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Invariance in Transverse Momentum of Photons in Double-slit Experiment

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Abstract: One of the intriguing mystery in modern physics is the quantum interference phenomena, which the behaviour of photons in double-slit experiment is still ambiguous. Instead of relying on the naive probabilistic point of view, Bohmian mechanics provides the ground base for interpreting quantum system in a deterministic way closely related to classical mechanics such as it constructs the photon trajectory for the double-slit set up. The appearance on the bending in the constructed photon trajectory seem to contravene the notable law of conservation of momentum. Here, we report on conservation of the transverse momentum of photon trajectories based on numerical solution of Bohmian mechanics in double-slit set up for single photon, pair of photons and ensemble of photons until interference pattern is produced. It is shown that the total transverse momentum in the system of Bohmian mechanics is invariance due to the non-local action of quantum potential.

Keyword: Bohmian mechanics; quantum potential; invariance; transverse momentum; double-slit.

1. Introduction

Quantum interference in double-slit experiment best represents the bizarre behaviour of particles in the quantum world for its simplicity and low-tech nature. The universality of this puzzling quantum phenomena was consistently demonstrated by particles of light, photons [1] and had been explained in well-established quantum predictions such as superposition principle [2] and wave function collapse [3]. However, these predictions seem incomplete as it relies heavily on the probabilistic nature of quantum system as the main foundation.

Alternatively, a more deterministic view such as in Bohmian mechanics interpretation [4,5] is an attempt to gain much of the intuition of classical mechanics in explaining quantum interference by stressing the main idea of a quantum particle with well-defined trajectory [6]. An ingenious derivation made by Bohm has led to two primary equations, the continuity equation and the quantum Hamilton-Jacobi equation, have been the fundamental pillars of constructing the trajectories. Consequently, the so-called quantum potential appeared in the formulation that is responsible for non-local action in Bohmian mechanics. The first numerically constructed Bohmian trajectories based on quantum potential was by [7] which followed by others for example from [8].

Although the Bohmian particle trajectories depend on the quantum potential which itself is the manifestation of wavefunction [4,5], they still follow the law of conservation of momentum. This notable law states that the total momentum of an isolated system is always constant, and it applies to both classical and non-classical system [9]. Thus, a propagating photon will remain on its initial speed and direction until acted upon by an external force. For that reason, the bending nature of Bohmian trajectories which appeared in numerous numerical analysis [7,8,10,11] and later appeared in experiments using photons [12,13] for the double-slit set up must be clearly explained. This seems a contradiction in Bohmian trajectories for double-slit set up is demonstrated through an illustration in Fig. 1.

Hence in this paper, the concept of quantum potential is shown as a perfect and satisfying explanation on the invariance in total transverse momentum by a system of ensembled photons based on Bohmian mechanics. The concept hold for cases such as, (i) the symmetrical spatial structure of photon pair trajectories because of same intensity from both slits [14] or (ii) the different in intensities between slits causing asymmetrical spatial structure of photon pair trajectories [15,16], and produce the invariance in the total transverse momentum. However, the first condition is

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presented in this paper for best visualisation of the invariance. We reported that conservation of momentum seems to be violated for the single Bohmian trajectory of a photon only due to the ignorance of the total transverse momentum contribution because of the non-local action of the other photons of the ensemble in the double-slit set up.

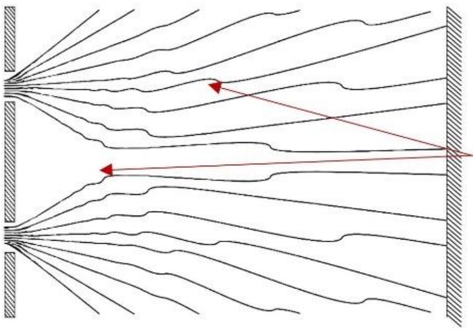

Fig. 1 Illustration on Bohmian trajectories in double-slit experiment showing bending trajectories without any appearance external forces shown by red arrows.

