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An Improved Confidence Interval for the Difference between Standard Deviations of Normal Distributions Using a Ranked Set Sampling

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Abstract: In this paper, a confidence interval is derived for the difference between the standard deviations of normal distributions using the method of variance of estimates recovery. This confidence interval was improved using the estimators of the standard deviations using ranked set sampling (RSS) instead of the standard method using simple random sampling (SRS). An evaluation of the performance of the proposed confidence interval based on RSS compared to the existing one based on SRS was conducted via a Monte Carlo simulation study. The results revealed that the proposed confidence interval based on RSS performed more efficient than the existing one based on SRS in terms of the coverage probability and average length. A confidence interval comparison is also illustrated using a real data example in the area of medical science.

Keywords: Interval estimation, measure of dispersion, coverage probability, average length, sampling technique

1. Introduction

The standard deviation (SD), calculated as the square root of the variance of a population, is a measure of the amount of variation or dispersion in a data : a low SD indicates that the values tend to be close to the mean of the data whereas a high one indicates that the values are spread out over a wider range. It has been widely applied in various fields by many authors e.g. Singh et al., 2004; Huang & Chen, 2005; Bonett, 2006; Maravelakis & Castagliola, 2009. Mead (1966) first presented a quick method of estimating the SD, while an estimator selection for the SD of a normal distribution was later provided by Freund (1987). Tatum (1997) suggested a new approach for the robust estimation of the process SD, while Tsai & Wu (2008) proposed a heuristic method based on an adjusted weighted SD for constructing an R chart for a skewed process. A new confidence interval for the SD of non-normal distributions was proposed by Niwitpong & Kirdwichai (2008), and Frost et al. (2013) proposed the interval estimation of the SD of a gamma population. Last, Niwitpong (2015) presented confidence intervals for the SD and the difference between SDs of normal distributions with known coefficients of variation.

Examples of related works on population variance are as follows. Tate & Klett (1959) proposed optimal confidence intervals for the variance of a normal distribution, while Levy & Narula (1974) derived the shortest confidence interval for the ratio of two variances in normal distributions. Shorrock (1990) later constructed confidence intervals for normal variance that depend on the sample mean and have the same length as the shortest interval depending only on the sample variance. Goutis & Casella (1991) discussed confidence intervals for the variance of a normal distribution with an unknown mean that offered an improvement on the usual short interval based on the sample

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variance alone. Cojbasic & Tomovic (2007) suggested confidence intervals for the population variance and the difference in the variances of two populations based on the ordinary t-statistic combined with a bootstrap method, while the generalized confidence interval for the difference between the variances of normal populations were constructed by Phonyiem & Niwitpong (2012). Last, Mahmoudi & Mahmoudi (2013) proposed the confidence interval for the ratio of variances in two independent populations based on the asymptotic distribution for the ratio of sample variances.

Measuring the variables of interest for some phenomenon can either be too expensive or not easy perform (or both), whereas it is easy to rank them. Ranked set sampling (RSS), which can be used to estimate the population mean and SD, is a powerful sampling alternative to simple random sampling (SRS) in these situations. Estimating the population mean using RSS instead of SRS was first introduced by McIntyre (1952). Later, Takahashi & Wakimoto (1968) proposed the idea using mathematical theory to support their concept. The sample mean obtained from RSS is unbiased and has smaller variance compared with that obtained from SRS using the same sample size (Takahashi & Wakimoto, 1968, see Dell & Clutter (1972), Samawi & Muttlak (1996), and Samawi (1999) for discussions on this). Stokes (1980) constructed an estimator for the variance of ranked set sample data that is asymptotically unbiased and asymptotically more efficient than using SRS with the same number of observations. MacEachern et al. (2002) presented an alternative estimator for the variance that is unbiased and more efficient than Stokes' (1980) estimator (although asymptotically equivalent to it) when the underlying distribution is non-normal and the ranking of the elements is not perfect; the variance estimator performed very well for small to moderate sample sizes. In addition, Albatineh et al. (2017) showed that RSS instead of SRS improved the performance of confidence intervals for the signal-to-noise ratio as measured by the coverage probability.

