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# Effects of Joule Heating and Viscous Dissipation on MHD Marangoni Convection Boundary Layer Flow

Rohana Abdul Hamid[a], Norihan Md. Arifin[b], Roslinda Nazar[c], Fadzilah Md. Ali[b]

[a]Institute for Mathematical Research, Universiti Putra Malaysia, 43400 UPM Serdang, Selangor, Malaysia.

[b]Department of Mathematics, Faculty of Science, Universiti Putra Malaysia, 43400 UPM Serdang, Selangor, Malaysia.

[c]School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia.

*Corresponding emails: rosa_han86@yahoo.com, norihan@math.upm.edu.my*

**Abstract**

An analysis is performed to study the effects of the Joule heating and viscous dissipation on the magnetohydrodynamics (MHD) Marangoni convection boundary layer flow. The governing partial differential equations are reduced to a system of ordinary differential equations via the similarity transformations. Numerical results of the similarity equations are obtained using the Runge-Kutta-Fehlberg method. Effects of the magnetic field parameter, and the combined effects of the Joule heating and the viscous dissipation are investigated and the numerical results are tabulated in tables and figures. It is found that the magnetic field reduces the fluid velocity but increases the fluid temperature. On the other hand, the combined effects of the Joule heating and viscous dissipation have significantly influenced the surface temperature gradient.

**Keywords:** Marangoni convection; boundary layer; MHD; Joule heating; viscous dissipation.

# 1. INTRODUCTION

In recent years, many researchers have investigated the Marangoni convection and applied such convection into their problems. Marangoni convection is induced by the variations of the surface tension gradients at the surface of immiscible fluids (see Golia and Viviani, 1986). Kang and Kashiwagi (2002) have studied the effect of the Marangoni convection in the ammonia-water absorption process. Tan (2005) investigated the gas diffusion in liquids that can cause the Marangoni convection. Further, Christopher and Wang (2001) studied the Prandtl number effect on the Marangoni convection.

On the other hand, the study of magnetohydrodynamics (MHD) is important in the heat and mass transport process. Merkin and Kumaran (2010) have analyzed the unsteady MHD boundary layer flow on a shrinking surface. Rashad and Bakier (2009) investigated the effects of magnetic field on the non-Darcy forced convection boundary layer flow past a permeable wedge in a porous medium with uniform heat flux. An analysis of MHD boundary layer flow over a moving vertical cylinder has been studied by Amkadni and Azzouzi (2006). Furthermore, there are many attempts to study the effect of the magnetic field on the Marangoni convection. Witkowski and Walker (2001) and Munakata et al. (2002) have employed the magnetic field on the Marangoni convection in the floating zone (FZ) silicone melt. On the other hand, the effects of the magnetic field, chemical reaction, and heat generation on the thermosolutal Marangoni convection have been studied by Al-Mudhaf and Chamkha (2005). Exact analytical solutions of thermosolutal Marangoni flows in the presence of magnetic field has been examined by Magyari and Chamka (2007, 2008).

The effects of viscous dissipation and Joule heating are usually characterized by the Eckert number and the product of the Eckert number and the magnetic parameter, respectively, and both effects are important in nuclear engineering (see Alim et al., 2007). Duwairi (2005) has presented the effects of Joule heating and viscous dissipation on the forced convection flow in the presence of thermal radiation. Aissa and Mohammadein (2005) have analyzed the effects of the magnetic parameter, Joule heating, viscous dissipation and heat generation on the MHD micropolar fluids that past through a stretching sheet.

The present study focuses on the effects of Joule heating and viscous dissipation on the MHD Marangoni convection. We also investigate the influence of the magnetic field parameter on the boundary layer flow and heat transfer. The results show that the Joule heating and viscous dissipation as well as the magnetic parameter have significant effects on the Marangoni convection boundary layer flow.

