

An Improvement of Numerical Result of Crashing CPM/PERT Network

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Abstract

The research introduces and develops a mathematical modeling technique with linearized Taylor's first order expansion and solve by using the simplex method. The main objective is to minimize the pessimistic time of the activity which is lie on the critical path by investing additional amounts of money to the project. 7 different amounts of money which is \$5000, \$10000, \$15000, \$20000, \$25000, \$30000 and \$35000 will be invest to the project, to show the increase amount of money invest in the project will tend to minimize the pessimistic time to decrease the expected time and project duration. Then at the same time, it is also reduces its variance and standard deviation. As the result of the research, it will bring to the increase of the probability or percentage of completing the project on or before the completion time. The PERT and normal distribution will display the differences between of the amounts of money that will invest to the project.

Keywords: Critical Path Method; Critical Path Analysis; Program Evaluation and Review Technique

1 INTRODUCTION

In 1958, a new managerial planning and control technique was developed, which was called PERT. PERT (Program Evaluation and Review Technique) is a manager's tool for defining and coordinating moves for completing a project's objectives on time. It uses is not restricted to the business world. It can be applied to any endeavor which requires planned, controlled, and integrated work patterns.

PERT designed to provide management information on actual and impending problems in completing a project. If the duration of the activities is not known with certainty, the PERT can be used to estimate the probability that the project will be completed by a deadline. In essence, the ability of PERT is to predict future performance and potential future problems through frequency reporting marked its major departure from previous planning and control techniques which relied heavily on historical data.

Besides that, network crashing developed with the critical path method (CPM) for planning and controlling large scale projects. Crashing in CPM means that by selecting the lowest cost slope activity or activities, which will shorten the critical path. This procedure is repeated until the project has been shortened sufficiently or the cost to shorten the project exceeds the benefits to be derived. The idea of crashing a project was not developed in PERT where stochastic time estimates were used. Probability estimates for completing a project at a specific date were the closest that PERT came to addressing the problem.

Crashing PERT networks was investigated by a number of researchers, such as Cho and Yum [1], Ghaleb and Adnan [3], Johnson and Schou [8], Keefer and Verdini [9], Saman [11]. The concept of crashing in CPM is applied to PERT networks in order to reduce the project duration of the project, and also to increase the probability of completing the project on or before completion time with the additional amounts of money that will be invest to activity that lie on the critical path.

2 MODEL CONSTRUCTION

According to the traditional PERT technique the probability of a certain project meeting a specific schedule time T_s , for a particular event is equal to $\phi(Z)$, the probability can be describe as follows:

$$Z = \frac{T_s - T_E}{\sigma_E} \quad (1)$$

where the total expected duration of the project T_E , is the sum of the expected time along the critical path.

$$T_E = \sum_{i=1}^n t_i = t_1 + t_2 + \dots + t_n \quad (2)$$

According to [2], the variability in the activity time estimates is approximated equations (3).

$$\sigma_i = \frac{b_i - a_i}{6}, \sigma_E = \sqrt{\sum_{i=1}^n \sigma_i^2} = \sqrt{\sigma_1^2 + \sigma_2^2 + \dots + \sigma_n^2} \quad (3)$$

σ_i represents the standard deviation for the activity and σ_E represents the standard deviation for the project along the critical path. While the expected time and variance of the activity on the critical path is computed by the following formula:

$$t_i = \frac{a_i + 4m_i + b_i}{6}, \sigma_i^2 = \frac{(b_i - a_i)^2}{36} \quad (4)$$

where a is a optimistic activity time, m is a most likely activity time, b is a pessimistic activity time. The amount of money will be invested in each critical path activity is r_i , where $i = 1, 2, \dots, n$ and n is the total number of activities which lie on the critical path. The amount of money invested in that critical path activities are M units of money.

$$M = \sum_{i=1}^n r_i = r_1 + r_2 + r_3 + \dots + r_n \quad (5)$$

Then the pessimistic time of the activity will reduce from b to \hat{b} , and the total expected project duration will reduce from T_E to \hat{T}_E , and similar with the standard deviation of the project reduce from σ_E to $\hat{\sigma}_E$ lie on the critical path. The new of expected time, variance of the activity and standard deviation for the activity that lie on the critical path will be as follows

$$\hat{t}_i = \frac{a_i + 4m_i + \hat{b}_i}{6}, \hat{\sigma}_i^2 = \frac{(\hat{b}_i - a_i)^2}{36}, \hat{\sigma}_i = \frac{\hat{b}_i - a_i}{6} \quad (6)$$

