Journal of Science and Technology

BPGD-AG: A New Improvement Of Back-Propagation Neural Network Learning Algorithms With Adaptive Gain

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Abstract

The back propagation algorithm is one of the popular learning algorithms to train self learning feed forward neural networks. However, the convergence of this algorithm is slow mainly because the algorithm required the designers to arbitrarily select parameters such as network topology, initial weights and biases, learning rate value, the activation function, value for gain in activation function and momentum. An improper choice of theses parameters can result the training process comes to as standstill or get stuck at local minima. Previous research demonstrated that in a back propagation algorithm, the slope of the activation function is directly influenced by a parameter referred to as 'gain'. In this paper, the influence of the variation of 'gain' on the learning ability of a back propagation neural network is analysed. Multi layer feed forward neural networks have been assessed. Physical interpretation of the relationship between the gain value and the learning rate and weight values is given. Instead of a constant 'gain' value, we propose an algorithm to change the gain value adaptively for each node. The efficiency of the proposed algorithm is verified by means of simulation on a function approximation problem using sequential as well as batch modes of training. The results show that the proposed algorithm significantly improves the learning speed of the general back-propagation algorithm.

Keywords : Neural Networks; Gain; Activation function; Learning rate; Training Efficiency.

1 INTRODUCTION

We consider standard multi-layer feed forward neural networks that have an input layer of neurons, a hidden layer of neurons and an output layer of neurons, and in that every node in a layer is connected to every other node in the adjacent forward layer. The back-propagation algorithm has been the most popular and most widely implemented for training these types of neural network. When using the back-propagation algorithm to train a multilayer neural network, the designer is required to arbitrarily select parameter such as the network topology, initial weights and biases, a learning rate value, the activation function, and a value for the gain in the activation function. An improper choice of any of these parameters can result in slow convergence or even network paralysis where the training process comes to a virtual standstill. Another problem is the tendency of the steepest descent technique, which is used in the training process, to get stuck at local minima.

In recent years, a number of research studies have attempted to overcome these problems. Theses involved the development of heuristic techniques, based on studies of properties of the general back-propagation algorithm. These techniques include such idea as varying the learning rate, using momentum, gain tuning of activation function. Perantonis et. al. [1] proposed an algorithm for efficient learning in feed forward neural networks using momentum acceleration. Kamarthi et. al. [2] presents a universal acceleration technique for the back-propagation algorithm based on extrapolation of each individual interconnection weight. Research has also focused on the use of standard numerical optimisation techniques. Moller [3] explained how conjugate gradient algorithm could be used to train multi-layer feed forward neural networks while Lera et. al. [4] described the use of Levenberg-Marquardt algorithm for training multi-layer feed forward neural networks. However, most of these methods are quite complex, require excessive memory and are computationally very expensive.

In order to improve the performance of the back-propagation algorithm an algorithm has been proposed in this paper to change the gain value adaptively. It is shown that changing the 'gain' value adaptively for each node can significantly reduce the training time. In order to verify the efficiency of the proposed method, and to compare it with the general back-propagation algorithm, we perform simulation experiments on a function approximation problem using the sequential as well as batch modes of training.

2 EFFECT OF THE GAIN PARAMETER ON THE PERFORMANCE OF A NEURAL NETWORK

An activation function is used for limiting the amplitude of the output of a neuron. It generates an output value for a node in a predefined range as the closed unit interval [0, 1] or alternatively [-1, +1]. This value is a function of the weighted inputs of the corresponding node. The most commonly used activation function is the logistic sigmoid activation function. Alternative choices are the hyperbolic tangent, linear, step activation functions. For the j^{th} node, a logistic sigmoid activation function which has a range of [0, 1] is a function of the following variables, viz.

$$o_j = \frac{1}{1 + e^{-c_j a_{net,j}}} \tag{1}$$

Where,

$$a_{net,j} = \left(\sum_{i=1}^{l} w_{ij} o_i \right) + \theta_j \tag{2}$$

Where,

o_j	Output of the j^{th} unit.
w_{ij}	weight of the link from unit i to unit j.
$a_{net,j}$	net input activation function for the j^{th} unit.
θ_j	bias for the j^{th} unit.
c_j	gain of the activation function.

The value of the gain parameter, c_j, directly influences the slope of the activation function. For large gain values ($c >> 1$), the activation function approaches a ‘step function’ whereas for small gain values ($0 < c << 1$), the output values change from zero to unity over a large range of the weighted sum of the input values and the sigmoid function approximates a linear function.