In this study, improving the confidence interval for the difference between SDs of normal distributions using the estimators of the population means and SDs via RSS instead of SRS. Because a theoretical comparison is not possible, a simulation study was conducted to compare the performance of these confidence intervals based on RSS and SRS to discover which one was closet to the nominal confidence level and had the shortest average length.

The rest of the paper is organized as follows. Estimating the population mean and SD using RSS is discussed in Section 2. Confidence intervals for the SD of a normal distribution are reviewed in Section 3. Section 4 contains the proposed confidence intervals for the difference between the SDs of normal distributions based on RSS. Simulation study details and a discussion of the results is presented in Section 5.

2. Estimators of the Population Mean and Standard Deviation Using RSS

Performing RSS to obtain sample size n from a population is of interest. First, SRS is used to select a sample k observations and rank them in order on the attribute of interest. The smallest observation $X_{[1]1}$ is retained and the remaining $k-1$ units are discards. A second SRS sample of size k is selected from the population and ranked in exactly the same way; the second smallest observation $X_{[2]1}$ is selected and the rest discarded. This process is continued for $X_{[3]1}, X_{[4]1}, ..., X_{[k]1}$, and so $X_{[1]1}, X_{[2]1}, ..., X_{[k]1}$ represents the first balanced RS sample of size k. To obtain a balanced RSS of size $n = km$, we repeat the process for m independent cycles, thereby yielding the balanced RSS of size n reported in Table 1.

Table 1 - Balanced RSS with m cycles and set size k.

Cycle 1	$X_{[1]1}$	$X_{[2]1}$	$X_{[3]1}$	$\cdots$	$X_{[k]1}$
Cycle 2	$X_{[1]2}$	$X_{[2]2}$	$X_{[3]2}$	$\cdots$	$X_{[k]2}$
Cycle 3	$X_{[1]3}$	$X_{[2]3}$	$X_{[3]3}$	$\cdots$	$X_{[k]3}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Cycle m	$X_{[1]m}$	$X_{[2]m}$	$X_{[3]m}$	$\cdots$	$X_{[k]m}$

The complete balanced RSS with set size k for m cycles can be expressed as $\{X_{[r]i} : r = 1, 2, ..., k;\ i = 1, 2, ..., m\}$. The term $X_{[r]i}$ is the r^{th} judgment order statistic from the i^{th} cycle, which is the observation determined as the r^{th} order statistic from one of the k sets in the i^{th} cycle. Assuming that the underlying distribution has a finite mean μ_x and variance σ_x^2, the estimator of σ_x^2 proposed by Stokes (1980) is written as

$$\hat{\sigma}_x^2 = \frac{1}{km-1}\sum_{i=1}^{m}\sum_{r=1}^{k}(X_{[r]i} - \hat{\mu}_x)^2,$$

where $\hat{\mu}_x = \frac{1}{km}\sum_{i=1}^{m}\sum_{r=1}^{k} X_{[r]i}$. This is a biased estimator of σ_x^2, but it is asymptotically unbiased as either k or m get larger. In addition, he showed that the RSS estimator $\hat{\mu}$ has more precision than the sample mean $\bar{X}$ obtained using

SRS because of the independence of the order statistics composing the ranked set sample and also showed that $Var(\bar{X}) \geq Var(\hat{\mu}_x)$.

