which has 216 observations, has a mean price of 78.58 and a median of 75.08 for the sample period, showing a somewhat right-skewed distribution. Prices range from 14.28 to 138.74, indicating significant volatility, as seen by a standard deviation of around 25.69. The series has minor right skewness (0.224) and leptokurtosis (2.195), indicating larger tails and higher peakedness than a normal distribution. The Jarque-Bera test indicates a divergence from normalcy (p = 0.022), highlighting the importance of robust modeling methodologies to capture the intricacies of crude oil price movements.

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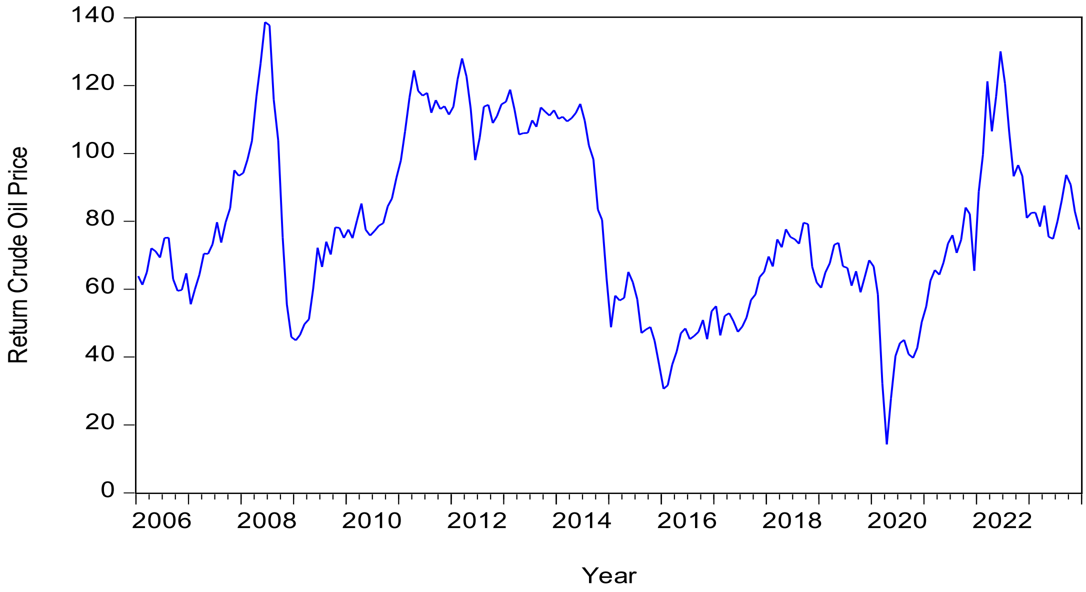

Fig. 1 *Time series plot of return crude oil price (2006 – 2023)*

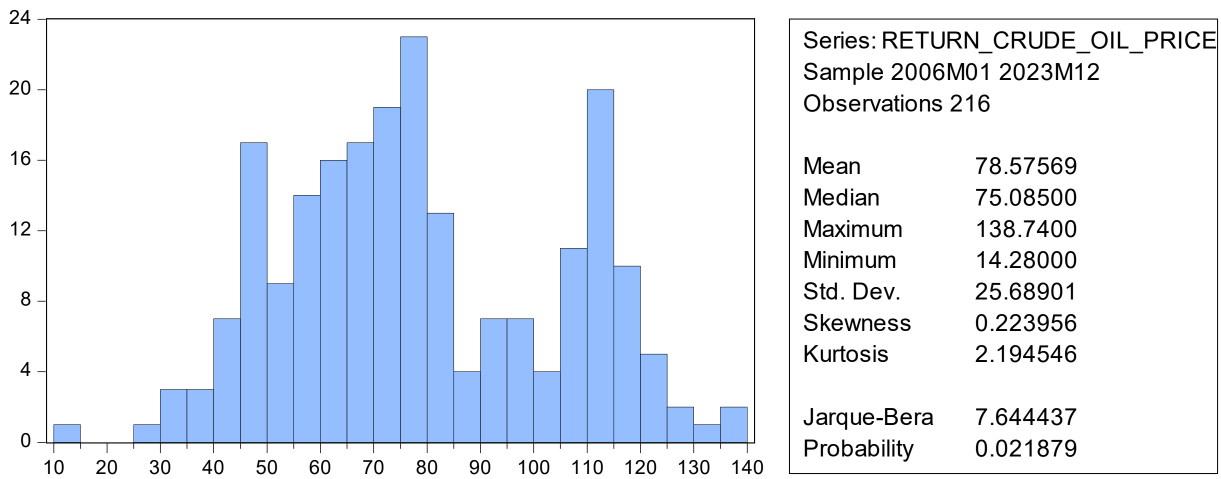

Fig. 2 *Histogram and descriptive statistics of return crude oil price (2006 – 2023)*