Most of the application oriented papers on neural networks tend to advocate that neural networks operate like a ‘magic black box’, which can simulate the “learning from example” ability of our brain with the help of network parameters such as weights, biases, gain, hidden nodes etc. There are very few publications, or textbooks, which give physical interpretation for various parameters used in the network. Also, a unit value for gain has generally been used for most of the research reported in the literature but a few authors have researched the relationship of the gain parameter with other parameters used in back-propagation algorithms. The recent results [5] show that learning rate, momentum constant and gain of the activation function have a significant impact on training speed. However, higher values of learning rate and/or gain cause instability [6]. Thimm et. al. [7] also proved that a relationship between the gain value, a set of initial weight values, and a learning rate value exists. Looney [8] suggested to adjust the gain value in small increments during the early iterations and to keep it fixed somewhere around halfway through the learning. Eom et. al. [9] proposed a method for automatic gain tuning using a fuzzy logic system.

However, the authors have not come across publications in the literature that have implemented adaptive gain variation as proposed in this research work.

3 THE PROPOSED ALGORITHM USING SEQUENTIAL MODE OF TRAINING.

We propose an algorithm for changing the gain value adaptively for each node which is implemented for the sequential mode of training. The following subsection describes the algorithm. The advantages of using an adaptive gain value have been explored. Gain update expressions as well as weight and bias update expressions for output and hidden nodes have also been proposed. These expressions have been derived using same principles as used in deriving weight updating expressions.

The sequential mode of training requires an immediate updating of weights, biases and gains after the presentation of training example. An epoch is said to be complete after the presentation of the entire training set. A sum squared error value is calculated after the presentation of each training example and compared with the target error. Training is done on an epoch-by-epoch basis until the sum squared error value falls below the desired target error value.

The following iterative algorithm is proposed by the authors for the sequential mode of training. Weight, biases and gains are calculated and updated for each training example, which is being presented to the network.

For a given epoch,
    For each example,
    Step 1
    Calculate the weight and bias values using the previously converged gain value.
    Step 2
    Use the weight and bias value calculated in step (1) to calculate the new gain value.
Repeat Steps (1) and (2) for each example on an epoch-by-epoch basis until the error on the entire training data set reduces to a predefined value.

The gain update expression for a gradient descent method is calculated by differentiating the following error term *E* with respect to the corresponding gain parameter.

The network error *E* is defined as follows:

$$E = \frac{1}{2}\sum (t_k - o_k(o_j, c_k))^2 \tag{3}$$

For output unit, $\frac{\partial E}{\partial c_k}$ needs to be calculated whereas for hidden units. $\frac{\partial E}{\partial c_j}$ is also required. The respective gain values would then be updated with the following equations.

$$\Delta c_k = \eta\,(-\frac{\partial E}{\partial c_k}) \tag{4}$$

$$\Delta c_j = \eta\,(-\frac{\partial E}{\partial c_j}) \tag{5}$$

$$\frac{\partial E}{\partial c_k} = -\left(t_k - o_k\right) o_k (1 - o_k) \left(\sum w_{jk} o_j + \theta_k \right) \tag{6}$$

Therefore, the gain update expression for links connecting to output nodes is:

$$\Delta c_k(n+1) = \eta\,(t_k - o_k)\,o_k(1-o_k)\left(\sum w_{jk} o_j + \theta_k\right) \tag{7}$$

$$\frac{\partial E}{\partial c_j} = \left[-\sum_k c_k w_{jk} o_k (1-o_k)(t_k - o_k)\right] o_j (1-o_j)\left(\left(\sum_j w_{ij} o_i\right) + \theta_j\right) \tag{8}$$

Therefore, the gain update expression for the links connecting hidden nodes is:

$$\Delta c_j(n+1) = \eta\left[-\sum_k c_k w_{jk} o_k (1-o_k)(t_k - o_k)\right] o_j (1-o_j)\left(\left(\sum_j w_{ij} o_i\right) + \theta_j\right) \tag{9}$$

Similarly, the weight and bias expressions are calculated as follows:

The weight update expression for the links connecting to output nodes with a bias is:

$$\Delta w_{jk} = \eta\,(t_k - o_k)\,o_k(1-o_k)\,c_k o_j \tag{10}$$

Similarly, the bias update expressions for the output nodes would be:

$$\Delta \theta_k = \eta\,(t_k - o_k)\,o_k(1-o_k)\,c_k \tag{11}$$

The weight update expression for the links connecting to hidden nodes is:

$$\Delta w_{ij} = \eta\left[\sum_k c_k w_{jk} o_k (1-o_k)(t_k - o_k)\right] c_j o_j (1-o_j)\,o_i \tag{12}$$