A balanced RSS was applied in the present study. The estimator of the variance of an RSS presented by MacEachern et al. (2002) (an improvement on the one proposed by Stokes (1980)) was adopted in the simulation studies because it performs very well for small and large ranked set samples. It is given by

$$\hat{\sigma}_x'^2 = \frac{1}{km-1}\left[(k-1)MST + (mk-k+1)MSE\right],$$

where MST and MSE are the mean square treatment and mean square error, respectively, from an analysis of variance performed on the ranked set sample data with a judgment class used as the factor given by

$$MST = \frac{1}{k-1}\sum_{i=1}^{m}\sum_{r=1}^{k}(X_{[r]i} - \hat{\mu}_x)^2 - \frac{1}{k-1}\sum_{i=1}^{m}\sum_{r=1}^{k}(X_{[r]i} - \bar{X}_{[r].})^2, \;\; MSE = \frac{1}{k(m-1)}\sum_{r=1}^{k}\sum_{i=1}^{m}(X_{[r]i} - \bar{X}_{[r].})^2,$$

where $\bar{X}_{[r].} = \frac{1}{m}\sum_{i=1}^{m} X_{[r]i}$. Thus, the improved estimator of the population SD is

$$\hat{\sigma}_x' = \sqrt{\frac{1}{km-1}\left[(k-1)MST + (mk-k+1)MSE\right]}. \quad (1)$$

3. The Confidence Interval for the SD of a Normal Distribution

Let $X_1,...,X_n$ be a random sample of size n from a normal distribution with mean μ_x and variance σ_x^2 and $Y_1,...,Y_m$ be a random sample of size m from a normal distribution with mean μ_y and variance σ_y^2. In this section, the confidence interval for the SD of a normal distribution is reviewed. The $(1-\alpha)100\%$ confidence interval for the SD σ_x of a normal distribution based on the chi-squared statistic is given by

$$CI_0 = \left(\sqrt{\frac{(n-1)S_x^2}{\chi^2_{1-\alpha/2,n-1}}}, \sqrt{\frac{(n-1)S_x^2}{\chi^2_{\alpha/2,n-1}}}\right), \quad (2)$$

where $\chi^2_{\alpha/2,n-1}$ and $\chi^2_{1-\alpha/2,n-1}$ are the $(\alpha/2)100^{th}$ and $(1-\alpha/2)100^{th}$ percentiles of a chi-square distribution with $n-1$ degrees of freedom, respectively.

4. Confidence Intervals for the Difference Between the SDs of Normal Distributions

In this section, the method of variance estimates recovery (MOVER) approach is used to construct the confidence intervals for the difference between the SDs of normal distributions. It is based on estimating the confidence interval for the functions of parameters in the form of $\theta_1 + \theta_2$ and θ_1/θ_2. This method was introduced by Donner & Zou)2012(and was applied to construct confidence interval by Newcombe)2016(, Sangnawakij & Niwitpong)2017(, and Thangjai & Niwitpong)2019(. The idea of this approach is to find the separate confidence intervals for two single parameters and then recover the variance estimates from the confidence intervals afterward to form the confidence interval for the function of parameters.

The MOVER method is based on the central limit theorem (CLT) to find the confidence interval for $\theta_1 + \theta_2$. Therefore, the general form of a two-sided confidence interval under the assumption of independence between estimators $\hat{\theta}_1$ and $\hat{\theta}_2$ is given by

$$\left[L,U\right] = \left[(\hat{\theta}_1 + \hat{\theta}_2) \mp Z_{1-\alpha/2}\sqrt{Var(\hat{\theta}_1) + Var(\hat{\theta}_2)}\right],$$

