Thermal and Solutal Mixed Marangoni Boundary Layers with Suction or Injection Effects

Noraini Ahmad*, Seripah Awang Kechil, Norma Mohd Basir

Faculty of Computer and Mathematical Sciences, Universiti Teknolog MARA
40450 UiTM Malaysia, Shah Alam, Selangor Darul Ehsan, Malaysia.

*Corresponding email: noraini.ahmad@rocketmail.com

Abstract

A steady two-dimensional Marangoni boundary-layer flow over a permeable flat surface is considered. The surface tension is assumed to vary linearly with temperature and solute concentration. The transformed similarity boundary layer equations are obtained and solved numerically by Runge-Kutta-Fehlberg with shooting technique. The Marangoni and external pressure gradient effects that generated in boundary layer flow are assessed. As the suction/ injection parameter increases from injection to suction, the velocity, temperature and concentration boundary layers thickness decrease but injection increases them. The thermosolutal surface tension ratio increases the velocity boundary thickness but decreases the temperature and concentration boundary layers.

Keywords: marangoni convection; surface tension, similarity solution; suction or injection effect

1. INTRODUCTION

Marangoni convection occurs when there is temperature or concentration differences on the surface of the fluid and the fluid will flow from region having low surface tension (high temperature region) to region having high surface tension (cold temperature region). In thermocapillary convection, the surface tension varies with temperature. However, small amounts of certain surfactant additives are also known to drastically change the surface tension. Surface-tension induced convection is important in study of nucleate and bubble growth dynamics because it causes undesirable effects in industrial processes [1,2,3].

Theoretical investigations on Marangoni flows include the works of by Christopher and Wang [3], Magyari and Chamkha [4,5], Zueco and Bég [6], Al-Mudhaf and Chamkha [7]. Christopher and Wang [2] examined the effect of Prandtl number to see the relative thickness of momentum and thermal boundary layers. The influence of temperature exponent using the power-law function of temperature in nucleate and vapour bubble growth was examined by Christopher and Wang [3]. Magyari and Chamkha [4,5] found the exact analytical solutions for the MHD thermosolutal Marangoni convection in the presence of heat and mass generation or consumption. Their analytical results showed that the thermosolutal surface tension ratio increases the wall velocity and mass flow rate. The effects of gravity, magnetic field and external pressure gradients on the Marangoni convection boundary layers have been considered by Zueco and Bég [6]. Al-Mudhaf and Chamkha [7], Hamid et al. [8] and Ahmad et al. [9] studied the effect of suction, injection on Marangoni boundary layer flow.

In this paper, we extend the work of Christopher and Wang [3] to include the effects of surface tension gradient due to insoluble surfactant on the development of momentum, thermal and concentration of Marangoni boundary layers flow.

2. PROBLEM FORMULATION

Consider the steady two-dimensional mixed Marangoni boundary layer flow with an external pressure gradient over a flat plate. The surface is assumed to be permeable in order to allow for possible suction or injection at the wall. The temperature variation is in the form of power law function and the boundary layer develops along the surface due to the coupled Marangoni convection. The governing equations are the balance laws of mass, momentum, energy and concentration given by,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+u_e\frac{du_e}{dx}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}, \tag{4}$$

where u and v are velocity components along the x and y axes, respectively, T is the fluid temperature, C is the solute concentration, D is the mass diffusivity, α is the thermal diffusivity, ν is the kinematic viscosity and $u_e(x)$ is the velocity of the external flow. The surface tension σ at the interface is assumed to vary linearly with temperature and surfactant concentration in the form,

$$\sigma=\sigma_0-\gamma_T(T-T_\infty)-\gamma_C(C-C_\infty), \tag{5}$$

where σ_0 is the reference surface tension, $\gamma_T=\left.\frac{\partial\sigma}{\partial T}\right|_C$ and $\gamma_C=\left.\frac{\partial\sigma}{\partial C}\right|_T$.