After investing the amount of money in the activities which lie on the critical path, the new total expected duration of the project and standard deviation for

the project along the critical path is equal to

$$\hat{T}_E = \sum_{i=1}^n \hat{t}_i, \hat{\sigma}_E = \sqrt{\sum_{i=1}^n \hat{\sigma}_i^2} \quad (7)$$

Then the new probability of realizing the terminal node will be

$$\hat{Z} = \frac{T_s - \hat{T}_E}{\hat{\sigma}_E} = \frac{T_s - \sum_{i=1}^n \hat{t}_i}{\sqrt{\sum_{i=1}^n \hat{\sigma}_i^2}} \quad (8)$$

Since the reduction of the expected time and the variance of the activities that lie on the critical path are depend on the amount of the money that will invest in each activity. Therefore, it is means that the decrease in both expected time and variance of the activities will be a function ϕ and ψ of the additional investment. Then the new expected duration and variance of the activities for a certain activity will be

$$\hat{t}_e = t_e - \phi(r_i), \hat{\sigma}_i^2 = \sigma_i^2 - \psi(r_i) \quad (9)$$

From [3], the equation of new expected duration and variance of the activities becomes:

$$\hat{t}_e = t_e + q_i r_i \quad -\frac{t_e}{r_i} < q_i < 0 \quad (10)$$

$$\hat{\sigma}_i^2 = \sigma_i^2 + s_i r_i \quad -\frac{\sigma_i^2}{r_i} < s_i < 0 \quad (11)$$

where $s_i = q_i$. By substitute the equations (10) and (11) to equation (8), Then \hat{Z} becomes

$$\hat{Z} = \frac{T_s - \{t_1 + q_1 r_1 + t_2 + q_2 r_2 + \dots + t_n + q_n r_n\}}{\sqrt{\{\sigma_1^2 + s_1 r_1 + \sigma_2^2 + s_2 r_2 + \dots + \sigma_n^2 + s_n r_n\}}} = \frac{T_s - \left\{ \sum_{i=1}^n t_i + \sum_{i=1}^n q_i r_i \right\}}{\sqrt{\left\{ \sum_{i=1}^n \sigma_i^2 + \sum_{i=1}^n s_i r_i \right\}}} \quad (12)$$

According to [3], $\sum_{i=1}^n t_i$ and $\sum_{i=1}^n \sigma_i^2$ are constant because same before and after crashing. Therefore let $\sum_{i=1}^n t_i$ be C_1 and $\sum_{i=1}^n \sigma_i^2$ be C_2 . Then

$$\hat{Z} = \frac{T_s - \left\{ C_1 + \sum_{i=1}^n q_i r_i \right\}}{\sqrt{C_2 + \sum_{i=1}^n s_i r_i}} \quad (13)$$

where

$$-\frac{t_i}{r_i} < q_i < 0, \quad -\frac{\sigma_i}{r_i} < s_i < 0, \quad -\frac{\sigma_i^2}{r_i} < s_i < 0 \quad (14)$$

Let \bar{r}_i is the upper limit for the amount of money to be invested in each activity that lie on the critical path. And the range of the r_i will be as $0 \leq r_i \leq \bar{r}_i$. then we found that

$$-\frac{t_i}{\bar{r}_i} < -\frac{t_i}{r_i} < q_i < 0, \quad -\frac{\sigma_i}{\bar{r}_i} < -\frac{\sigma_i}{r_i} < s_i < 0, \quad -\frac{\sigma_i^2}{\bar{r}_i} < -\frac{\sigma_i^2}{r_i} < s_i < 0$$

therefore the new range of s_i and q_i will develop as below, when the amount of money invested r_i increase until upper limit for the amount of money to be invested in each activity \bar{r}_i .

$$-\frac{t_i}{\bar{r}_i} < q_i < 0, \quad -\frac{\sigma_i}{\bar{r}_i} < s_i < 0, \quad -\frac{\sigma_i^2}{\bar{r}_i} < s_i < 0 \quad (15)$$

Equation (13) is the required mathematical model, which represents the objective function and the constraints. The value of q_i and s_i are specified by expert having high knowledge in the nature of project activities. The variable in this model is r_i . The whole model can be rewritten as follows

$$Max \hat{Z} = \frac{T_s - C_1 - \sum_{i=1}^n q_i r_i}{\sqrt{C_2 + \sum_{i=1}^n s_i r_i}} \text{ such that } 0 \leq r_i \leq \bar{r}_i \text{ and } \sum_{i=1}^n r_i \leq M \quad (16)$$