where $Var(\hat{\theta}_1)$ and $Var(\hat{\theta}_2)$ are the unknown variances of $\hat{\theta}_1$ and $\hat{\theta}_2$, respectively. Zou et al. (2009) assumed that $\left[l_i,u_i\right]$ are the $(1-\alpha)100\%$ confidence intervals for θ_i, $i=1,2$. Furthermore, they pointed out that the value of $l_1 + l_2$ is similar to L and $u_1 + u_2$ is similar to U. To estimate $Var(\hat{\theta}_i)$ using the CLT and under the conditions $\theta_1 = l_1$ and $\theta_2 = l_2$, the estimated variances recovered from l_i to obtain L can be derived as $Var(\hat{\theta}_i) \approx \frac{(\hat{\theta}_i - l_i)^2}{Z^2_{1-\alpha/2}}$. On the other side, under the conditions $\theta_1 = u_1$ and $\theta_2 = u_2$, the estimated variances recovered from u_i to obtain U can be derived as $Var(\hat{\theta}_i) \approx \frac{(u_i - \hat{\theta}_i)^2}{Z^2_{1-\alpha/2}}$. Thus, we can obtain the $(1-\alpha)100\%$ confidence interval for $\theta_1 + \theta_2$ by replacing the corresponding estimated variances into the confidence interval $\left[L,U\right]$. Similarly, the confidence interval for the difference between the parameters can be developed by changing $\theta_1-\theta_2$ into the form $\theta_1+(-\theta_2)$ and then recovering the variance estimates by following the above approach.

Following the concept of Donner and Zou (2012), the $(1-\alpha)100\%$ confidence interval for the difference between SDs $\sigma_x-\sigma_y$ of normal distributions based on the MOVER approach is given by

$$CI=\left[(\hat{\sigma}_x-\hat{\sigma}_y)-\sqrt{(\hat{\sigma}_x-l_x)^2+(u_y-\hat{\sigma}_y)^2},\ \ (\hat{\sigma}_x-\hat{\sigma}_y)+\sqrt{(u_x-\hat{\sigma}_x)^2+(\hat{\sigma}_y-l_y)^2}\right],$$

where $[l_x,u_x]$ and $[l_y,u_y]$ are the $(1-\alpha)100\%$ confidence intervals for σ_x and σ_y shown in Equation (2), respectively. In this study, the two confidence intervals for the difference between the SDs $\sigma_x-\sigma_y$ based on the sampling methods SRS and RSS are as follows:

1) The $(1-\alpha)100\%$ confidence interval for $\sigma_x-\sigma_y$ of normal distributions based on SRS is

$$CI_{SRS}=\left[(\hat{\sigma}_x-\hat{\sigma}_y)-\sqrt{(\hat{\sigma}_x-l_x)^2+(u_y-\hat{\sigma}_y)^2},\ \ (\hat{\sigma}_x-\hat{\sigma}_y)+\sqrt{(u_x-\hat{\sigma}_x)^2+(\hat{\sigma}_y-l_y)^2}\right], \quad (3)$$

where $\hat{\sigma}_x=c_nS_x$ and $\hat{\sigma}_y=c_mS_y$ are the unbiased estimators of σ_x and σ_y, respectively,

$$S_x=\sqrt{(n-1)^{-1}\sum_{i=1}^{n}(X_i-\bar{X})^2},\ \ S_y=\sqrt{(m-1)^{-1}\sum_{i=1}^{m}(Y_i-\bar{Y})^2},$$

$$c_n=\sqrt{\frac{2}{n-1}}\left(\Gamma\left(\frac{n}{2}\right)\Big/\Gamma\left(\frac{n-1}{2}\right)\right),\ \ c_m=\sqrt{\frac{2}{m-1}}\left(\Gamma\left(\frac{m}{2}\right)\Big/\Gamma\left(\frac{m-1}{2}\right)\right),$$

$$[l_x,u_x]=\left[\sqrt{\frac{(n-1)S_x^2}{\chi^2_{1-\alpha/2,n-1}}},\sqrt{\frac{(n-1)S_x^2}{\chi^2_{\alpha/2,n-1}}}\right],\ \text{and}\ [l_y,u_y]=\left[\sqrt{\frac{(m-1)S_y^2}{\chi^2_{1-\alpha/2,m-1}}},\sqrt{\frac{(m-1)S_y^2}{\chi^2_{\alpha/2,m-1}}}\right].$$