We first revisited the mathematical derivation of quantum potential from the original work of [4,5] that was later extended by [17]. We then further extended it in this study to show the relation between quantum potential and momentum hence its significance. The work of [14] was selected in this study for convenience stated before. Lastly, a thorough analysis on the transverse momentum of the single photon, pair of photons and ensemble of photons was clarified by means of symmetrical spatial structure of photon pair trajectories to display the manifestation of invariance in total transverse momentum in Bohmian trajectories hence obeying the law of conservation of momentum.

2. Theory

The so-called quantum potential is the aftermath of the development of Bohmian mechanics. It emerges from the derivation of the time dependent wavefunction ψ in its polar form (1),

$$\psi(r,t)=\sqrt{\rho(r,t)}e^{iS/\hbar} \tag{1}$$

with $\rho(r,t)=\psi\psi^*$ the probability density, and S the quantum phase given by (2),

$$S=\frac{\hbar}{2i}\ln\left(\frac{\psi}{\psi^*}\right) \tag{2}$$

Substituting the wavefunction in (1) into the usual Schrodinger equation (3) reduces into two separate equations known as the continuity equation (4) and the quantum Hamilton-Jacobi equation (5).

$$i\hbar\frac{d\psi}{dt}=-\frac{\hbar^2}{2m}\nabla^2\psi+V\psi \tag{3}$$

$$\frac{d\rho}{dt}+\nabla\cdot\left(\rho\frac{\nabla S}{m}\right)=0 \tag{4}$$

$$\frac{dS}{dt}+\frac{|\nabla S|^2}{2m}+V+Q=0 \tag{5}$$

The Q in equation (5) is the quantum potential which can be shown by the derived expression of (6) [17],

$$Q=-\frac{\hbar^2}{2m}\left\{\mathrm{Re}\left(\frac{\nabla^2\psi}{\psi}\right)+\left[\mathrm{Im}\left(\frac{|\nabla\psi|}{\psi}\right)\right]^2\right\} \tag{6}$$

To get the relation between momentum **p** and quantum potential Q which bridge classical dynamics to quantum dynamics based on (6), the momentum **p** is firstly defined in one dimension, the x-axis (the generalisation in 3-dimensional space is straightforward) as $\mathbf{p_x}=m\mathbf{v_x}$. From the equation of motion (7) [17], the transverse component of the gradient of S, ∇S_x is made the subject becoming the equation (8) and (9),

$$\mathbf{v_x}=\frac{\nabla S_x}{m}=\frac{1}{m}\mathrm{Re}\left\{\frac{\hat{p}\psi_x}{\psi_x}\right\} \tag{7}$$

$$\nabla S_x=\mathrm{Re}\left\{\frac{\hat{p}\,\psi_x}{\psi_x}\right\} \tag{8}$$

$$\nabla S_x=m\mathbf{v_x}=\mathbf{p_x} \tag{9}$$

with $\hat{p}=-i\hbar\nabla$ being the momentum operator. The equation of motion (7) shows the direct relation between momentum $\mathbf{p_x}$ and ∇S_x through (9). Next, after rearranging the $\hat{p}$ in (8) thus,

$$\frac{\nabla S_x}{-i\hbar}=\mathrm{Re}\left\{\frac{\nabla\psi_x}{\psi_x}\right\} \tag{10}$$

Operating ∇ on both side of equation (10),

$$-\frac{1}{i\hbar}\nabla^2 S_x = \mathrm{Re}\left\{\frac{\nabla^2\psi_x}{\psi_x}\right\} \qquad (11)$$

Equation (11) is then substituted into the time-dependent quantum potential (due to the presence of time-dependent wavefunction) (6) resulting in (12),

$$Q_x = \frac{\hbar}{2im}\nabla^2 S_x - \frac{\hbar^2}{2m}\left[\mathrm{Im}\left(\frac{|\nabla\psi_x|}{\psi_x}\right)\right]^2 \qquad (12)$$

Disregarding the second part of equation (12) shows the proportional relation between quantum potential and quantum phase as in (13),

$$Q_x \propto \nabla^2 S_x \qquad (13)$$

The relation (13) defines that the non-local effect of the quantum potential towards photons is related proportionally to the Laplacian of the quantum phase of the system. Further from (9), thus relation (13) could also be deduced to (14),

$$Q_x \propto \nabla\cdot\mathbf{p_x} \qquad (14)$$

or what we call the divergence in momentum fields, $\nabla\cdot\mathbf{p_x}$. The momentum can be represented as an analogy of a vector momentum field and the action of quantum potential on particle's momentum is dependence on the particle's position at a time. This is very true by the definition of divergence by which the vector field flow behaves like a source at a given point. Hence, each point in the momentum field assigns the magnitude of the momentum of a particle at that specific point. It is thus a first-order approximation on the relation between the classical quantity momentum, $\mathbf{p}$ with the quantum quantity quantum potential, Q.