3 LINEARIZING THE MATHEMATICAL

The mathematical model from equation (16) will be linearized by using the Taylor's series 1st order expansion. Then, the $Max \hat{Z}$ will be as follow

$$\frac{T_s - C_1 - \sum_{i=1}^n q_i \bar{r}_i}{\sqrt{C_2 + \sum_{i=1}^n s_i \bar{r}_i}} + \left(-\frac{1}{2} \frac{\left(T_s - C_1 - \sum_{i=1}^n q_i \bar{r}_i \right) \left(\sum_{i=1}^n s_i \right)}{\left(C_2 + \sum_{i=1}^n s_i \bar{r}_i \right)^{\frac{3}{2}}} - \frac{\sum_{i=1}^n q_i}{\sqrt{C_2 + \sum_{i=1}^n s_i \bar{r}_i}} \right) (r_i - \bar{r}_i) \quad (17)$$

subject to $0 \leq r_i \leq \bar{r}_i$ and $\sum_{i=1}^n r_i \leq M$.

According to [3], from the equation (17), the first term is constant and depends on \bar{r}_i , and then let first term be α . The equation is a linear equation, it can be solve by using simplex method. Since α is constant, then it can be written as follows subject to

$$Max \hat{Z} = \left(-\frac{1}{2} \frac{\left(T_s - C_1 - \sum_{i=1}^n q_i \bar{r}_i \right) \left(\sum_{i=1}^n s_i \right)}{\left(C_2 + \sum_{i=1}^n s_i \bar{r}_i \right)^{\frac{3}{2}}} - \frac{\sum_{i=1}^n q_i}{\sqrt{C_2 + \sum_{i=1}^n s_i \bar{r}_i}} \right) (r_i - \bar{r}_i) \quad (18)$$

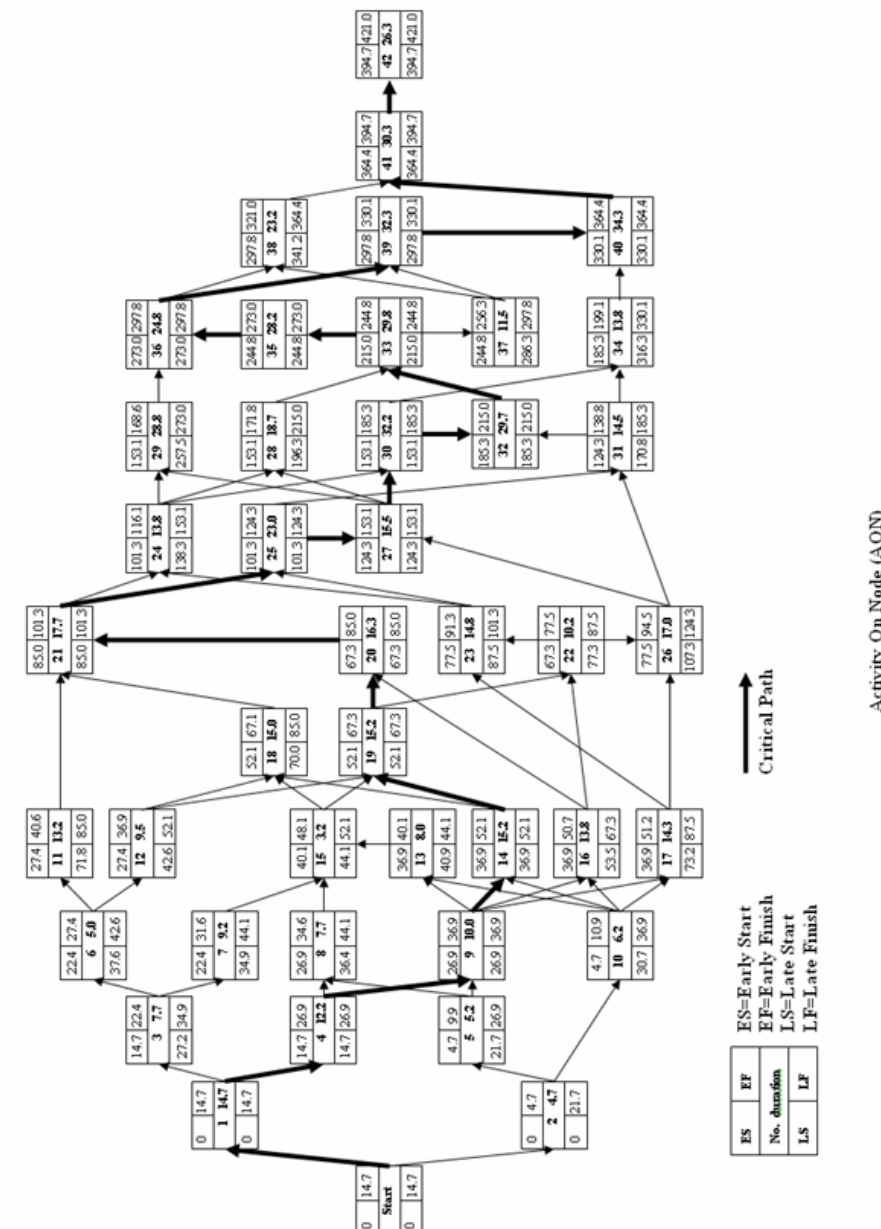
subject to $0 \leq r_i \leq \bar{r}_i$ and $\sum_{i=1}^n r_i \leq M$.