2) The $(1-\alpha)100\%$ confidence interval for $\sigma_x-\sigma_y$ of normal distributions based on RSS is

$$CI_{RSS}=\left[(\hat{\sigma}'_x-\hat{\sigma}'_y)-\sqrt{(\hat{\sigma}'_x-l_x)^2+(u_y-\hat{\sigma}'_y)^2},\ \ (\hat{\sigma}'_x-\hat{\sigma}'_y)+\sqrt{(u_x-\hat{\sigma}'_x)^2+(\hat{\sigma}'_y-l_y)^2}\right], \quad (4)$$

where $\hat{\sigma}'_x$ and $\hat{\sigma}'_y$ are the improved estimators of the SD based on RSS calculated using the Equation (1).

5. Simulation Study

In this study, a confidence interval based on using RSS is derived for the difference between the SDs of normal distributions. Because a theoretical comparison between it and the standard one based on SRS is not possible, a Monte Carlo simulation study was designed using R version 3.6.2 statistical software (Ihaka and Gentleman, 1996) and conducted to compare the performances of the proposed and standard confidence intervals (the code for the simulation study is included in Appendix A). The confidence intervals were compared in terms of their coverage probabilities and the average lengths of their performances. Two sets of normal data were generated with means $\mu_x=\mu_y=10$ and SDs $(\sigma_x,\sigma_y)=$ (1,1), (1,3), (1,5), (3,1), (3,3), (3,5), (5,1), (5,3), and (5,5). To observe the behavior of small, moderate, and large sample sizes, $(n,m)=$ (10,10), (20,20), (30,30), (50,50) and (100,100) were used, and the number of simulations was fixed at 10,000. The nominal confidence level $1-\alpha$ was varied as 0.90, 0.95 and 0.99.

The results of the study are reported in Tables 2-4. The confidence interval based on SRS, CI_{SRS}, had empirical coverage probabilities close to the nominal confidence levels for all situations. However, the empirical coverage probabilities of the confidence interval based on RSS, CI_{RSS}, were better than the nominal confidence levels, meaning that it can cover the true parameter values better and so is a more reasonable approach. Additionally, the empirical coverage probabilities of both confidence intervals were independent of the values of the sample sizes and the SDs. Regarding the average length comparisons, the average lengths of CI_{RSS} were shorter than those of CI_{SRS} for all scenarios. In addition, the average lengths of both confidence intervals became shorter when the sample sizes increased. On the other hand, those of both confidence intervals increased according to the values of the difference between the SDs. The use of RSS compared to SRS improved the average length in the range of 1-10% depending on the sample size and the value of the SD.

6. A Real Data Example

The birth weight of babies in grams were used to illustrate the application of the confidence intervals proposed in the previous section. The first data-set comprising the birth weights of 189 babies was obtained from a book written by Hosmer and Lemeshow (2000) (https://rdrr.io/cran/TRSbook/man/BIRTH.WEIGHT.html). The second data-set comprising the birth weights of 107 babies was obtained from Secher et al. (1987) (https://rdrr.io/cran/ISwR/man/secher.html).

The sample mean and SD of the first and second data-set were 2,944.66 and 729.022, and 2,739.09 and 690.307, respectively. The coefficients of skewness and kurtosis of the first and second data-sets were -0.2102 and -0.0814, and -0.0882 and 0.3982, respectively. Histograms, density plots, box-and-whisker plots and normal quantile-quantile plots for both data-sets are displayed in Fig. 1 and 2, respectively. Fig. 3 shows the results of the Shapiro-Wilk normality test using R. It was found that both data-sets were in excellent agreement with a normal distribution. Samples of sizes (25,25) and (50,50) were randomly selected from each data sets using SRS and RSS. The 95% confidence intervals for the difference between the SDs were calculated and are reported in Table 5. Similar to the simulation results, the confidence interval based on RSS was more efficient than that based on SRS in terms of average length.