The boundary conditions at the surface are

$$\mu\left.\frac{\partial u}{\partial y}\right|_{y=0}=-\left.\frac{\mathrm{d}\sigma}{\mathrm{d}T}\right|_C\left.\frac{\partial T}{\partial x}\right|_{y=0}-\left.\frac{\mathrm{d}\sigma}{\mathrm{d}C}\right|_T\left.\frac{\partial C}{\partial x}\right|_{y=0}, \tag{6}$$

$$v(x,0)=-v_0,\ \ T(x,0)=T(0,0)+Ax^{k+1}, \tag{7}$$

$$C(x,0)=C(0,0)+A^*x^{k+1}. \tag{8}$$

and at the free stream, the boundary conditions are

$$u(x,\infty)=0, \tag{9}$$

$$T(x,\infty)=T_\infty=T(0,0)\quad\text{or}\quad\left.\frac{\partial T}{\partial y}\right|_{y=\infty}=0, \tag{10}$$

$$C(x,\infty)=C_\infty=C(0,0)\quad\text{or}\quad\left.\frac{\partial C}{\partial y}\right|_{y=\infty}=0, \tag{11}$$

where A and A^* are the temperature and concentration gradient coefficient, respectively, and k is the constant exponent of the temperature and concentration, and v_0 is a constant which refers to suction or injection.

Following [6], we introduce the similarity transformations

$$\eta=C_1x^d y,\ \ f(\eta)=C_2x^a\psi(x,y), \tag{12}$$

$$\theta(\eta)=\frac{[T(x,y)-T(0,0)]x^h}{A}, \tag{13}$$

$$\phi(\eta)=\frac{[C(x,y)-C(0,0)]x^h}{A^*}, \tag{14}$$

where η is the similarity variable, ψ is the stream function defined by $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$, $f(\eta)$ is the dimensionless stream function, θ is the dimensionless temperature function and ϕ is the dimensionless concentration function. The exponents d, a and h in (12) and (14) are related to the temperature and concentration gradient exponent, *k*, given by the relation [6],

$$d = \frac{k-1}{3},\ a = \frac{-2-k}{3},\ h = -1-k. \tag{15}$$

By applying (12)-(14) on the governing equations (1)- (4) and boundary conditions (6) – (11), we obtained a coupled nonlinear system of ordinary differential equations,

$$f''' - (d-a)(f'^2 - 1) - aff'' = 0, \tag{16}$$

$$\theta'' - \Pr\left(af\theta' - hf'\theta\right) = 0, \tag{17}$$

$$\phi'' - Sc(af\phi' - hf'\phi) = 0, \tag{18}$$

where $\Pr$ is the Prandtl number and Sc is the Schmidt number. The transformed boundary conditions are

$$f''(0) = (-1-k)(1+\varepsilon),\ f(0) = f_w,\ \theta(0) = 1,\ \phi(0) = 1, \tag{19}$$

$$f'(\infty) = 0,\ \theta(\infty) = 0 \text{ or } \theta'(\infty) = 0,\ \phi(\infty) = 0 \text{ or } \phi'(\infty) = 0, \tag{20}$$

with $f_w > 0$ is the constant of suction parameter , $f_w < 0$ is the constant of injection parameter and ε is the thermosolutal surface tension ratio.

3. RESULTS AND DISCUSSION

The system of nonlinear ordinary differential equation (16) and -18) subject to boundary conditions (19) and (20) are solved numerically using Runge-Kutta-Fehlberg method with shooting technique. The three unknown conditions, $f'(0)$, $\theta'(0)$ and $\phi'(0)$ are determined.

Table 1: Comparison values of $f'(0)$, $-\theta'(0)$ and $-\phi'(0)$

	Al-Mudhaf and Chamkha (2005)	Present
$f'(0)$	1.587671	1.587401
$-\theta'(0)$	1.442203	1.442066
$-\phi'(0)$	1.220880	1.220711

Table 2: Effects of ε on $f'(0)$, $-\theta'(0)$, and $-\phi'(0)$

ε	$f'(0)$	$-\theta'(0)$	$-\phi'(0)$
0	1.7885555881	1.2608835684	1.0937432457
1	2.4660513411	1.4365160819	1.2404014544
3	3.6042256176	1.6923289378	1.4544116268
5	4.5838291117	1.8855580010	1.6163084822

Table 1 shows the numerical values of the wall velocity $f'(0)$, heat transfer coefficient $-\theta'(0)$ and mass transfer coefficient $-\phi'(0)$ when Pr = 0.78, Sc = 0.6, $f_w = 0$, temperature gradient exponent $k = 1$ and the pressure gradient is absent $(-dp/dx = u_e(x)du_e/dx = 0)$. It can be observed that the numerical values of the Runge-Kutta Fehlberg with shooting technique are in good agreement with the results form the implicit finite-difference method of Al-Mudhaf and Chamkha [7].