4 NUMERICAL RESULT

Here are the optimistic (a), most likely (m), pessimistic (b) time and predecessors (*) of the 42 activities.

Table 4.1 : a, m, b and predecessors of the 42 activities

No.	*	a	m	b	No.	*	a	m	b	No.	*	a	m	b
1	-	5	12	35	15	7, 8, 13	5	8	11	29	24, 27	11	16	18
2	-	2	5	6	16	9, 10	12	14	15	30	24, 27	30	32	35
3	1	5	8	9	17	9, 10	12	14	18	31	25, 26	12	14	19
4	1	10	12	15	18	12, 14, 15	12	15	18	32	30, 31	23	29	39
5	2	2	5	9	19	12, 14, 15	12	15	19	33	28, 32	25	29	38
6	3	2	5	8	20	16, 19	15	18	19	34	30, 31	12	14	15
7	3	5	9	14	21	11, 18, 20	15	16	19	35	33	22	28	35
8	4, 5	5	8	9	22	16, 19	9	10	12	36	29, 35	23	24	30
9	4, 5	8	9	16	23	17, 22	12	13	19	37	33	10	11	15
10	2	5	6	8	24	21, 23	12	15	17	38	36, 37	23	23	24
11	6	12	13	15	25	21, 23	21	22	29	39	36, 37	30	32	36
12	6	5	9	16	26	17, 22	15	17	19	40	34, 39	30	35	36
13	9, 10	2	3	5	27	25, 26	26	29	31	41	38, 40	30	30	32
14	9, 10	13	15	18	28	24, 27	15	19	21	42	41	25	26	29



After that, PERT and CPM are compute to each of the 42 activity. And the data show that the total expected project duration to finish the whole project is 421 days.

Table 4.2 : Values t , EL, EF, LS, LF, TF, FF, σ , σ^2 of the CPM/PERT of the 42 activities

Activity No.	Expected Time	Early		Late		Total Float	Free Float	σ	σ^2
		Start	Finish	Start	Finish				
1	14.7	0	14.7	0	14.7	0	0	5	25
2	4.7	0	4.7	17	21.7	17	0	0.7	0.4
3	7.7	14.7	22.4	27.2	34.9	12.5	0	0.7	0.4
4	12.2	14.7	26.9	14.7	26.9	0	0	0.8	0.7
5	5.2	4.7	9.9	21.7	26.9	17	17	1.2	1.4
6	5	22.4	27.4	37.6	42.6	15.2	0	1	1
7	9.2	22.4	31.6	34.9	44.1	12.5	8.5	1.5	2.3
8	7.7	26.9	34.6	36.4	44.1	9.5	5.5	0.7	0.4
9	10	26.9	36.9	26.9	36.9	0	0	1.3	1.8
10	6.2	4.7	10.9	30.7	36.9	26	26	0.5	0.3
11	13.2	27.4	40.6	71.8	85	44.4	44.4	0.5	0.3
12	9.5	27.4	36.9	42.6	52.1	15.2	15.2	1.8	3.4
13	3.2	36.9	40.1	40.9	44.1	4	0	0.5	0.3
14	15.2	36.9	52.1	36.9	52.1	0	0	0.8	0.7
15	8	40.1	48.1	44.1	52.1	4	4	1	1
16	13.8	36.9	50.7	53.5	67.3	16.6	16.6	0.5	0.3
17	14.3	36.9	51.2	73.2	87.5	36.3	26.3	1	1
18	15	52.1	67.1	70	85	17.9	17.9	1	1
19	15.2	52.1	67.3	52.1	67.3	0	0	1.2	1.4
20	17.7	67.3	85	67.3	85	0	0	0.7	0.4
21	16.3	85	101.3	85	101.3	0	0	0.7	0.4
22	10.2	67.3	77.5	77.3	87.5	10	0	0.5	0.3
23	13.8	77.5	91.3	87.5	101.3	10	10	1.2	1.4
24	14.8	101.3	116.1	138.3	153.1	37	37	0.8	0.7
25	23	101.3	124.3	101.3	124.3	0	0	1.3	1.8
26	17	77.5	94.5	107.3	124.3	29.8	29.8	0.7	0.4
27	28.8	124.3	153.1	124.3	153.1	0	0	0.8	0.7
28	18.7	153.1	171.8	196.3	215	43.2	43.2	1	1
29	15.5	153.1	168.6	257.5	273	104.4	104.4	1.2	1.4
30	32.2	153.1	185.3	153.1	185.3	0	0	0.8	0.7
31	14.5	124.3	138.8	170.8	185.3	46.5	46.5	1.2	1.4
32	29.7	185.3	215	185.3	215	0	0	2.7	7.1
33	29.8	215	244.8	215	244.8	0	0	2.2	4.7
34	13.8	185.3	199.1	316.3	330.1	131	131	0.5	0.3
35	28.2	244.8	273	244.8	273	0	0	2.2	4.7
36	24.8	273	297.8	273	297.8	0	0	1.2	1.4
37	11.5	244.8	256.3	286.3	297.8	41.5	41.5	0.8	0.7
38	23.2	297.8	321	341.2	364.4	43.4	43.4	0.2	0
39	32.3	297.8	330.1	297.8	330.1	0	0	1	1
40	34.3	330.1	364.4	330.1	364.4	0	0	1	1
41	30.3	364.4	394.7	364.4	394.7	0	0	0.3	0.1
42	26.3	394.7	421	394.7	421	0	0	0.7	0.4