## 2. MATHEMATICAL PROBLEM

We consider the steady, two-dimensional, laminar boundary layer flow of an electrically-conducting fluid over a flat surface. The surface is assumed to be in the presence of surface tension due to temperature. Further, a strong magnetic field of strength B0 is applied normal to the surface which then produces the magnetic forces along the surface. Under the above assumptions and with the usual boundary layer approximations, the governing equations are (see El-Hakiem et al., 1999) as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\Delta B_0^2}{\rho}u, \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\Delta B_0^2}{\rho c_p}u^2+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2, \qquad (3)$$

subject to the boundary conditions

$$v(x,0)=0,\ \ T(x,0)=T_\infty+Ax^2,\ \ \mu\frac{\partial u}{\partial y}=-\frac{d\sigma}{dT}\frac{\partial T}{\partial x}\ \text{at}\ y=0 \qquad (4)$$
$$u(x,\infty)=0,\ \ T(x,\infty)=T_\infty\ \text{as}\ y\to\infty$$

where $u$ and $v$ are the velocity components in the $x$ and $y$ directions, respectively, ν is the kinematic viscosity, Δ is the electric conductivity, σ is the surface tension, $B_0$ is the uniform magnetic field strength, $\rho$ is the density of the fluid, $c_p$ is the specific heat at constant pressure, $T$ is the fluid temperature, $T_\infty$ is the fluid temperature far from the surface, $A$ is the temperature gradient coefficient and $\mu$ is the dynamic viscosity.

Further, we use the similarity transformations as presented by Al-Mudhaf and Chamkha (2005) and the standard definition of the stream function such that $u=\partial\psi/\partial y$ and $v=\partial\psi/\partial x$ to obtain the similarity solution of the problems. The similarity transformations are given by

$$f(\eta) = C_2 x^{-1} \psi(x, y), \qquad \theta(\eta) = \frac{[T(x, y) - T_\infty] x^{-2}}{A}, \qquad \eta = C_1 y \qquad (5)$$

where

$$C_1 = \sqrt[3]{\frac{\rho A (d\sigma / dT)}{\mu^2}}, \qquad C_2 = \sqrt[3]{\frac{\rho^2}{\mu A (d\sigma / dT)}}, \qquad (6)$$

We apply (5) and (6) into the equations (1), (2) and (3) and get the following system of equations:

$$f''' + ff'' - (f')^2 - M^2 f' = 0, \qquad (7)$$

$$\frac{1}{\Pr}\theta'' - 2f'\theta + f\theta' + Ec[M^2 (f')^2 + (f'')^2] = 0, \qquad (8)$$

with the boundary conditions

$$f''(0) = -2, \quad f(0) = 0, \quad \theta(0) = 1, \quad f'(\infty) = 0, \quad \theta(\infty) = 0. \qquad (9)$$

where $M^2=\Delta B_0^2 C_2/\rho C_1$ is the magnetic field parameter, Pr is the Prandtl number and $Ec=C_1^2/Ac_p C_2^2$ is the Eckert number. It should be mentioned here again that the viscous dissipation effect is examined using the Eckert number, *Ec* while the product of the Eckert number and the magnetic field parameter, *M* gives the Joule heating.

## 3. RESULTS AND DISSCUSSION

The nonlinear ordinary differential equations (7) and (8) subject to the boundary conditions (9) were solved numerically using the Runge-Kutta-Fehlberg fourth-fifth order (RKF45) method using Maple 12 and the algorithm RKF45 in Maple has been well tested for its accuracy and robustness (Aziz, 2009). In this method, it is most important to choose the appropriate finite value of the edge of boundary layer, $n \to \infty$ ·(say $n_\infty$ ) that is between 4 to 10, which is in accordance with the standard practice in the boundary layer analysis. We begin with some initial guess value of $n_\infty$ and solve the equations (7) and (8) subject to the boundary conditions (9) with some particular set of parameters to obtain the surface velocity $f'(0)$ and the temperature gradient $-\theta'(0)$. The solution process is repeated until further changes (increment) in $n_\infty$ do not lead to any changes in the values of $f'(0)$ and $-\theta'(0)$, or in other words, the results are independent of the value of $n_\infty$. The initial step size employed is $h = \Delta n = 0.1$. The present data is then validated by comparing it with the previously published work as depicted in Table 1. It is seen that the present results are in very good agreement with the results obtained by Al-Mudhaf and Chamkha (2005). Then, we investigate the effects of the magnetic field parameter, $M$ on the surface velocity $f'(0)$ and the surface temperature gradient $-\theta'(0)$ as display in Table 2. In the presence of the Prandtl number, Joule heating and viscous dissipation, it is shown that the increase of $M$ values tend to reduce both the surface velocity and the surface temperature gradient. Thus, the heat transfer rate at the surface also reduces as $M$ increases. Moreover, in Table 3, the effect of the Prandtl number, Pr on the surface temperature gradient when $M = 1$ and $Ec = 0.2$ is presented. As mentioned by Thirumaleshwar (2006), the Prandtl number values for gases are in the range of 0.7 to 1 while for water it is 1.7 to 13.7. The Prandtl number for heavy oils takes the values of 50 to 100 000. It is observed that the surface temperature gradient is increasing as Pr increases.