4.1 Stationarity Test

The Augmented Dickey-Fuller (ADF) test for stationarity is shown below in Table 1. After the first difference, the p-value of 0.0000 is below the 5% level of significance, which is strong proof against the null hypothesis showing that separating the series just once gets rid of any trend or unit root. In time series analysis, this stable series is better for modeling and making predictions. Figure 3 presents a graph of the series that has been divided by itself.

Hypothesis:
H_0: Returns Series shows the presence of a unit root
H_1: Returns Series do not show the presence of a unit root

Table 1 *ADF unit root test*

ADF TEST	t-statistics	CV 10%	CV 5%	CV 1%	Prob
Crude Oil **I(0)**	-2.844334	-2.573985	-2.874932	-3.461030	0.0539
Crude Oil **I(1)**	-10.75046	-2.573985	-2.874932	-3.461030	0.0000

Note: CV = Critical value at 1 lag, α = 0.05

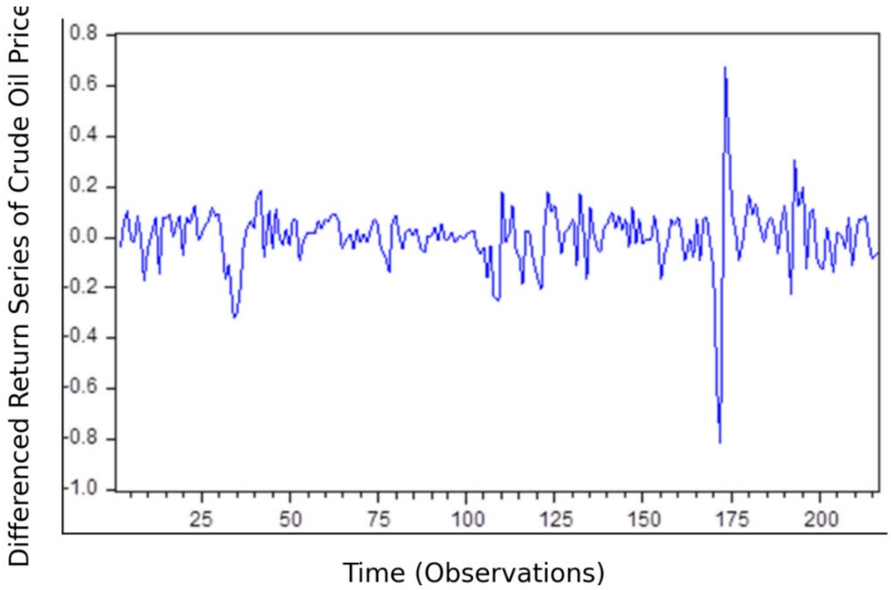

Fig. 3 *Graph of differenced crude oil price return*

4.2 Arch Effect Test for Volatility

From Table 2, we can see that the ARCH Effect test results are strong proof that there is no heteroskedasticity, which is false. The F-statistics and the Chi-Square statistics both have low p-values, which means that there is strong evidence of autoregressive conditional heteroskedasticity in the time series data. This means that the error term's variance doesn't stay the same over time. Instead, it changes over time, showing patterns of changing volatility or volatility clusters in the series.

Hypothesis
H_0: There is no ARCH effect in the residual series.
H_1: There is an ARCH effect in the residual series.