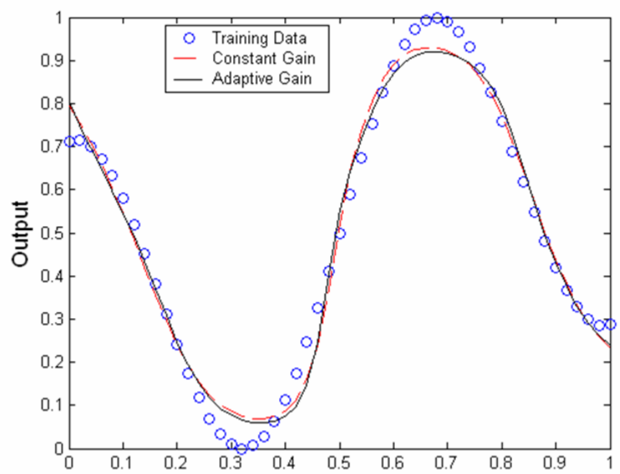
Similarly, the bias update expressions for the hidden nodes would be:

4 VALIDATIONS ON THE 'COSINE CURVE' AND 'SINE CURVE' EXAMPLES USING SEQUENTIAL MODE

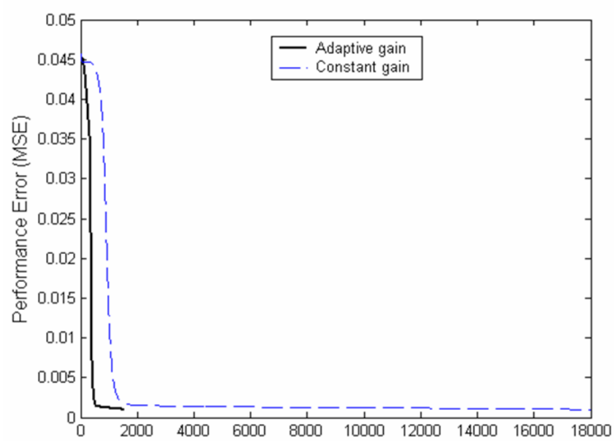
The results of our proposed algorithm were validated on a standard feed forward neural network with one hidden layer having five hidden nodes. The training data set is created based on Bishop [10] by using the same function $y = x + \cos(3 * pi * x)$ and $y = x + \sin(2 * pi * x)$ *where* $x \in [0,1]$. The network is trained using the learning rate value of 0.3 to achieve a target error of 0.001. The Gradient Descent training algorithm was employed in a sequential mode with adaptive changes in weight, bias and gain values. The initial weight and bias values were chosen as small random numbers in the range [-1, +1].

The network is trained with an adaptive gain with an initial value of unity for the gain parameter for all output as well as hidden nodes. In Figure 1(a) the network output (continuous curve) is shown against the training data points (circles) using the function $y = x + \cos(3 * pi * x)$. The output of the network using constant unit gain value is also plotted in Figure 1(a) (dotted curve). The gain values for the five hidden nodes at the end of training are 1.5831, 0.9126, 1.9954, 1.9959 and 1.1594 respectively. The gain value for output node at the end of training is 1.9941.

As shown in Figure 1(b) the speed of convergence of the proposed algorithm is high due to the modified gain values. The network required 1537 epochs to achieve the target error using the proposed adaptive gain algorithm in sequential mode, whereas using the same set of initial weight and biases the network required 17966 epochs to achieve the target error using constant unit gain value during training.

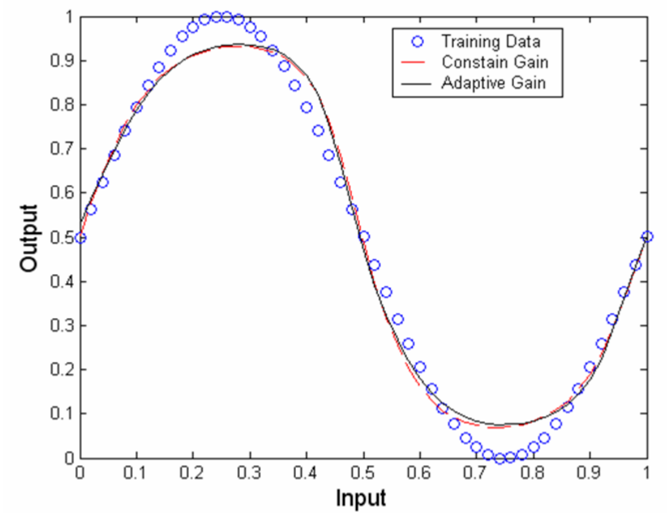


(a)