In Table 4.2, have 18 activities that have a total float of zero, therefore

there have 18 activities lie on the critical path. Such activity is critical to the completion of the project on time. The 18 activities are 1→4→9→14→19→20→21→25→27→30→32→33→35→36→39→40→41→42.

Table 4.3 : Activity’s name, t , EL, EF, LS,LF, TF, FF, σ , σ^2 of the CPM/PERT of the 18 activities that lie on the critical path.

No.	Activity	Expected Time	Early		Late		σ	σ^2	Total	Free
			Start	Finish	Start	Finish			Float	
1	1	14.7	0	14.7	0	14.7	5	25	0	0
2	4	12.2	14.7	26.9	14.7	26.9	0.8	0.7	0	0
3	9	10	26.9	36.9	26.9	36.9	1.3	1.8	0	0
4	14	15.2	36.9	52.1	36.9	52.1	0.8	0.7	0	0
5	19	15.2	52.1	67.3	52.1	67.3	1.2	1.4	0	0
6	20	17.7	67.3	85	67.3	85	0.7	0.4	0	0
7	21	16.3	85	101.3	85	101.3	0.7	0.4	0	0
8	25	23	101.3	124.3	101.3	124.3	1.3	1.8	0	0
9	27	28.8	124.3	153.1	124.3	153.1	0.8	0.7	0	0
10	30	32.2	153.1	185.3	153.1	185.3	0.8	0.7	0	0
11	32	29.7	185.3	215	185.3	215	2.7	7.1	0	0
12	33	29.8	215	244.8	215	244.8	2.2	4.7	0	0
13	35	28.2	244.8	273	244.8	273	2.2	4.7	0	0
14	36	24.8	273	297.8	273	297.8	1.2	1.4	0	0
15	39	32.3	297.8	330.1	297.8	330.1	1	1	0	0
16	40	34.3	330.1	364.4	330.1	364.4	1	1	0	0
17	41	30.3	364.4	394.7	364.4	394.7	0.3	0.1	0	0
18	42	26.3	397.7	421	397.7	421	0.7	0.4	0	0
Total		421					54			

From Table 4.3, the total expected duration of the project, $T_E = 421$ and variation in the project duration, $\sum_{i=1}^n \sigma_i^2 = 54$. If scheduled duration of the project, $T_S = 415$, then the probability of finishing the project within the completion date 415 days can be compute as: $Z = -0.82$

The Figure 4.3 shows that the probability of completing the project within 415 days is equal to the area under the normal curve, $Z = -0.82$. From the tables of areas under the standardized normal curve from $-\infty$ to $-z$ or PERT Factor Tables [5], a Z value -0.82 gives the probability of 0.2061, which mean there is a 20.6% chance of completing the project on or before 415 days.

In order to increase the probability of completing the project on or before 415 days, hence the total amount of money will be invest on all activities that lie on the critical path, M . 7 different total amount of money is test to demonstrate how the investments increase the probability of completing the

project within 415 days. The 7 different total amount of money to be invested are \$5000, \$10000, \$15000, \$20000, \$25000, \$30000, \$35000.