Table 2 *Heteroskedasticity test: ARCH*

F-statistic	129.6257	Prob. F(1,211)		0.0000
Obs*R-squared	81.05751	Prob. Chi-Square(1)		0.0000
		Variance Equation		
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.005845	0.003489	1.675475	0.0953
RESID^2(-1)	0.616906	0.054184	11.38533	0.0000
R-squared	0.380552	Mean dependent var		0.015273
Adjusted R-squared	0.377616	S.D. dependent var		0.062694
S.E. of regression	0.049460	Akaike info criterion		-3.165957
Sum squared resid	0.516169	Schwarz criterion		-3.134395
Log-likelihood	339.1744	Hannan-Quinn criteria		-3.153202
F-statistic	129.6257	Durbin-Watson stat		1.794419
Prob(F-statistic)	0.000000			

4.3 Model Selection

Based on an examination of the Akaike Information Criterion values produced from fitting various GARCH models to returns from crude oil prices in Table 3, the best fitting and most parsimonious specification is the GARCH(1,1) process. This simplest formulation achieves the lowest AIC and BIC of -1.867243 and -1.788856 respectively, indicating superior in-sample balance of log-likelihood and model complexity relative to higher-order alternatives. Further, no other GARCH order tested was able to surpass the performance of this parsimonious

GARCH(1,1) specification according to the AIC and BIC model selection criteria. Therefore, for purposes of capturing volatility dynamics in crude oil returns through an optimized lag structure, the GARCH(1,1) can be considered the preferred model based on these results.

Table 3 *Model selection for GARCH model for the crude oil price*

GARCH Order	Log-likelihood	AIC	BIC	Hannan
c (1,1)	205.7286	-1.867243	-1.788856	-1.835571
c (2,1)	206.6138	-1.866175	-1.772111	-1.828169
c (2,2)	205.5118	-1.846621	-1.736879	-1.802280
c (3,1)	176.5817	-1.577504	-1.467762	-1.533163
c (3,2)	206.3289	-1.844920	-1.719501	-1.794245
c (3,3)	202.4108	-1.799170	-1.658074	-1.742160
c (4,1)	206.8214	-1.849502	-1.724083	-1.798827
c (4,2)	203.9414	-1.813408	-1.672312	-1.756399
c (4,3)	203.6723	-1.801603	-1.644829	-1.738259
c (4,4)	194.7529	-1.709329	-1.536878	-1.639651

Note: AIC = Akaike Information Criterion, BIC = Bayes Information Criterion

4.4 Autocorrelation (ACF) and Partial Autocorrelation (PACF)

The autocorrelation and partial correlation analysis in Table 4 also provide empirical evidence consistent with the structure of a GARCH(1,1) model specification. Notably, the positive and significant autocorrelation at lag 1 indicates volatility clustering and short-term memory effects. However, the significant negative autocorrelation observed at lag 2 suggests potential mean-reverting or dampening influences on volatility at longer horizons. This duality of short-term persistence coupled with longer-term dampening tendencies aligns well with the conditional variance equation of a GARCH(1,1) model. The GARCH(1,1) can parsimoniously capture both the immediate impact of lagged volatility as well as the reverting dynamics represented by the significant negative lag 2 autocorrelation. Therefore, estimating a GARCH(1,1) appears well-motivated by the empirical autocorrelation patterns exhibited in the data.

Table 4 *ACF and PACF*

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
.|* |	.|* |	1	0.202	0.202	8.9128	0.003
*|. |	*|. |	2	-0.114	-0.161	11.737	0.003
*|. |	.|. |	3	-0.115	-0.060	14.646	0.002
*|. |	.|. |	4	-0.072	-0.055	15.802	0.003
.|. |	.|. |	5	0.012	0.017	15.835	0.007
.|. |	.|. |	6	-0.006	-0.038	15.843	0.015
.|. |	.|. |	7	-0.002	0.001	15.844	0.027
.|. |	.|. |	8	-0.012	-0.019	15.876	0.044
.|. |	.|. |	9	-0.057	-0.057	16.617	0.055
.|. |	.|. |	10	-0.054	-0.040	17.275	0.068
.|. |	.|. |	11	-0.052	-0.052	17.883	0.084
.|. |	.|. |	12	0.012	0.012	17.917	0.118

4.5 Model Efficient Statistic and Goodness of Fit Criterion of GARCH Variants

Table 5 displays the model efficiency statistics and goodness-of-fit criteria for several GARCH model variations applied to crude oil price data. The optimal model for modeling crude oil data in the GARCH model is the one with the highest log likelihood and the lowest Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) values. Based on the given goodness of fit statistics, the output provides a comparison of the standard GARCH, EGARCH, and TGARCH models fitted to the returns series, evaluating their relative abilities to adequately capture the time-varying conditional volatility. According to key goodness-of-fit measures such as the maximum log-likelihood, Akaike Information Criterion, and Schwarz criterion, the EGARCH specification stands out as the preferred characterization of the conditional variance dynamics. With the highest log-likelihood value of 213.4793 and lowest AIC -1.930040 and BIC -1.835976 scores among the models tested, the EGARCH formulation achieves an optimal balance between explanatory power and complexity in representing the underlying volatility process as implied by these statistical measures. Therefore, the EGARCH model can be considered the best-fitting parameterized structure for modeling time-varying risk in this return series based on these model selection diagnostics.