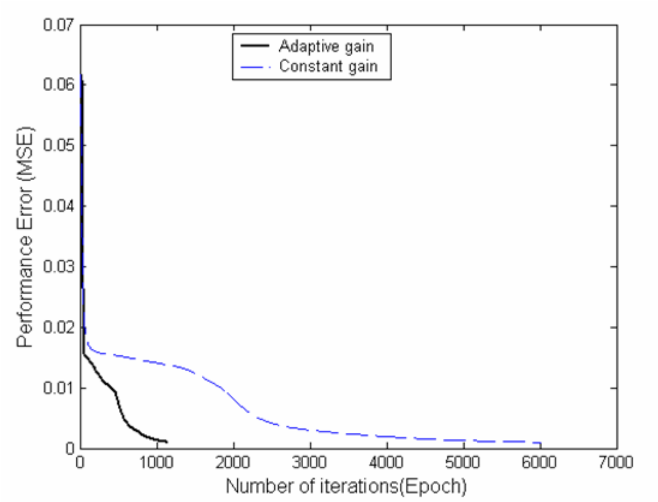


(b)

Figure 1 : Output of the neural network training to learn a cosine curve with and without using the adaptive gain algorithm in sequential mode of training (a), and convergence speed for the cosine function with and without using the adaptive gain algorithm (b)



(a)



(b)

Figure 2 : Output of the neural network training to learn a sine curve with and without using the adaptive gain algorithm in sequential mode of training (a), and convergence speed for the sine function with and without using the adaptive gain algorithm (b).

In Figure 2(a) the network output (continuous curve) is shown against the training data points (circles) y = x + sin(2*pi*x). The output of the network using constant unit gain value is also plotted in Figure 1(b) (dotted curve). The gain values for the five hidden nodes at the end of training are 1.3048, 0.7119, 1.9958, 1.5527 and 1.9985 respectively. The gain value for output node at the end of training is 1.9956. Again, the result showed that the speed of convergence is high due to the modified gain values. As shown in figure 2(b) the network required 1154 epochs to achieve the target error using the proposed adaptive gain algorithm in sequential mode, whereas using the same set of initial weight and biases the network required 6014 epochs to achieve the target error using constant unit gain value during training.

Comparing both the curves in Figure 1(a) and Figure 2(a) it can be seen that the training performance of the adaptive gain algorithm is similar to that using constant gain value. However the speed of convergence of adaptive gain algorithm is very high as compared to that using constant gain value as shown in Figure 1(b) and Figure 2(b). There is a dramatic improvement in the learning speed of back-propagation algorithm.

5 THE PROPOSED ALGORITHM USING BATCH MODE OF TRAINING

We propose an algorithm for changing the gain value adaptive for each node which is implemented for the batch mode of training. The following subsection describes the algorithm. The advantages of using an adaptive gain value have been explored. The gain update expressions as well as weight and bias update expressions for output and hidden nodes have already been derived in Section 3. The only difference is that in the batch mode of training the weight, bias and gain updation terms are calculated and summed for all the training examples. Then the weights, biases and gains are updated after one complete presentation of the entire training set. An epoch is said to be complete after the presentation of the entire training set. A sum squared error value is calculated after the presentation of the training set and compared with the target error. Training is done on an epoch-by-epoch basis until the sum squared error falls below the desired target value.

The following iterative algorithm is proposed by the authors for the batch mode of training. Weights, biases and gains are calculated and updated for the entire training set, which is being presented to the network.

For a given epoch,

    For each example,

    Step 1

    Calculate the weight and bias and gain updation terms using the previously converged gain value.

    Step 2

    Use the weight and bias values calculated in Step (1) to calculate the gain updation value

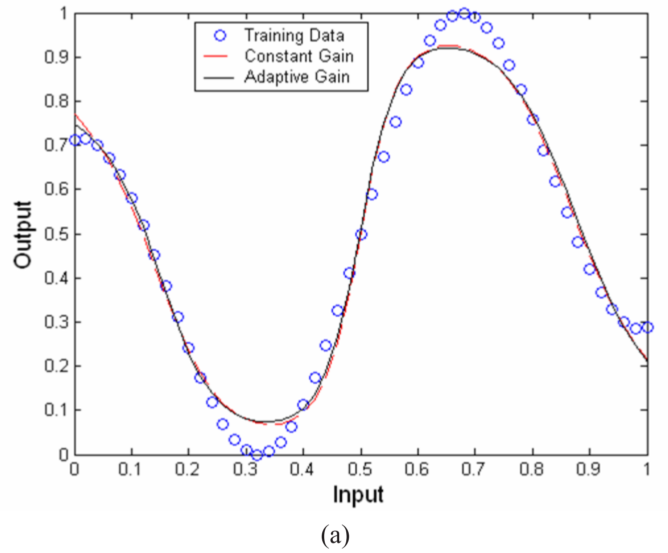
    Repeat Steps (1) and (2) for each example and sum all the weights, biases and gain updation terms.