After that the total amount of money to be invest to each activity that lie on the critical path are specified by experts as follows:

$$\begin{aligned} \bar{r}_1 &= 5000 & \bar{r}_4 &= 1000 & \bar{r}_9 &= 3000 & \bar{r}_4 &= 2000 & \bar{r}_9 &= 1000 & \bar{r}_0 &= 2000 \\ \bar{r}_2 &= 1200 & \bar{r}_3 &= 1800 & \bar{r}_2 &= 800 & \bar{r}_9 &= 750 & \bar{r}_3 &= 3200 & \bar{r}_3 &= 2000 \\ \bar{r}_3 &= 1500 & \bar{r}_5 &= 1100 & \bar{r}_9 &= 2200 & \bar{r}_0 &= 500 & \bar{r}_4 &= 850 & \bar{r}_2 &= 6000 \end{aligned}$$

Then the value of q_i and s_i are specify by experts having high knowledge in the nature of project activities and satisfy the equation (15) from above.

Table 4.4 : Value of q_i and s_i

No.	$-\frac{t_i}{\bar{r}_i}$	$-\frac{\sigma_i^2}{\bar{r}_i}$	$-\frac{\sigma_i}{\bar{r}_i}$	$s_i = q_i$
1	-0.002933334	-0.005	-0.001	-0.0001
4	-0.01216667	-0.00069444	-0.00083333	-0.0003
9	-0.003333333	-0.000592593	-0.000444443	-0.00011
14	-0.007583335	-0.00034722	-0.000416665	-0.00023
19	-0.01516667	-0.00136111	-0.00116667	-0.00041
20	-0.008833335	-0.00022222	-0.000333335	-0.00005
21	-0.013611108	-0.000370367	-0.000555558	-0.00033
25	-0.012777778	-0.000987656	-0.000740739	-0.00036
27	-0.036041663	-0.00086805	-0.001041663	-0.00018
30	-0.042888893	-0.00092592	-0.001111107	-0.00045
32	-0.009270834	-0.002222222	-0.000833334	-0.00036
33	-0.014916665	-0.00234722	-0.001083335	-0.00055
35	-0.018777778	-0.003129627	-0.001444447	-0.00066
36	-0.022575755	-0.001237373	-0.001060609	-0.0007
39	-0.014696968	-0.000454545	-0.000454545	-0.0002
40	-0.068666666	-0.002	-0.002	-0.00025
41	-0.035686271	-0.000130718	-0.000392153	-0.0003
42	-0.004388888	-0.000740733	-0.000111112	-0.00004

By substituting the values of q_i , s_i and \bar{r}_i in the developed model $Max\hat{Z}$, then it can be written as follow subject to

$$\begin{aligned} \text{Max } \hat{Z} = & 0.0000129690 r_1 + 0.0000387660 r_4 + 0.0000142219 r_9 + 0.0000298069 r_{14} \\ & + 0.0000530859 r_{19} + 0.0000064377 r_{20} + 0.0000427168 r_{21} + 0.0000468143 r_{25} \\ & + 0.0000231941 r_{27} + 0.0000581885 r_{30} + 0.0000472495 r_{32} + 0.0000721174 r_{33} \\ & + 0.0000863658 r_{35} + 0.0000912310 r_{36} + 0.0000259097 r_{39} + 0.0000322030 r_{40} \\ & + 0.0000387344 r_{41} + 0.0000051632 r_{42} \end{aligned}$$

subject to

$$\begin{aligned} 0 \leq r_1 \leq 5000 \quad & 0 \leq r_4 \leq 1000 \quad 0 \leq r_9 \leq 3000 \quad 0 \leq r_{14} \leq 2000 \quad 0 \leq r_{19} \leq 1000 \\ 0 \leq r_{20} \leq 2000 \quad & 0 \leq r_{21} \leq 1200 \quad 0 \leq r_{25} \leq 1800 \quad 0 \leq r_{27} \leq 800 \quad 0 \leq r_{30} \leq 750 \\ 0 \leq r_{32} \leq 3200 \quad & 0 \leq r_{33} \leq 2000 \quad 0 \leq r_{35} \leq 1500 \quad 0 \leq r_{36} \leq 1100 \quad 0 \leq r_{39} \leq 2200 \\ 0 \leq r_{40} \leq 500 \quad & 0 \leq r_{41} \leq 850 \quad 0 \leq r_{42} \leq 6000 \end{aligned}$$

$$r_1 + r_4 + r_9 + r_{14} + r_{19} + r_{20} + r_{21} + r_{25} + r_{27} + r_{30} + r_{32} + r_{33} + r_{35} + r_{36} + r_{39} + r_{40} + r_{41} + r_{42} \leq M$$