3. Method

We performed an analysis on the numerically constructed Bohmian trajectories of photons in double-slit experiment [14] in term of transverse momentum evolution. As stated by [14], the photon trajectories were numerically computed by considering the wavefunction described by a coherent superposition of two Gaussian wave packets (15),

$$\psi(x,t) \propto e^{-\frac{(x-x_o)^2}{4\sigma_o\sigma_t}} + e^{-\frac{(x+x_o)^2}{4\sigma_o\sigma_t}} \qquad (15)$$

with x the position, x_o the initial position, σ_o the initial width of the wave packets and σ_t the width at a time t given by $\sigma_t=\sigma_o[1+i\frac{\hbar t}{2m\sigma_o}]$ [14]. The expression (15) was inserted into the equation of motion in Bohmian mechanics (16) (substitution of (2) into (7)) with **v** the velocity of the particle.

$$\mathbf{v} = \frac{\hbar^2}{2im}\nabla\ln\left(\frac{\psi}{\psi^*}\right) \qquad (16)$$

Equation (16) thus was the basis of constructing Bohmian trajectories by [14]. The distribution of the magnitude of the probability density in the double-slit set up will not be stressed in our study.

The Bohmian trajectories were analysed and extracted for its trajectories only by using the eraser tools in Adobe Illustrator CS6. The background colour was left behind. From all trajectories, three pairs of trajectories were selected where the pairs originate from both upper and lower slits. The eraser tools in Adobe Photoshop CS6 was then used to produce three graphs with three selected pairs labelled Pair 1, Pair 2 and Pair 3 respectively (Fig. 2 (b)(i), (c)(i) and (d)(i)). Fig. 2 (a)(i) shows the single trajectory from the lower slit of Pair 1.

To extract data points along the trajectories, Webplot Digitizer 3.8 was used. Each trajectory has respectively 176 data points extracted. The data was then imported to Origin Pro 2016 to calculate the tangent value for each data points. The tangent values of two immediate points were then plotted in the graphs of tangent, $\frac{dx}{dt}$ against time, t for all set of selected trajectories (Fig. 2 (a)(ii), (b)(ii), (c)(ii) and (d)(ii)). As from the general equation of momentum, $\mathbf{p_x}=m\frac{dx}{dt}$ where m is assumed to be photon's effective mass, given by $m=\frac{E}{c^2}$ where E is photon's energy and c is the speed of light, thus, it can be interpreted that the graphs produced were representation of the evolution of transverse momentum $\mathbf{p_x}$ of photons against time, t.