Update the weights, biases and gains using the summed updation terms and repeat this procedure on an epoch-by-epoch basis until the error on the entire training data set reduce to a predefined value.

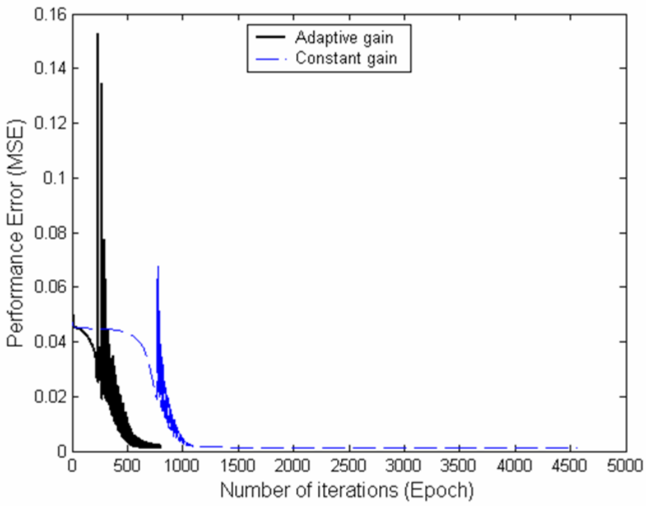
6 VALIDATION ON THE 'COSINE CURVE' AND 'SINE CURVE' EXAMPLES USING BATCH MODE

The results of our proposed algorithm were validated on a standard feed forward neural network with one hidden layer having five hidden nodes. The training data set is created based on Bishop [10] by using the function $y = x + \cos(3 * pi * x)$ and $y = x + \sin(2 * pi * x)$ where x ϵ [0, 1]. The network is trained using 0.3 as the learning rate value to achieve a target error equal to 0.001. The Gradient Descent training algorithm was employed in a batch mode with adaptive changes in weight, bias and gain values. The initial weight and bias values were chosen as small random numbers in the range [-1, +1]. The network is trained with an adaptive gain with an initial value of unity for the gain parameter for all output as well as hidden nodes. In Figure 3(a) the network output (continuous curve) is shown against the training data points (circles) using the function $y = x + \cos(3 * pi * x)$. The output of the network using constant unit gain value is also plotted in Figure 3(a) (dotted curve).

In Figure 3(b) the network required 803 epochs to achieve the target error using the proposed adaptive gain algorithm in batch mode, whereas using the same set of initial weights and biases the network required 4559 epochs to achieve the target error using unit gain value during training.



(a)

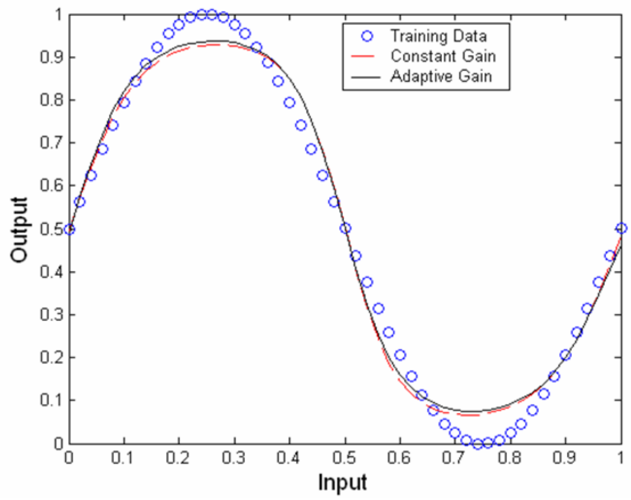


(b)