Then solve the linear equation from above by using simplex method, when $M = \{5000, 10000, 15000, 20000, 25000, 30000, 35000\}$. The probability of completing the project by 415 days is show as follows:

Table 4.5 : The probability of completing the project by 415 days when $M = \{0, 5000, 10000, 15000, 20000, 25000, 30000, 35000\}$

No.	M	$\sum_{i=1}^n \sigma_i^2$	T_S	T_E	σ_E	Z		(%)
1	0	53.996	415	421	7.348197	-0.81653	≈ -0.82	20.61
2	5000	45.66		417.957	6.757218	-0.43761	≈ -0.44	33
3	10000	39.468		416.075	6.282356	-0.17111	≈ -0.17	43.25
4	15000	37.409		414.493	6.11629	0.082893	≈ 0.08	53.19
5	20000	35.957		413.461	5.996416	0.256653	≈ 0.26	60.26
6	25000	33.159		412.932	5.758385	0.359128	≈ 0.36	64.06
7	30000	30.337		412.538	5.507903	0.446994	≈ 0.45	67.36
8	35000	30.112		412.338	5.48744	0.485108	≈ 0.49	68.79

Then the results are display with the probability density function (pdf) and cumulative distribution function (cdf) of normal distribution.

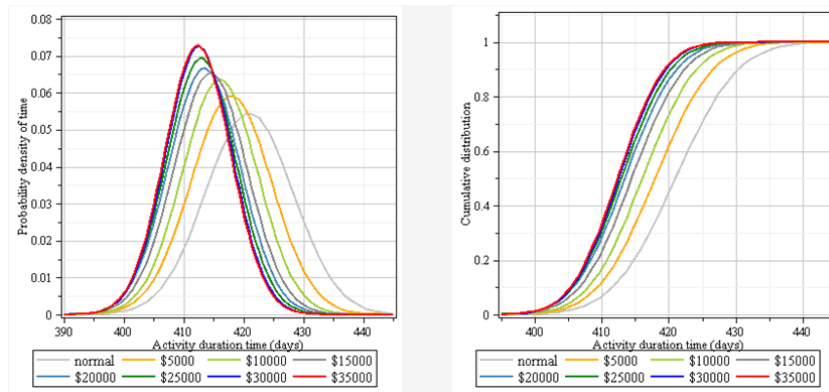


Figure 4.1 : Pdf of normal distribution **Figure 4.2 : Cdf of normal distribution**

Figure 4.1 shows that the expected duration of the project decreases simultaneously with the increase of the amount of money that to be invest to activities that lie on the critical path. Moreover, the bell-curve of the project becomes more peak when the amount of the money to be invest to the project increase accordingly. Besides that, figure 4.2 shows that the cumulative distribution’s curve becomes more and more slope with the increase of the amount of money to be invest to the activity that lie on the critical path.

5 DISCUSSION AND CONCLUSION

The probability of completing the project within 415 days was increased from 20.61%, to 33%, 43.25%, 53.19%, 60.26%, 64.06%, 67.36% and 68.79% to the investment of the amounts of money \$5000, \$10000, \$15000, \$20000, \$25000, \$30000 and \$35000.

From the Table 4.5, the expected duration of the project was successfully reduced from 421 days to 412 days upon the increased in the amounts of money from \$5000 to \$35000 to the project. Besides that, Figure 4.3-4.10(figures on next page) show that the area under the normal distribution was become larger when the expected duration of the project was reduced small enough to the scheduled times.

Unfortunately, the results of the probability of completing the project within 415 days for the additional investment of \$30000 was differ with the [3]. That is 67.36% and 35%. However, according to the table from [10], probability that project will be completed sooner than the expected duration are 1%-50%, while probability that project will be completed later than

expected duration are 50%-99%. Moreover, from [2],[4],[6] and [7], there exist an example shows that probability of the project will be complete later than expected duration are 50%-99%.