Table 2 - Empirical coverage probabilities and average lengths of 90% confidence intervals for the difference between the SDs of normal distributions

(n,m)	(σ_x,σ_y)	Coverage Probabilities		Average Lengths	
		CI_{SRS}	CI_{RSS}	CI_{SRS}	CI_{RSS}
(10,10)	(1,1)	0.9152	0.9420	1.3988	1.2612
	(1,3)	0.9068	0.9404	2.9576	2.6933
	(1,5)	0.9052	0.9325	4.6399	4.2533
	(3,1)	0.9060	0.9405	2.9595	2.6969
	(3,3)	0.9123	0.9452	4.2036	3.7964
	(3,5)	0.9104	0.9448	5.6675	5.1186
	(5,1)	0.9066	0.9382	4.6619	4.2497
	(5,3)	0.9105	0.9432	5.6673	5.1257
	(5,5)	0.9090	0.9433	7.0056	6.2977
(20,20)	(1,1)	0.9068	0.9384	0.8502	0.8115
	(1,3)	0.9053	0.9400	1.8425	1.7682
	(1,5)	0.9021	0.9346	2.9287	2.8292
	(3,1)	0.9028	0.9352	1.8396	1.7698
	(3,3)	0.9059	0.9447	2.5488	2.4320
	(3,5)	0.9094	0.9417	3.4720	3.3281
	(5,1)	0.8957	0.9394	2.9350	2.8223
	(5,3)	0.9061	0.9417	3.4754	3.3201
	(5,5)	0.9014	0.9394	4.2529	4.0624
(30,30)	(1,1)	0.9038	0.9372	0.6615	0.6416
	(1,3)	0.8989	0.9415	1.4433	1.4113
	(1,5)	0.8998	0.9417	2.3128	2.2586
	(3,1)	0.8987	0.9363	1.4444	1.4105
	(3,3)	0.9100	0.9368	1.9881	1.9234
	(3,5)	0.9031	0.9381	2.7042	2.6256
	(5,1)	0.9011	0.9386	2.3094	2.2567
	(5,3)	0.9040	0.9380	2.7050	2.6292
	(5,5)	0.9042	0.9409	3.3086	3.2079
(50,50)	(1,1)	0.8962	0.9379	0.4931	0.4837
	(1,3)	0.8974	0.9382	1.0852	1.0699
	(1,5)	0.8977	0.9369	1.7413	1.7196
	(3,1)	0.9003	0.9371	1.0871	1.0710
	(3,3)	0.9014	0.9394	1.4798	1.4510
	(3,5)	0.9019	0.9384	2.0233	1.9895
	(5,1)	0.8966	0.9391	1.7422	1.7196
	(5,3)	0.9025	0.9380	2.0222	1.9899
	(5,5)	0.8998	0.9398	2.4648	2.4191
(100,100)	(1,1)	0.8914	0.9352	0.3383	0.3353
	(1,3)	0.8959	0.9326	0.7519	0.7459
	(1,5)	0.8928	0.9281	1.2088	1.1995
	(3,1)	0.8991	0.9338	0.7513	0.7459
	(3,3)	0.8976	0.9322	1.0155	1.0068
	(3,5)	0.8931	0.9336	1.3938	1.3814
	(5,1)	0.8988	0.9366	1.2087	1.2002
	(5,3)	0.8956	0.9328	1.3923	1.3811
	(5,5)	0.9019	0.9360	1.6934	1.6783

Table 3 - Empirical coverage probabilities and average lengths of 95% confidence intervals for the difference between the SDs of a normal distributions

