

Analysis of Plasmonic Structure using Finite Element Method

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Abstract: Plasmonic structure was investigated by modelling the different shape of v-groove shape and curve shape in three dimension (3D) using Finite Element Method (FEM). We simulate and compare the shape with electric and magnetic wave propagation in term of total electric energy and total magnetic energy using gold as a metal material and glass as a dielectric material. The result of the reflection and transmission of transverse electric (TE) wave and transverse magnetic (TM) wave with the angle of incidence are shown depending on the shape of plasmonic structure by using analytic solution based on Fresnel equation.

Keyword: Plasmonic structure; Finite Element Method; Angle of Incidence; Fresnel Equation.

1. Introduction

Many researchers explore the applications of plasmonic structure that can give potential benefit like solar cells, biochemical sensing, optical computing and cancer treatments [1]. Plasmonics which is the process of interaction between the free electrons at metallic interfaces and the electromagnetic field [2] exactly can do it in nanoscale. In plasmonic structure, plasmon acts as the waves that produce from the interaction between the free electron and electromagnetic wave [3]. Maxwell's equation is the equations that describe all (classical) electromagnetic phenomena.

$$\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} \quad (1)$$

$$\nabla \times \mathbf{H} = \mathbf{J} - \frac{\partial \mathbf{D}}{\partial t} \quad (2)$$

$$\nabla \cdot \mathbf{D} = \rho \quad (3)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (4)$$

where, electric field \mathbf{E} , magnetic field \mathbf{H} , electric flux density \mathbf{D} , magnetic flux density \mathbf{B} , and electric current density \mathbf{J} , and charge ρ [2].

In this paper, the plasmonic structure in three dimensions is numerically solved by using FEM. The different shapes of plasmonic structure which are v-groove shape and curve shape were analyzed in term of electric and magnetic wave propagation with reflection and transmission. The materials are gold on the surface and glass of the bottom of the surface.

The rest of the paper is organized as follows. Modelling and methodology are outlined in section 2, numerical result and discussion in section 3 and conclusion in section 4.

2. Modelling Plasmonic Structure

The schematic of a plasmonic structure with different shape is shown in Fig. 1. In this article, the different of shape are investigated under fixed size for both shapes in three dimensions (3D). The shape was compared to determine their ability toward the application of plasmonic.

In order to obtain the reflection and transmission for TE and TM, Fresnel's equation are used to compute the solution with angle of incident and the existence of the Brewster angle [4].

2.1 Fresnel's equation on reflection and transmission coefficient

The equation of a reflection and transmission coefficient for s-polarization and p-polarization [5,6] was used and can be defined as

$$r_s = \frac{n_1 \cos \theta_{inc} - n_2 \cos \theta_{tra}}{n_1 \cos \theta_{inc} + n_2 \cos \theta_{tra}} \quad (5)$$

$$r_p = \frac{n_2 \cos \theta_{inc} - n_1 \cos \theta_{tra}}{n_1 \cos \theta_{tra} + n_2 \cos \theta_{inc}} \quad (6)$$

$$t_s = \frac{2n_1 \cos \theta_{inc}}{n_1 \cos \theta_{inc} + n_2 \cos \theta_{tra}} \quad (7)$$

$$t_p = \frac{2n_1 \cos \theta_{inc}}{n_1 \cos \theta_{tra} + n_2 \cos \theta_{inc}} \quad (8)$$

where r_s is reflection coefficient, transverse electric (TE), r_p is reflection coefficient, transverse magnetic (TM), t_s is transmission coefficient, TE, t_p is transmission coefficient, TM, and n is refractive index.

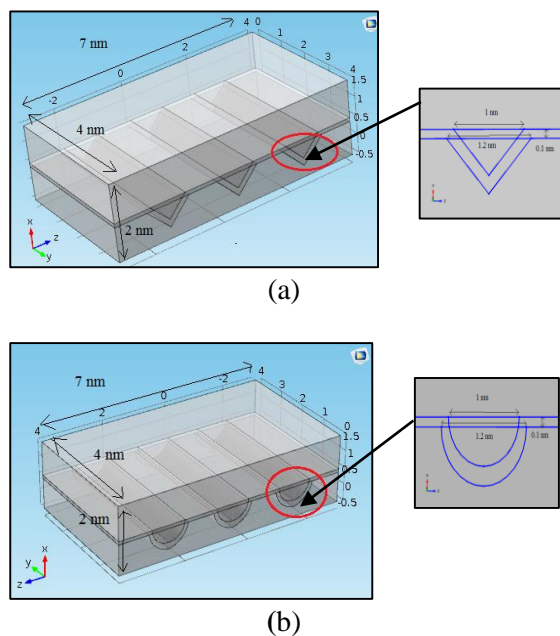


Fig. 1 The schematic of a plasmonic structure in (a) v-groove shape and (b) curve shape.