Table 5 *Model efficient statistic and goodness of fit criterion of GARCH variants*

Model	GARCH	EGARCH	TGARCH
R-squared	0.877009	0.877355	0.877861
Adjusted R-squared	0.876432	0.876780	0.877288
Log-likelihood	205.7286	213.4793	210.4610
Durbin-Watson stat	1.558733	1.537980	1.563129
AIC	-1.867243	-1.930040	-1.901963
Schwarz criterion	-1.788856	-1.835976	-1.807899

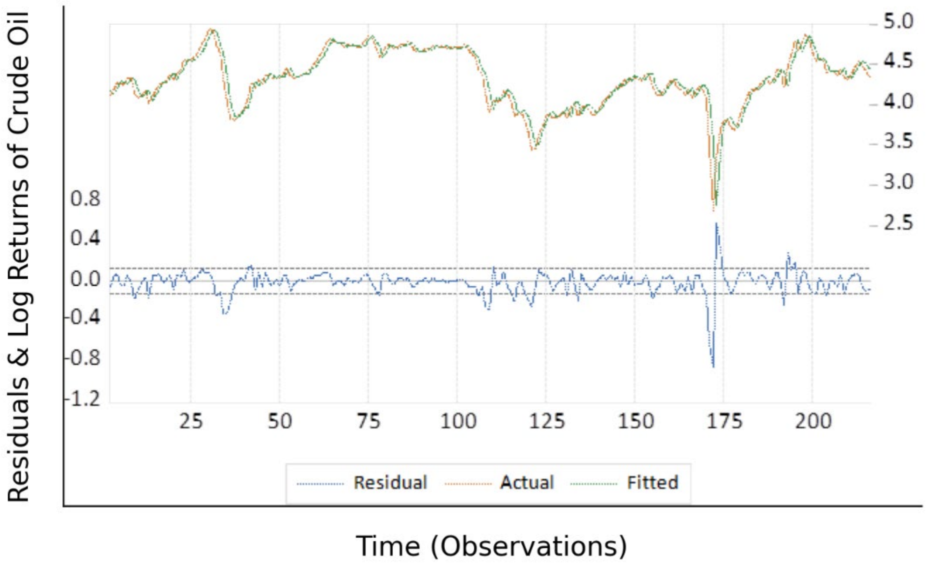

Fig. 4 *Residuals-Actual-Fitted plot for EGARCH (1, 1)*

Table 6 *Test for serial correlations*

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
.|. |	.|. |	1	0.008	0.008	0.0152	0.902
.|. |	.|. |	2	0.010	0.010	0.0393	0.981
.|. |	.|. |	3	-0.026	-0.026	0.1832	0.980
.|. |	.|. |	4	0.014	0.014	0.2245	0.994
.|. |	.|. |	5	-0.062	-0.062	1.0795	0.956
.|. |	.|. |	6	-0.020	-0.020	1.1696	0.978
.|. |	.|. |	7	0.030	0.033	1.3732	0.986
.|. |	.|. |	8	-0.056	-0.060	2.0727	0.979
.|. |	.|. |	9	0.067	0.069	3.0880	0.961
.|. |	.|. |	10	-0.029	-0.031	3.2755	0.974
.|. |	.|. |	11	0.033	0.027	3.5284	0.982
.|. |	.|. |	12	-0.004	0.004	3.5326	0.990

From the p-values of the ACF and PACF across the twelve lags of the model residuals series, it can be concluded that the residuals are not serially correlated since the p-values are greater than the level of significance (0.05). Therefore, the EGARCH (1, 1) model has been diagnosed to be a good fit for the Crude Oil Price monthly data.