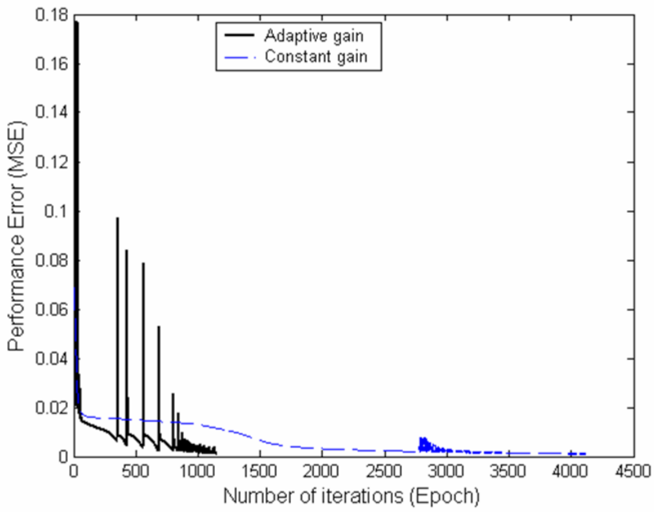
Figure 3 : Output of the neural network training to learn a cosine curve with and without using the adaptive gain algorithm in batch mode of training (a), and convergence speed for the cosine function with and without using the adaptive gain algorithm in batch mode of training (b).

In Figure 4(a) the network output (continuous curve) is shown against the training data points (circles). The output of the network using constant unit gain value is also plotted in Figure 4(a) (dotted curve). Again, the result showed that the speed of convergence of the proposed algorithm is high due to the modified gain values. As shown in Figure 4(b) the network required 1148 epochs to achieve the target error using the proposed adaptive gain algorithm in batch mode, whereas using the same set of initial weight and biases the network required 4114 epochs to achieve the target error using constant unit gain value during training.

Comparing both the curves in Figure 3(a) and Figure 4(a) it can be seen that the training performance of the proposed adaptive gain algorithm is similar to that using constant gain value. However the speed of convergence of the proposed adaptive gain algorithm is very high as compared to that using constant gain value as shown in Figure 3(b) and Figure 4(b). The results proved that there is a dramatic improvement in the learning speed of back-propagation algorithm.



(a)



(b)

Figure 4 : Output of the neural network training to learn a sine curve with and without using the adaptive gain algorithm in batch mode of training (a), and convergence speed for the sine function with and without using the adaptive gain algorithm in batch mode of training (b).

7 EFFECT OF NUMBER OF HIDDEN NODES, TARGET NODES, TARGET ERROR AND INITIAL WEIGHT AND BIAS VALUES ON NETWORK TRAINING.

A standard feed forward neural network with one hidden layer having fifteen hidden nodes is now used to validate the results of our approach. The network is trained using the learning rate value of 0.3 and momentum value of 0.4 (in batch mode) to achieve a target error of 0.0001. The network was trained using the Gradient Descent training algorithm in both sequential and batch modes of training with adaptive changes in weight, bias and gain values. The network output for sequential mode (continuous curve) and batch mode (dotted curve) is shown against the training data points (circles) in Figure 5. It can be observed that outputs form both modes are almost the same and they overlap each other. Also, it can be seen that as the number of hidden nodes in the network is increased and the target error is reduced further, the trained network over fits the data and the curve passes through almost all the data points.

The initial weights and biases were chosen as normally distributed random numbers in the range [-1, +1]. The same network is also trained on the same training data set using uniformly distributed random numbers in the range [0, 1]. However, there was 50% reduction in the training speed when using normally distributed random numbers. Hence, we conclude that the speed of training is also affected by the choice of initial weights and biases.

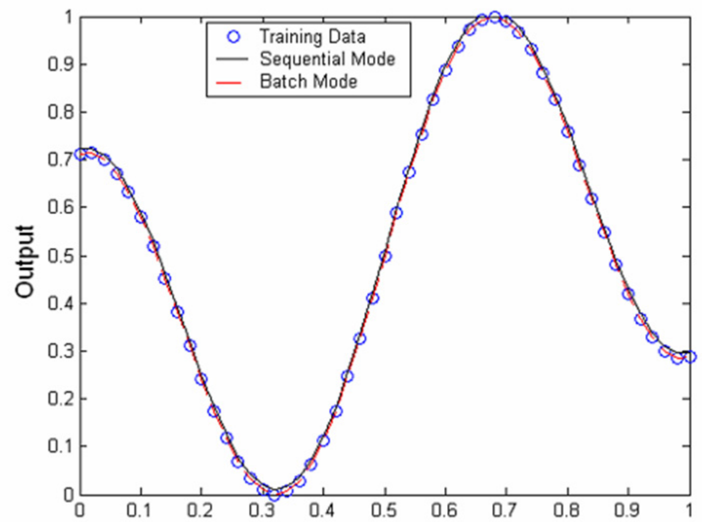
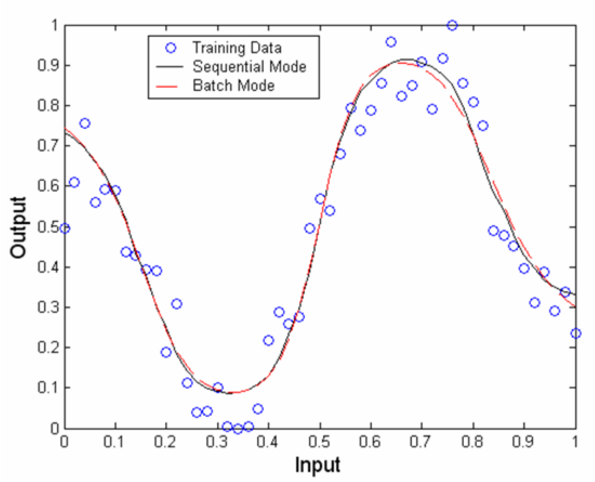