3. Numerical Result and Discussion

Finite element method (FEM) was utilized for the numerical analysis of the proposed mesh and computed solution. By applying the FEM, we solved for the electric field and magnetic field in entire structure.

3.1 Mesh and the computed solution

In mesh solution, the number of nodes and element that generated depends on the geometry of the model because a larger element size will make a faster solution than smaller element size. However, for a more accurate solution, a smaller element size may be required.

In computed the solution, v-groove shapes has takes a shorter time compared curve shape. This is because curve shapes have many elements and nodes that need to solve especially in curve area compare with v-groove area as shown in Fig. 2.

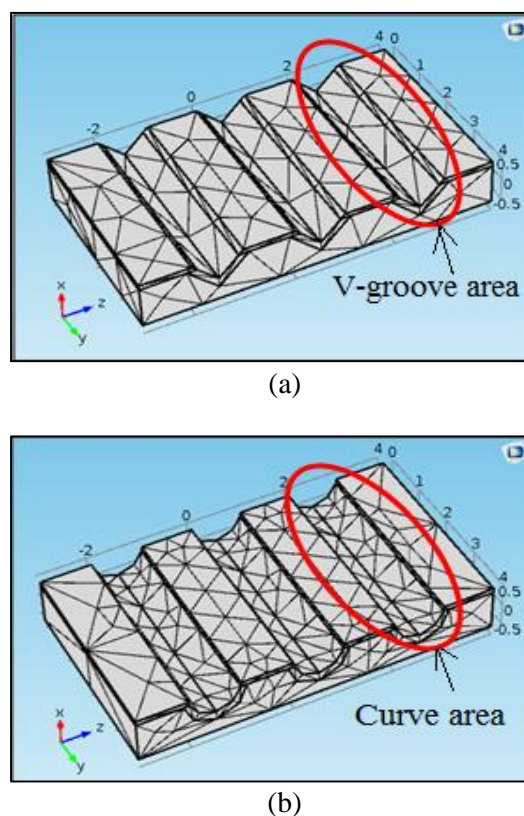


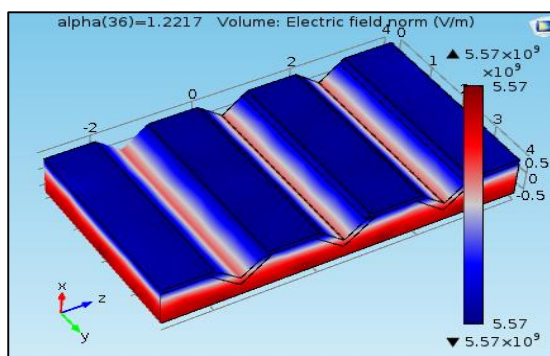
Fig. 2 Mesh solution for (a) V- groove shape and (b) Curve shape

To have a basic image of the plasmonic properties of v-groove and curve shape

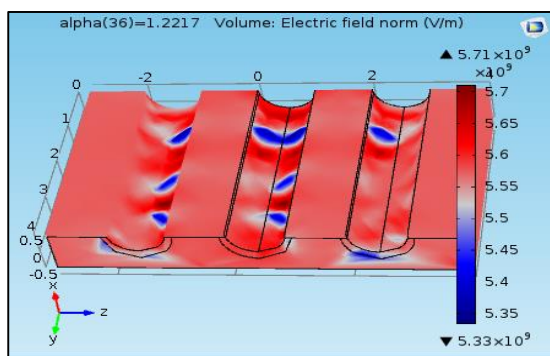
structure, a structure consist electric and magnetic field was investigated.

3.2 Electric field wave propagation

From this simulation, the structures that have a different shape as shown in Fig. 3 proved that the electric field intensity was influenced by plasmonic structure.