(n,m)	(σ_x,σ_y)	Coverage Probabilities		Average Lengths	
		CI_{SRS}	CI_{RSS}	CI_{SRS}	CI_{RSS}
(10,10)	(1,1)	0.9594	0.9749	1.7611	1.5882
	(1,3)	0.9526	0.9757	3.7047	3.3717
	(1,5)	0.9565	0.9751	5.8053	5.3262
	(3,1)	0.9584	0.9747	3.6949	3.3834
	(3,3)	0.9567	0.9790	5.2699	4.7745
	(3,5)	0.9571	0.9768	7.1340	6.4453
	(5,1)	0.9524	0.9750	5.7938	5.3063
	(5,3)	0.9594	0.9755	7.1148	6.4619
	(5,5)	0.9611	0.9779	8.7820	7.9520
(20,20)	(1,1)	0.9532	0.9766	1.0401	0.9952
	(1,3)	0.9536	0.9740	2.2442	2.1567
	(1,5)	0.9501	0.9722	3.5629	3.4386
	(3,1)	0.9471	0.9723	2.2404	2.1542
	(3,3)	0.9577	0.9790	3.1203	2.9813
	(3,5)	0.9543	0.9759	4.2338	4.0589
	(5,1)	0.9489	0.9703	3.5533	3.4274
	(5,3)	0.9558	0.9744	4.2510	4.0652
	(5,5)	0.9559	0.9765	5.1944	4.9682
(30,30)	(1,1)	0.9503	0.9767	0.8025	0.7783
	(1,3)	0.9520	0.9747	1.7460	1.7042
	(1,5)	0.9477	0.9718	2.7885	2.7258
	(3,1)	0.9513	0.9750	1.7448	1.7062
	(3,3)	0.9537	0.9748	2.4068	2.3385
	(3,5)	0.9506	0.9738	3.2740	3.1871
	(5,1)	0.9483	0.9756	2.7891	2.7219
	(5,3)	0.9566	0.9727	3.2769	3.1872
	(5,5)	0.9505	0.9747	4.0133	3.8940
(50,50)	(1,1)	0.9553	0.9741	0.5925	0.5830
	(1,3)	0.9487	0.9739	1.3050	1.2866
	(1,5)	0.9484	0.9714	2.0930	2.0661
	(3,1)	0.9518	0.9758	1.3046	1.2859
	(3,3)	0.9479	0.9738	1.7814	1.7505
	(3,5)	0.9478	0.9761	2.4337	2.3941
	(5,1)	0.9499	0.9750	2.0923	2.0645
	(5,3)	0.9452	0.9724	2.4342	2.3919
	(5,5)	0.9518	0.9747	2.9666	2.9145
(100,100)	(1,1)	0.9488	0.9724	0.4055	0.4020
	(1,3)	0.9494	0.9699	0.8972	0.8926
	(1,5)	0.9493	0.9690	1.4467	1.4349
	(3,1)	0.9493	0.9719	0.8991	0.8935
	(3,3)	0.9500	0.9751	1.2178	1.2051
	(3,5)	0.9506	0.9684	1.6670	1.6542
	(5,1)	0.9459	0.9725	1.4459	1.4350
	(5,3)	0.9494	0.9708	1.6680	1.6537
	(5,5)	0.9508	0.9711	2.0281	2.0104

Table 4 - Empirical coverage probabilities and average lengths of 99% confidence intervals for the difference between the SDs of a normal distributions