Table 2 illustrates the influence of the thermosolutal surface tension ratio ε on $f'(0), -\theta'(0),$ and $-\phi'(0)$. The increasing thermosolutal surface tension ratio ε increases $f'(0), -\theta'(0),$ and $-\phi'(0)$. The effects of f_w and ε on the velocity, temperature and concentration profiles are presented graphically in Figures 1- 4.

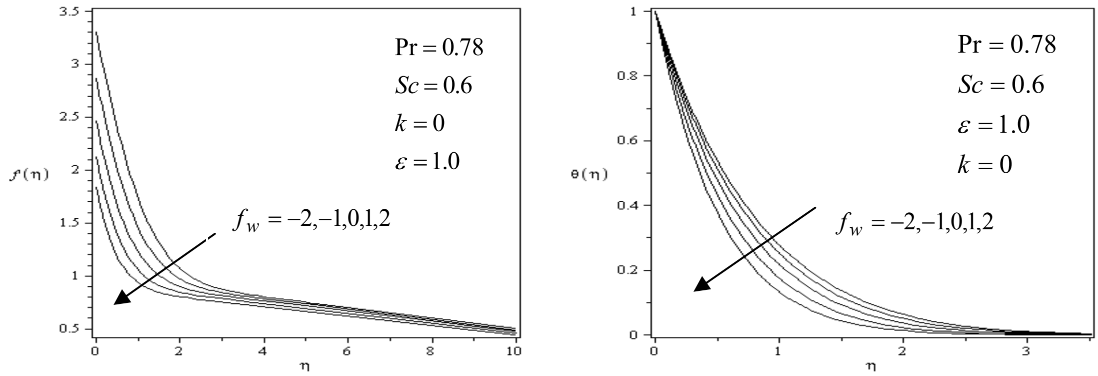

Figure 1: Effects of f_w on velocity and temperature profiles

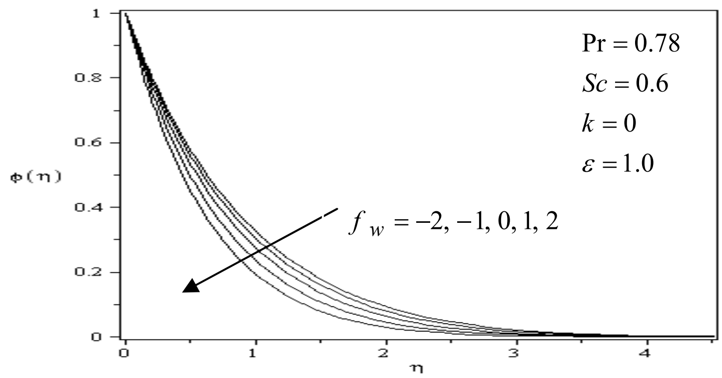

Figure 2: Effects of f_w on concentration profiles

Figure 1 presents the effects of suction or injection on velocity and temperature profiles and Figure 2 illustrates the effects of suction or injection on concentration profiles. Generally, suction decreases the velocity, temperature and concentration boundary layers thickness and injection shows the opposite effect. Figure 3 shows the influence of the thermosolutal surface tension ratio, ε on the velocity profiles. As ε increases, the Marangoni convection effect increases causing more induced flow. This induced flow starts at the surface and propagates in the boundary layer. Thus, the maximum velocity occurs at the wall. However, the increased velocity is caused by the increase in Marangoni convection effect followed by decreases in temperature and concentration as shown in Figure 4.

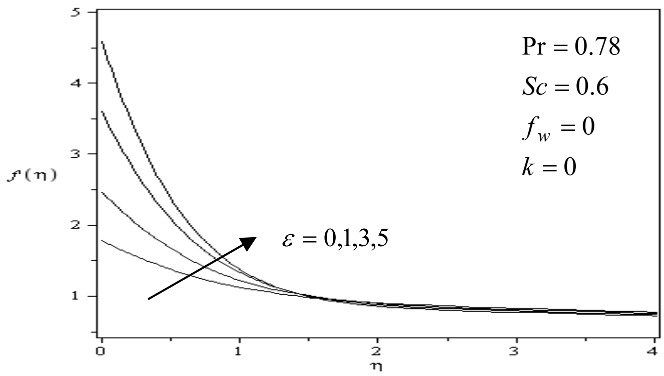

Figure 3: Effects of ε on velocity profiles.