(a)



(b)

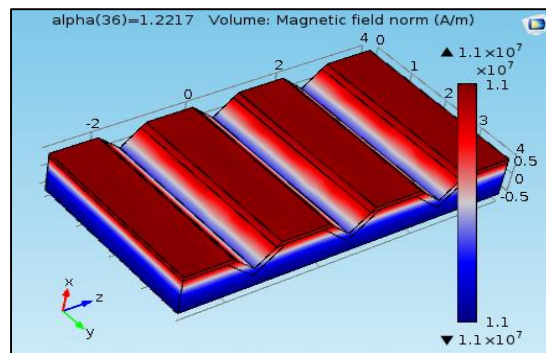
Fig. 3 Electric field in normal for (a) v-groove shape and (b) curve shape

From Fig. 3(a) and Fig. 3(b), the red color represent as the higher electric field intensity than the blue color. The intensity at the v-groove area as shown in Fig. 3(a) was moderated around 5.57185×10^9 V/m compare with the flat area. This means that the intensity of electric field at v-groove area still has electric field but not very high from flat area at bottom layer.

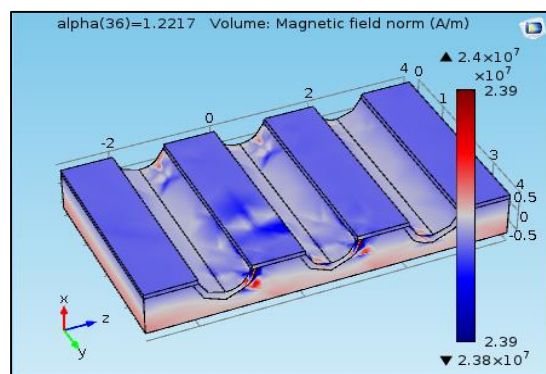
From Fig. 3(b), the intensity was different with v-groove shape. The higher intensity of electric field more focused in curve area which is about 5.7×10^9 V/m than the flat area although gold and glass layer already have a high intensity of about 5.5×10^9 V/m.

3.3 Magnetic field wave propagation

From Fig 4(a), the intensity at the v-groove area also was moderated about 1.10415×10^7 A/m compare with the flat area at upper layer like electric field intensity.



(a)



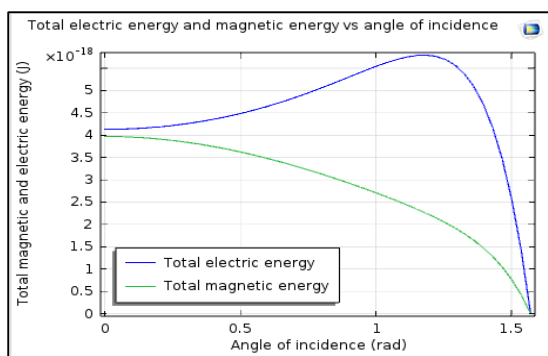
(b)

Fig. 4 Magnetic field in normal for (a) v-groove shape and (b) curve shape

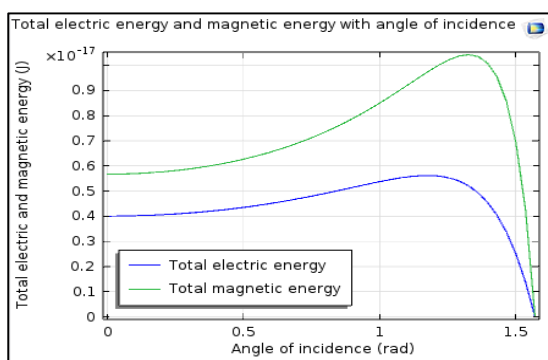
From Fig. 4(b), the lower intensity of magnetic field intensity more focused in flat area at gold layer while for the curve area, the intensity was moderated about 2.38800×10^7 A/m. So, in concluded, the intensity of electric field in glass layer was higher than gold layer.

3.4 Total electric energy and magnetic energy for v-groove and curve shape of plasmonic structure

The graph of total electric energy and magnetic energy for v-groove and curve shape was related with the intensity of electric and magnetic field as shown in Fig. 3, and Fig. 4.



(a)



(b)

Fig. 5 Total electric energy and magnetic energy with angle of incidence for (a) v-groove shape and (b) curve shape of plasmonic structure.

From Fig. 5(a), the total electric energy was decreased smoothly from 4×10^{-18} J when angle of incidence was increased. While, for the total magnetic energy, the value increased from 4.1×10^{-18} J to 6.0×10^{-18} J at 1.2 radian and declined suddenly when the angle of incidence close to 1.5 radian. However, the total electric energy was higher than total magnetic energy same like intensity of electric and magnetic energy.

From Fig. 5(b), the total electric energy and total magnetic energy was increased slowly from 0.40×10^{-17} J to 0.58×10^{-17} J at 1.2 radian for total electric field and 0.58×10^{-17} J to 1.5×10^{-17} J at 1.3 radian for total magnetic field. However, the value for electric and magnetic energy was fall suddenly when the angles of incidence close to 1.5 radians. However, the total magnetic energy was higher than total electric energy same as intensity of electric and magnetic energy.