Figure 5 : Output of the neural network trained to learn a cosine curve using the adaptive gain algorithm in both sequential (continuous curve) and batch mode (dotted curve) of training.

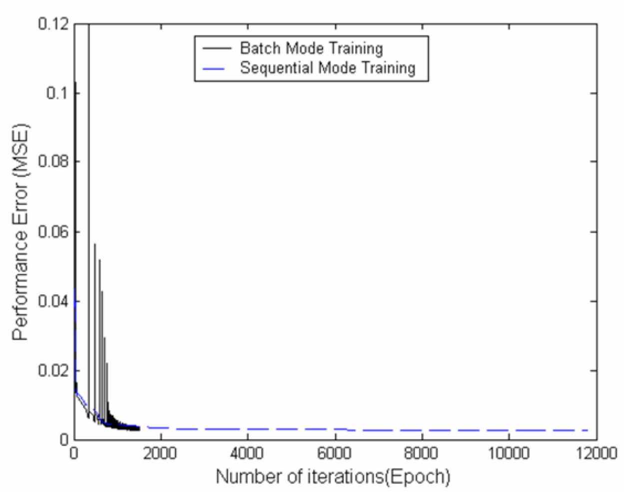
8 EFFECT OF NOISE

A standard feed forward neural network with one hidden layer having five hidden nodes is trained using a training data set which is created by using the function $y = x + \cos(3 * pi * x)$ and $y = x + \sin(2 * pi * x)$ where x ϵ [0,1], and adding 20% random Gaussian noise in it. The network is trained using the learning rate value of 0.3 and momentum value of 0.4 (in batch mode) to achieve a target error of 0.002. The initial weights and biases are small normally distributed random numbers in the range [-1, +1]. The network was trained using the Gradient Descent training algorithm in both sequential and batch modes of training with adaptive changes in weight, bias and gain values. The network output for sequential mode (continuous curve) and batch mode (dotted curve) is shown against the training data points (circles) in Figure 6(a) and Figure 7(a). It can be observed that network trained in both modes perform in a similar way.

For the function $y = x + \cos(3 * pi * x)$, the network required 7449 epochs to learn the target function using the sequential mode of training and only 609 epochs using the batch mode of training as shown in Figure 6(b). And for the function $y = x + \sin(2 * pi * x)$, the network required 11813 epochs to learn the target function using the sequential mode of training and 1505 epochs using the batch mode of training as shown in Figure 7(b). In practical both situations data is generally inherently noisy. The output from both modes for both functions shows that the network is also successful in learning the target function when there is noise present in the data. The results strengthen our belief in the better working of the adaptive gain algorithm.

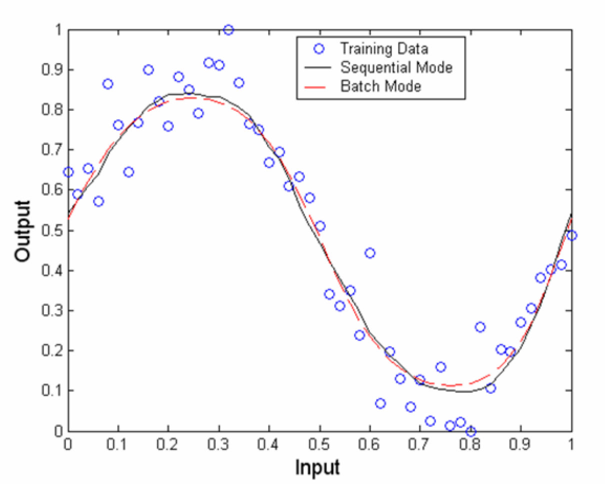


(a)