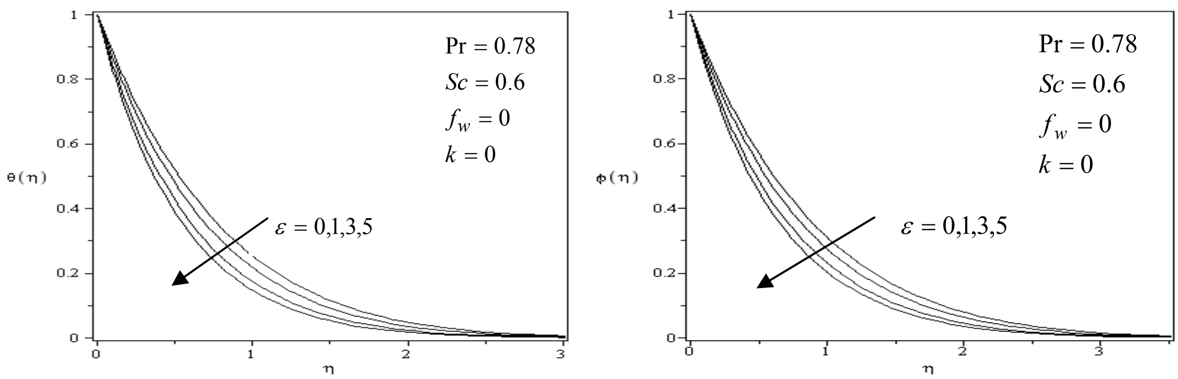

Figure 4: Effects of ε on temperature and concentration profiles

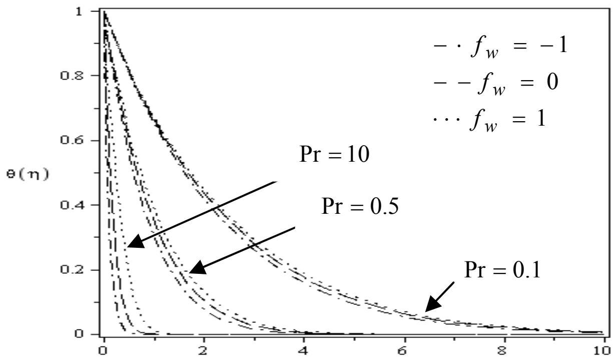

Figure 5: Effects of Pr on temperature profiles for various f_w when $Sc = 0.6$ and $k = \varepsilon = 0$.

In Figure 5, it can be seen that the thickness of the thermal boundary layer is influenced by the Prandtl number. Small values of Pr (<<1) correspond to liquid metals having high thermal conductivity and low viscosity but the value Pr >>1 corresponds to high viscosity oils. As the Prandtl number increases, the magnitude of the rate of heat transfer increases and the thermal boundary layers gets thinner for high Prandtl numbers. Suction reduces the rate of heat transfer.