3.5 The reflection and transmission of transverse electric wave (TE) and transverse magnetic wave (TM)

From Fig. 6, the graph shows that the reflection and transmission with value of analytic solution for TE wave for both shape was close and agree very well. When angle of incidence was increased, the reflection also increased but the transmission was decreased.

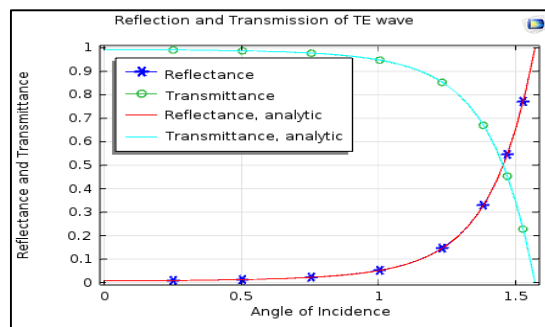
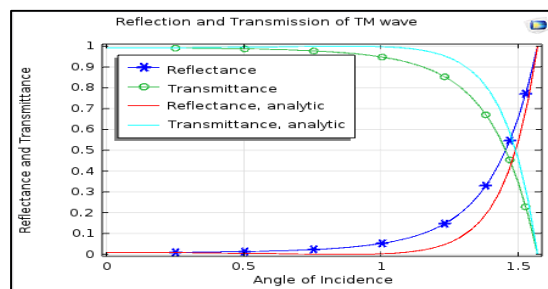
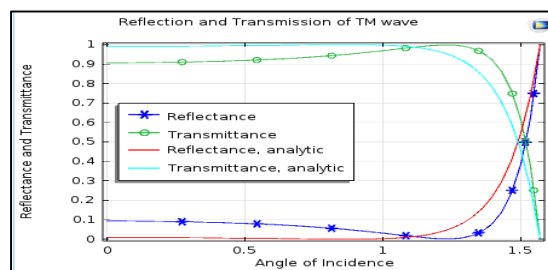


Fig. 6 Graph of reflection and transmission of TE wave for v-groove shape and curve shape.

From Fig. 7, the graph shows that the reflection and transmission with analytic solution for TM wave was not agree very well and different for both shapes.



(a)



(b)

Fig. 7 Graph reflection and transmission of TM wave for (a) v-groove shape and (b) curve shape

In Fig. 7 (a), reflection and transmission of TM wave for v-groove shape disagreed with analytic solution starting from 0.5 radians for the angle of incidence but almost close at around 1.5 radians. While, for the graph of reflection and transmission of TM wave for curve shape as shown in Fig. 7(b), it disagree with analytic solution starts from 0 radians and almost close at around 1 radian. However, it disagrees again until it achieve above 1.5 radians.

In reflection and transmission of TM wave, Brewster angle was used 0.87606 radians as the angle of incidence where the light with a particular polarization was perfectly transmitted through a transparent dielectric surface, with no reflection. At a certain angle were given at n_1 and n_2 , the value of reflection coefficient (R_p) goes to zero and a parallel polarization (p-polarized) incident ray is purely refracted. This statement is only true when the refractive indices of both materials are real numbers like air and glass material. For materials that absorb light, like metals and semiconductors, n is complex, and R_p does not generally go to zero. So, that why, the graph of reflection and transmission of TM wave not very agree well with analytic value.

4. Conclusion

In this paper, the plasmonic structure in three dimensions (3D) with different shape was observed in mesh and computed the solution. The resulting was numerically solved using the FEM. The result show that the memory of the computer used to support the data for mesh and computed the solution was influenced. The process of getting the result for the electric and magnetic field wave propagation was observed by simulation in three dimensions (3D) while, for the total of electric and magnetic field, it was observed by using a graph. The result shown the total electric energy for v-groove shape of plasmonic structure was higher than the total magnetic energy opposite with the curve shape. From this result, it was same as intensity of electric and magnetic energy as shown in simulation. Next, the graphs of the reflection and transmission with value of

analytic solution for TE wave for both shapes were similar and agree very well. However, the graph was shown the different between v-groove shape and curve shape in reflection and transmission with analytic solution for TM wave. This indicates that the structure of v-groove shape was most suitable in many applications because v-groove shape has a narrow area that can increase effectively of the light incident either in reflection or transmission process.

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