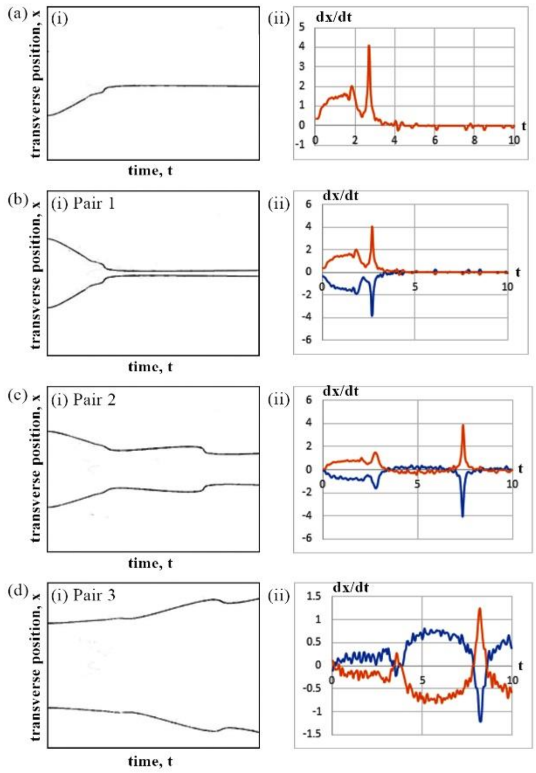

Fig. 2 The selected Bohmian trajectories from [14] (a)(i) single trajectory from lower slit and three pairs of trajectories from lower and upper slits-(b)(i), (c)(i) and (d)(i) labelled Pair 1, Pair 2 and Pair 3 respectively. (a)(ii), (b)(ii), (c)(ii) and (d)(ii) showing the graph of tangent of the curve (orange colour from the lower slit, blue colour form the upper slit) in (a)(i), (b)(i), (c)(i) and (d)(i) respectively which are equivalent to transverse momentum of photons with respect to time.

4. Discussion

Pair 1 (Fig. 2 (b)) was the central most trajectory pair with respect to the centre in the double-slit. Pair 2 (Fig. 2 (c)) and pair 3 (Fig. 2 (d)) was the middle and outermost trajectory pair in the double-slit, respectively. The appearance of tangent peaks was due to changes in momentum direction from pointing outward and then pointing back to the propagation direction of photons, which was related to the evolution of transverse momentum with time. The momentum change can be slow and can be quite sudden as shown by the broadness of the peak. It showed that the location and the distance between broad peaks and sharp peaks varies for each pair. Another notable feature in the momentum evolution was that after a specific time the momentum remained constant until the photon reached the detection plane in double-slit experiment.

The momentum evolution of the photon trajectories then was analysed at the single photon, the pair and the ensemble of photons. A single trajectory of photon in the double-slit set up (Fig. 2 (a)(ii)) seemed do not obey the law of conservation of momentum where the net transverse momentum is non-zero, $\mathbf{p}_x \neq 0$ at every instance of time during evolution fluctuation. This was due to the bending nature of the photon trajectory which undergoes continuous changes in its transverse momentum without any counter-reaction to conserve it.

However, if we paired the photon in Fig. 2 (a)(ii) with its equivalent pair from upper slit, (Fig. 2 (b)(ii)), also apply for the other pairs (Fig. 2 (c)(ii) and (d)(ii)), each complementary trajectory would add up their total transverse momentum contribution to zero, $\mathbf{p}_x = 0$ at any time. This conservation of momentum between two separate trajectories is due to non-local action that arises from quantum potential, Q_x as in equation (14) that act on the photon transverse momentum, $\mathbf{p}_x$. The appearance of non-local action is the manifestation of entanglement in quantum mechanics. Thus, it shows that the entanglement between the two photons in the two trajectories has assisted the conservation of momentum. In other words, the momentum is invariance at every single time. This leads us to a concept called envariance (environment assisted invariance) a discovered symmetry in the non-local quantum state [18] which later known as entanglement assisted invariance [19].

Hence, when all pair of trajectories as an ensemble of photons in the double-slit experiment was considered, it will result in conservation of the total transverse momentum, $\Sigma \mathbf{p}_x = 0$. The law of conservation of momentum holds in Bohmian trajectories given that the non-local action must be considered. Hence, the new definition of the law of conservation of momentum in quantum mechanics should consider the non-local action which leads to,

"Invariance of the total transverse momentum in an ensemble of photons is due to the action of non-local quantum potential"

This analysis also profoundly agreed with the gold standard in quantum mechanics which the physical quantity of momentum is conserved in a quantum system, given by the mathematical premise that says any observable commute with the Hamiltonian operator, H, its value is conserved $[\hat{p},H]=0$ [9].

5. Conclusion

We have shown that the conservation of momentum seems to be violated for the single Bohmian trajectory of photon only due to the ignorance of the total transverse momentum contribution of the ensemble, hence their non-local action. However, total transverse momentum is invariance in the system of Bohmian mechanics. Moreover, we also provide first-order mathematical relation between momentum and quantum potential. This research will have application in future quantum communication and quantum computing.

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