Table 7 *Forecast value of crude oil price returns*

Month/Year	Forecast	Year/Month	Forecast
January, 2024	77.2681	January, 2025	75.16229
February, 2024	76.9130	February, 2025	75.16229
March, 2024	76.7362	March, 2025	74.98942
April, 2024	76.5597	April, 2025	74.98942
May, 2024	76.2079	May, 2025	74.98942
June, 2024	76.0326	June, 2025	74.98942
July, 2024	75.8578	July, 2025	74.81695
August, 2024	75.6833	August, 2025	74.81695
September, 2024	75.6833	September, 2025	74.81695
October, 2024	75.5092	October, 2025	74.81695
November, 2024	75.3356	November, 2025	74.81695
December, 2024	75.3356	December, 2025	74.64488

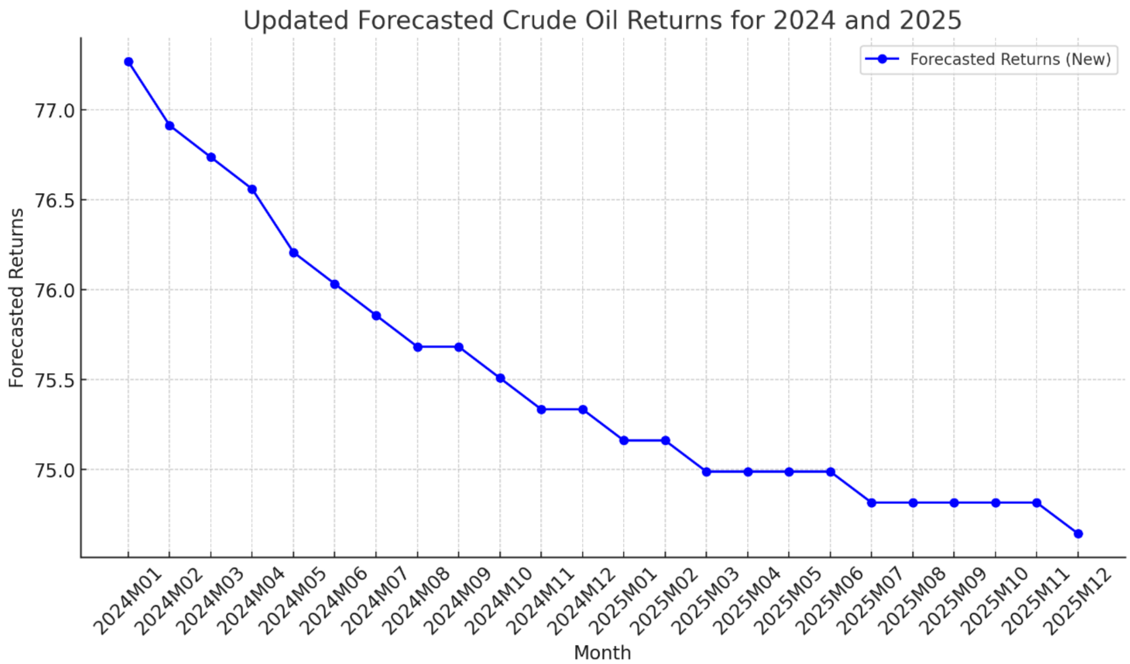

Fig. 5 *Forecast of crude oil price returns*

5. Discussion

The exploratory data analysis provided valuable initial insights into the volatility dynamics of Nigerian crude oil prices over the 2006-2023 period. The time series plot revealed periods of high variability punctuated by relatively calmer phases, indicative of volatility clustering. Furthermore, the histogram exhibited excess kurtosis and positive skew, suggesting fat-tailed distributions characteristic of time-varying volatility and non-normality. Strict statistical testing was necessary to validate the empirical patterns considering these initial findings. The non-stationarity null hypothesis was rejected at standard significance levels by the Augmented Dickey-Fuller (ADF) unit root test. The stationary returns series that followed first-order differencing laid the groundwork for reliable modeling and forecasting. The heteroskedasticity test showed strong support against the null hypothesis of constant variance, proving that autoregressive conditional heteroskedasticity (ARCH) effects do exist. This proved that GARCH-class models can do a good job of catching volatility clustering. The autocorrelation function (ACF) and the partial autocorrelation function (PACF) were looked at to help choose the right GARCH design. The strong positive autocorrelation at lag 1 and the strong negative autocorrelation at lag 2 are very similar to the shape of a GARCH(1,1) model. This model can simultaneously capture both the short-term persistence and potential longer-term dampening influences on volatility dynamics. Estimation of the GARCH(1,1) yielded significant coefficients on the ARCH and GARCH terms, further reinforcing its empirical relevance. This result is in agreement with [37-38]. Diagnostic checks on the standardized residuals from the fitted GARCH(1,1) model revealed no significant autocorrelations across 12 lags at conventional levels. This lack of systematic departures provided compelling evidence that the model adequately accounted for the temporal dependencies exhibited in the data. Furthermore, the residual plot demonstrated the model's ability to closely track the actual price movements, with residuals tightly clustered around zero. The model efficient statistics and goodness of fit criteria of the distribution fitted to the data revealed that EGARCH distribution is the best distribution for modeling Crude Oil price volatility with the highest log-likelihood value 213.4793 and lowest AIC -1.930040 and BIC -1.835976 scores amongst the three models tested which is in line with the study of [39]. The graph projects the anticipated price volatility of crude oil over the ensuing years to assess the forecasting effectiveness of the computed EGARCH (1,1) model. This forecast suggests that crude oil prices are expected to experience a significant decline over two years (2024-2025).