Meanwhile Figure 1 illustrates the variations of the surface temperature gradient with the Eckert number for different values of the Prandtl number i.e. Pr = 0.7, 7 and 100. It is worth mentioning that $Ec = 0$ means that there are no effects of Joule heating and viscous dissipation while $Ec > 0$ indicates the combined effects of Joule heating and viscous dissipation. The figure shows that as $Ec$ increases, the surface temperature gradient decreases. Further, when we consider higher Prandtl number, we notice further reduction of the surface temperature gradients. Figures 2 and 3 present the effects of the magnetic field parameter $M$ on the velocity profiles and temperature profiles, respectively. It is obvious from the Figure 2 that the increase of parameter $M$ decreases the velocity profiles.

Meanwhile, as seen in the Figure 3, the parameter $M$ tends to increase the temperature profiles. However, the temperature profile can be reduced using higher Prandtl number. Further, Figure 4 and 5 present the combined effects of the Joule heating and viscous dissipation on the temperature profiles when M = 1 for Pr = 0.7 and Pr = 7, respectively. It is noted that increasing the values of the Eckert number will produce an increases in the temperature profiles for both Prandtl number considered. Finally, it is worth mentioning that all the profiles presented in Figures 2 to 5 satisfy the boundary conditions (9), and thus support the numerical results obtained.

**Table 1:** Comparisons of the present results with those of Al-Mudhaf and Chamkha (2005).

| | Al-Mudhaf and Chamkha (2005) | | Present | |
|---|---|---|---|---|
| M | $f'(0)$ | $-\theta'(0)$ | $f'(0)$ | $-\theta'(0)$ |
| 0 | 1.587671 | 1.442203 | 1.587400 | 1.442076 |
| 1 | 1.315181 | 1.206468 | 1.314596 | 1.206059 |
| 2 | 0.9039450 | 0.7596045 | 0.9032119 | 0.7597828 |
| 3 | 0.6448883 | 0.4422402 | 0.6440222 | 0.4422687 |
| 4 | 0.4933589 | 0.2728471 | 0.4924782 | 0.2741291 |

**Table 2:** Effects of *M* on the surface velocity, $f'(0)$ and the surface temperature gradients, $-\theta'(0)$ when Pr = 0.7 and Ec = 0.2.

| M | $f'(0)$ | $-\theta'(0)$ |
|---|---|---|
| 0 | 1.587400 | 1.188189 |
| 0.5 | 1.508289 | 1.107814 |
| 1.0 | 1.314596 | 0.912075 |
| 1.5 | 1.093739 | 0.688528 |
| 2 | 0.903211 | 0.498001 |

**Table 3:** Effects of the Prandtl number Pr on the surface temperature gradients, $-\theta'(0)$ when M = 1 and *Ec* = 0.2.

| Pr | $-\theta'(0)$ |
|---|---|
| 0.7 | 0.912196 |
| 1 | 1.142406 |
| 5 | 2.729714 |
| 7 | 3.216857 |
| 10 | 3.815753 |
| 100 | 11.201822 |

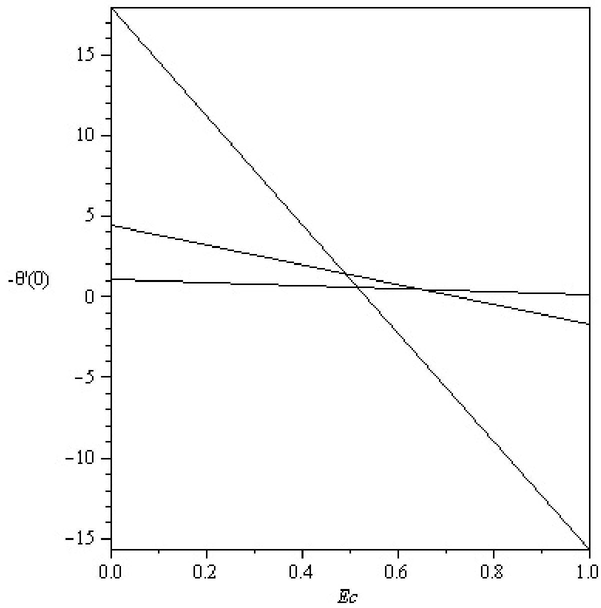


**Figure 1:** Variations of the surface temperature gradients, $-\theta'(0)$ with the Eckert number. *Ec* for different Prandtl number Pr.