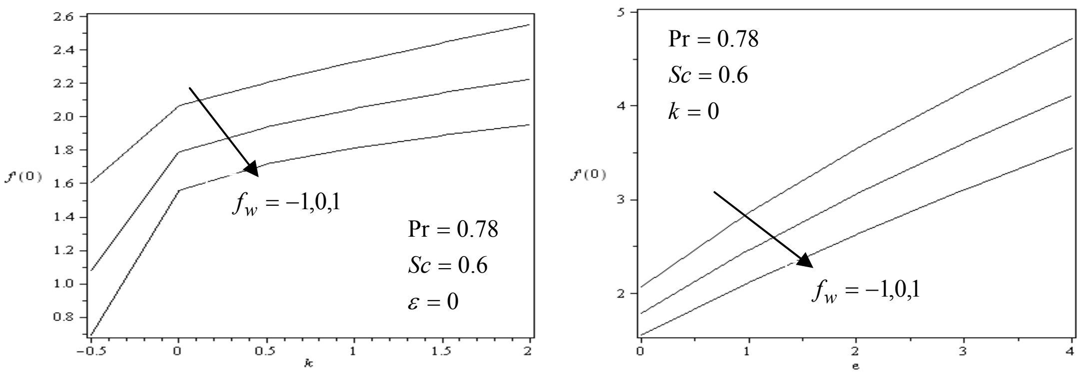

Figure 6: Effects of k and ε on surface velocity for various f_w

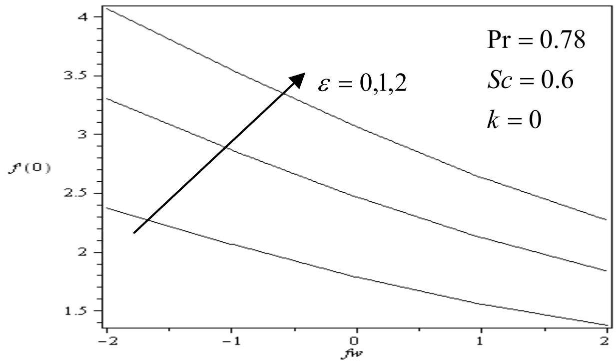

Figure 7: Effects of f_w on surface velocity for various ε

Figure 6 shows the effects of temperature exponent k and the thermosolutal surface tension ratio on the surface velocity for various f_w, respectively. The temperature exponent and the thermosolutal surface tension ratio increase the surface velocity thus reduce the velocity boundary layers but the increase from injection to suction reduces the surface velocity and thickens the velocity boundary layers. Figure 7 illustrates the effects of the thermosolutal surface tension ratio and suction or injection parameter on the surface velocity. Similar results can be observed that suction decreases the surface velocity but the wall velocity increases when thermosolutal surface tension ratio increases.

4. CONCLUSIONS

The steady Marangoni convection boundary layer flow in the presence of suction and injection effects was discussed and examined numerically. The basic governing equations in the form of partial differential equations were transformed to ordinary differential equations by similarity transformation. Numerical solutions of the similarity equations were obtained using Runge-Kutta with shooting technique. The effects of physical parameters on the velocity and concentration profiles were presented and evaluated. Suction decreases the velocity, temperature and concentration boundary layers thickness while injection increases them. The thermosolutal surface tension ratio increases the velocity boundary layer thickness but decreases the thickeness of the temperature and concentration boundary layers. The temperature exponent, thermosolutal surface tension ratio and injection increase the surface velocity leading to the reduced velocity boundary layer thickness.

Acknowledgment

The authors would like to acknowledge the financial support received from the Ministry of Higher Education of Malaysia and Universiti Teknologi MARA under the Fundamental Research Grant Scheme (FRGS) 600-RMI/ST/FRGS 5/3/Fst (9/2011) and the Young Lecturer's Scheme Scholarship.

REFERENCES

[1] Arafune, K. & Hirata, A. (1999). "Thermal and solutal Marangoni convection in In-Ga-Sb system in Journal of Crystal Growth, Vol. 197. pp. 811-817.

[2] Christopher, D.M. & Wang, B. (2001a). "Prandtl number effects for Marangoni convection over a flat surface" in International Journal of Thermal Sciences, Vol. 40. pp. 564-570.

[3] Christopher, D.M. & Wang, B. (2001b). "Similarity simulation for Marangoni convection around a vapor bubble during nucleation and growth" in International Journal of Heat and Mass Transfer, Vol. 44. pp. 799-810.

[4] Magyari E. & Chamkha, A.J. (2007). "Exact analytical solutions for thermosolutal Marangoni convection in the presence of heat and mass generation or consumption" in *Heat and Mass Transfer*, Vol. 43. pp. 965-974.

[5] Magyari, E. & Chamkha, A.J. (2008). "Exact analytical results for the thermosolutal MHD Marangoni boundary layer" in *International Journal of Thermal Sciences*, Vol. 47. pp. 848-857.

[6] Zueco, J. & Bég, O.A. (2011). "Network numerical simulation of hydromagnetic Marangoni mixed convection boundary layers" in Chemical Engineering Communications, Vol. 198. pp. 552-571.

[7] Al-Mudhaf, A. & Chamkha, A.J. (2005). "Similarity solutions for MHD thermosolutal Marangoni convection over a flat surface in the presence of heat generation or absorption effects" in Heat and Mass Transfer, Vol. 42. pp. 112-121.

[8] Hamid, R., Arifin, N., Nazar, R., & Ali, F. (2011). "Radiation effects on Marangoni convection over a flat surface with suction and injection" in Malaysian Journal of Mathematical Sciences, Vol. 5. No. 1 pp.13-25.

[9] Ahmad, N., Awang Kechil, S. & Basir, N.M. (2011). "Thermosolutal MHD Mixed Marangoni Boundary layer in the presence of suction or injection" in World Academy of Science, Engineering and Technology, Vol. 58. pp 459-464.