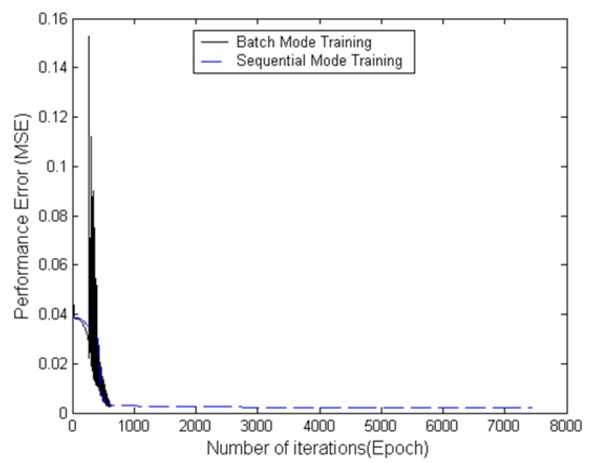


(b)

Figure 6 : Output of the neural network trained to learned a cosine curve with 20% random Gaussian noise using the adaptive gain algorithm in both sequential (continuous curve) and batch mode (dotted curve) of training (a), and convergence speed for the cosine function with using the adaptive gain algorithm in batch and sequential mode of training (b).



(a)



(b)

Figure 7 : Output of the neural network trained to learned a sine curve with 20% random Gaussian noise using the adaptive gain algorithm in both sequential (continuous curve) and batch mode (dotted curve) of training (a), and convergence speed for the sine function with using the adaptive gain algorithm in batch and sequential mode of training (b).

9 ADVANTAGES OF USING THE ADAPTIVE GAIN VARIATION

An algorithm has been proposed in this paper for the efficient calculation of the adaptive gain value in both sequential and batch modes of learning.

We proposed that the total learning rate value can be split into two parts – a local (nodal) learning rate value and a global (same for all nodes in a network) learning rate value. The value of parameter gain is interpreted as the local learning rate of a node in the network. The network is trained using a fixed value of learning rate equal to 0.3 which is interpreted as the global learning rate of the network. However, as the gain value was modified, the weights and biases were updated using the new value of gain. This resulted in higher values of gain which caused instability [7]. To avoid oscillations during training and to achieve convergence, an upper limit of 2.0 is set for the gain value. This will be explained in detail in our next publication. The method has been illustrated for Gradient Descent training algorithm using the sequential and batch modes of training. An advantage of using the adaptive gain procedure is that it is easy to introduce into a back-propagation algorithm and it also accelerates the learning process without a need to invoke solution procedures other than the Gradient Descent method. The adaptive gain procedure has a positive effect in the learning process by modifying the magnitude, and not the direction, of the weight change vector. This greatly increases the learning speed by amplifying the directions in the weight space that are successfully chosen by the Gradient-Descent method. However, the method will also be advantageous when using other faster optimisation algorithms such as Conjugate-Gradient method and Quasi-Newton method. These methods can only optimise an equivalent of the global learning rate (the step length). By introducing an additional local learning rate parameter, further increase in the learning speed can be achieved. Work is currently under progress to implement this algorithm using other optimisation methods.

10 CONCLUSION

While the back-propagation algorithm is used in the majority of practical neural networks application and has been shown to perform relatively well, there still exist areas where improvement can be made. We proposed an algorithm to adaptively change the gain parameter of the activation function to improve the learning speed. It was observed that the influence of variation in the gain value is similar to the influence of variation in the learning rate value. An algorithm has been proposed in this paper to change the gain value adaptively for each node.

In order to verify the effectiveness of the proposed algorithm, the function approximation problem was simulated and analysed using both sequential and batch modes of training. The results showed that the proposed adaptive gain algorithm has a better convergence rate and learning efficiency as compared to the general back-propagation algorithm.

The network also demonstrated the principles of over fitting vs. generalization as the number of hidden nodes in the network was increased and the target error was reduced further. The choice of normally distributed random numbers in the range [-1, +1] for the initial values of weights and biases in the network greatly speeded the training process.

In practical situations data is generally inherently noisy. A network trained with the adaptive gain algorithm in sequential as well as batch modes of training was also successful in learning the target function when there was noise present in the data. The results strengthen our belief in the better working of the adaptive gain algorithm.

ACKNOWLEDGMENT

The authors would like to thank Universiti Tun Hussein Onn Malaysia for supporting this research under the Short Term Research Grant.

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