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**Probability density function of normal distribution when
 $0 \leq M \leq 35000$**

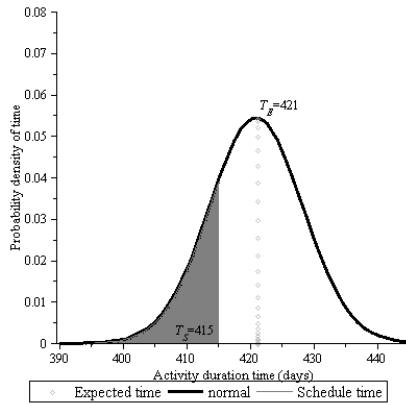


Figure 4.3 : Pdf for M=0

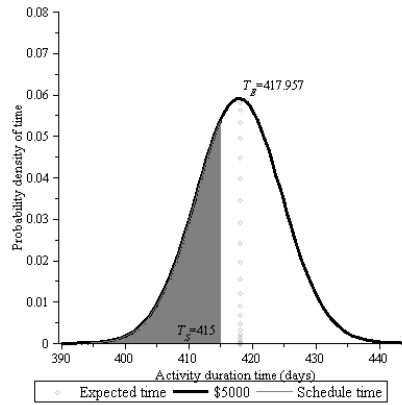


Figure 4.4 : Pdf for M=5000

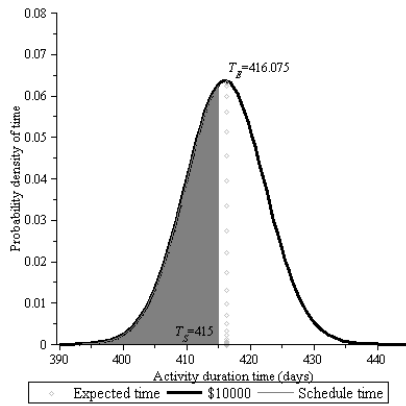


Figure 4.5 : Pdf for M=10000

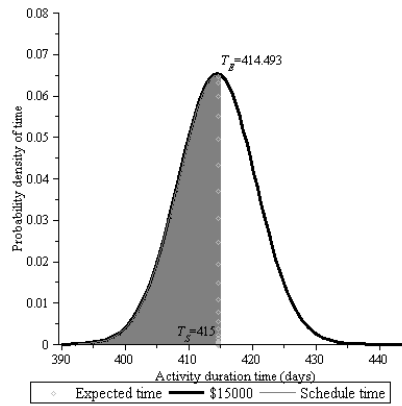


Figure 4.6 : Pdf for M=15000

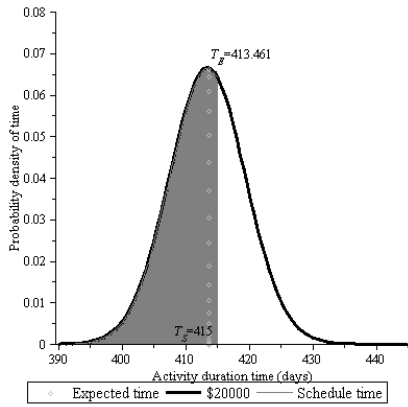


Figure 4.7 : Pdf for M=20000

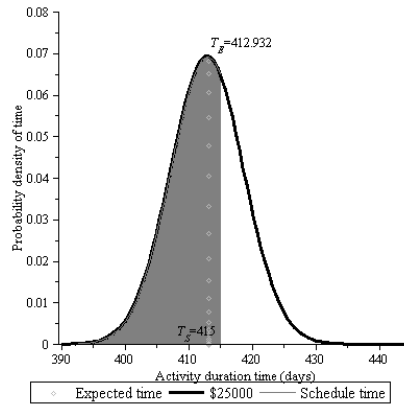


Figure 4.8 : Pdf for M=25000

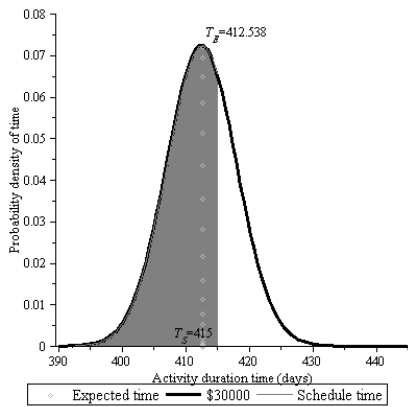


Figure 4.9 : Pdf for M=30000

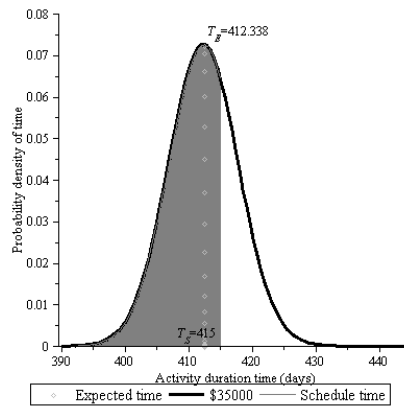


Figure 4.10 : Pdf for M=35000