6. Conclusion

This research used a standardized econometrics approach, the GARCH model, to estimate and forecast fluctuations in Nigerian crude oil prices between 2016 and 2023. The result indicated that the EGARCH (1, 1) model is appropriate for the specification of the data since it captured both short-run persistence and long-run volatility persistence forms. Estimation of the model showed significant coefficients for lagged squared residuals and conditional variances, indicating their importance in volatility modeling. Therefore, as envisaged in the graph presented in Figure 5, there is an anticipated decline in crude oil prices from 2024 to 2025. The following are some of the factors that may make the quantity demanded fall and therefore decrease the consumption rate; Demand may have reduced, availability may have increased, or other factors in the economy may affect the quantity demanded. The decrease of the price by a lower step is crucial to indicate that forecasts mean a sharp decline in the price of crude oil in the course of the given period. Therefore, the results of this study signify that the GARCH model is a proper tool for crude oil price forecasting and the results can be useful to all the participants in the crude oil market. Additional information or, other types of modeling approaches might be investigated in the subsequent research to improve the forecast. Based on the results of this study, the following recommendations are made: (i) To anticipate volatility in Nigerian crude oil prices, practitioners can use the EGARCH model. The empirical findings confirmed the capacity of EGARCH models to appropriately define conditional heteroskedasticity. (ii) include other variables: The present model depends solely on the autoregressive and GARCH components. Incorporating other explanatory variables, such as crude oil supply and demand, geopolitical events, or macroeconomic indicators, may improve the model's forecasting ability by capturing the influence of relevant external factors on volatility. (iii) Broaden Data Scope: While this study focused on Nigerian crude oil prices, future research might examine volatility trends across several crude oil benchmarks or energy commodities. This would give vital insights into the findings' generalizability and allow for a more comparable investigation of volatility patterns across other energy markets. (iv) Model Evaluation and Refinement: To keep up with the dynamic character of commodities markets, forecasting models must be evaluated and refined frequently. Periodic revision of model assumptions, diagnostic tests, and performance measures may assist in discovering areas for improvement while also ensuring that forecasting models remain relevant and accurate.

7. Limitation of the Study

Although some GARCH-type models were utilized in this study to investigate the volatility of crude oil prices, there are some limitations to consider: First, the study only included the investigation of three GARCH-type models; GARCH, TGARCH, and EGARCH. Secondly, the forecast provided only covers up to two years (2024-2025). GARCH models can be less reliable for long-term predictions due to the inherent limitations of capturing all potential future volatility drivers. Thirdly, the modeling of volatility was based only on past returns and the squared errors of the Return equation. Further research may include other variables (e.g. economic variables, geopolitical events) as the exogenous factors for increasing the model's predictive ability.

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Conflict of Interest

Authors declare that there is no conflict of interest regarding the publication of the paper.

Contribution of Authors

The authors confirm contribution to the paper as follows: ***study conception and design****: Samuel Olorunfemi Adams.* ***Data collection****: Samuel Olorunfemi Adams;* ***analysis and interpretation of results****: Praise Ifeanyichukwu Olive;* ***draft manuscript preparation****: Samuel Olorunfemi Adams, Praise Ifeanyichukwu Olive. All authors reviewed the results and approved the final version of the manuscript.*

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