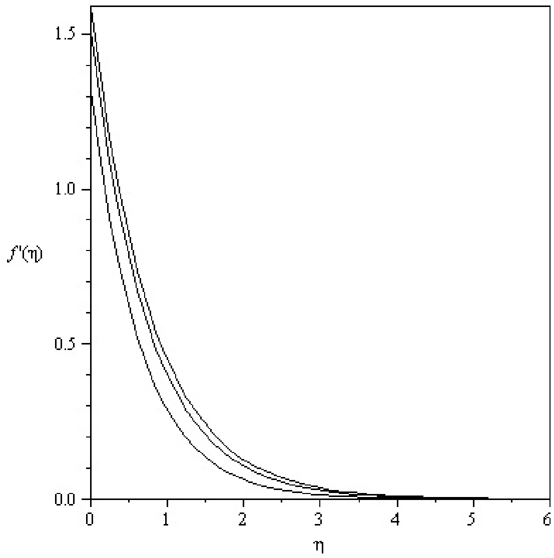


**Figure 2:** Effects of the magnetic field parameter, *M* on the velocity profiles

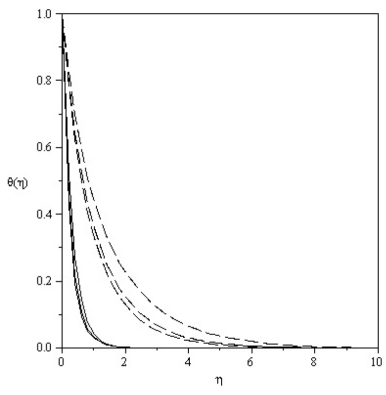


**Figure 3:** Effects of the magnetic field parameter, *M* on the temperature profiles when *Ec* = 0.2 for different Prandtl number Pr.

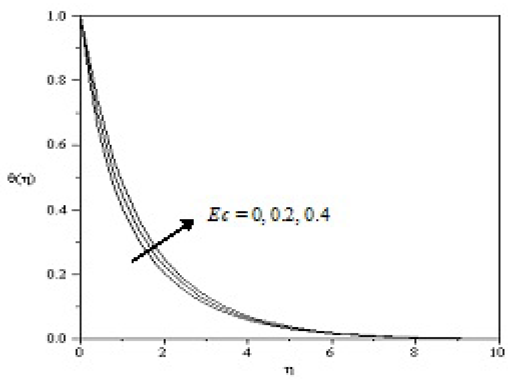


**Figure 4:** Effects of the Eckert number, *Ec* on the temperature profiles when Pr = 0.7, *M* = 1.

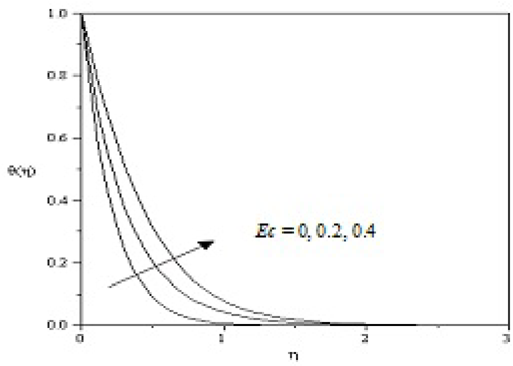


Figure 5: Effects of the Eckert number, *Ec* on the temperature profiles when Pr = 7, *M* = 1.

## 4. CONCLUSIONS

The problem of MHD Marangoni convection boundary layer flow with Joule heating and viscous dissipation has been solved numerically in this paper. From the results it can be concluded that in the presence of magnetic parameter *M*, the surface velocity and the velocity profiles can be reduced. On the other hand, the surface temperature gradients can be reduced by increasing the parameters *M* and *Ec* and with higher Prandtl number Pr.

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