(n,m)	(σ_x,σ_y)	Coverage Probabilities		Average Lengths	
		CI_{SRS}	CI_{RSS}	CI_{SRS}	CI_{RSS}
(10,10)	(1,1)	0.9926	0.9971	2.6396	2.4026
	(1,3)	0.9903	0.9972	5.4942	5.0164
	(1,5)	0.9916	0.9971	8.5279	7.8068
	(3,1)	0.9918	0.9967	5.5012	4.9990
	(3,3)	0.9928	0.9972	7.9076	7.2005
	(3,5)	0.9918	0.9957	10.6531	9.6665
	(5,1)	0.9911	0.9967	8.4697	7.8409
	(5,3)	0.9917	0.9965	10.6694	9.6947
	(5,5)	0.9924	0.9965	13.2283	11.9991
(20,20)	(1,1)	0.9918	0.9966	1.4597	1.3938
	(1,3)	0.9911	0.9969	3.1238	2.9964
	(1,5)	0.9924	0.9956	4.9314	4.7579
	(3,1)	0.9914	0.9961	3.1191	2.9923
	(3,3)	0.9907	0.9967	4.3777	4.1803
	(3,5)	0.9904	0.9959	5.9381	5.6825
	(5,1)	0.9898	0.9967	4.9280	4.7602
	(5,3)	0.9908	0.9975	5.9230	5.6858
	(5,5)	0.9921	0.9975	7.2928	6.9911
(30,30)	(1,1)	0.9907	0.9966	1.1004	1.0694
	(1,3)	0.9910	0.9961	2.3806	2.3227
	(1,5)	0.9896	0.9958	3.7882	3.7048
	(3,1)	0.9890	0.9972	2.3818	2.3244
	(3,3)	0.9914	0.9950	3.3033	3.2083
	(3,5)	0.9884	0.9966	4.4858	4.3663
	(5,1)	0.9905	0.9966	3.7917	3.6946
	(5,3)	0.9913	0.9962	4.4923	4.3678
	(5,5)	0.9899	0.9965	5.5063	5.3429
(50,50)	(1,1)	0.9893	0.9967	0.8001	0.7865
	(1,3)	0.9883	0.9961	1.7518	1.7271
	(1,5)	0.9891	0.9957	2.8060	2.7652
	(3,1)	0.9907	0.9963	1.7513	1.7278
	(3,3)	0.9894	0.9965	2.4009	2.3600
	(3,5)	0.9907	0.9963	3.2751	3.2230
	(5,1)	0.9901	0.9981	2.8017	2.7642
	(5,3)	0.9890	0.9962	3.2795	3.2241
	(5,5)	0.9899	0.9973	4.0011	3.9373
(100,100)	(1,1)	0.9893	0.9961	0.5402	0.5351
	(1,3)	0.9897	0.9961	1.1930	1.1850
	(1,5)	0.9899	0.9960	1.9156	1.9074
	(3,1)	0.9879	0.9965	1.1944	1.1863
	(3,3)	0.9900	0.9946	1.6195	1.6047
	(3,5)	0.9901	0.9963	2.2163	2.1990
	(5,1)	0.9901	0.9960	1.9194	1.9062
	(5,3)	0.9881	0.9965	2.2186	2.1998
	(5,5)	0.9903	0.9960	2.6991	2.6761

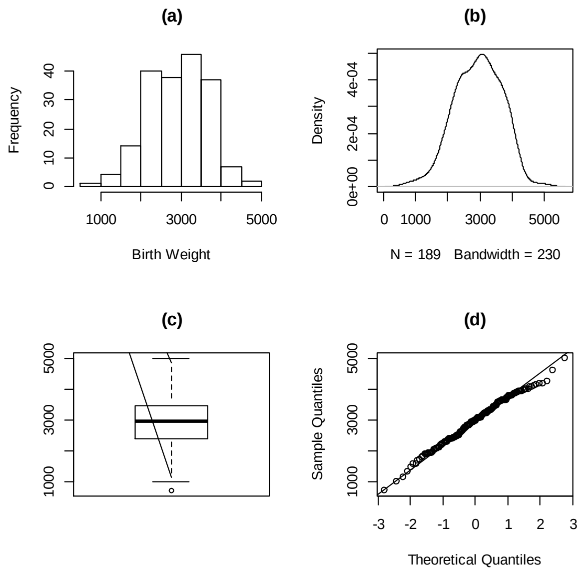

Fig. 1 - (a) Histogram, (b) density plot, (c) Box-and-Whisker plot and (d) normal quantile-quantile plot of the birth weight of babies in the first data-set.

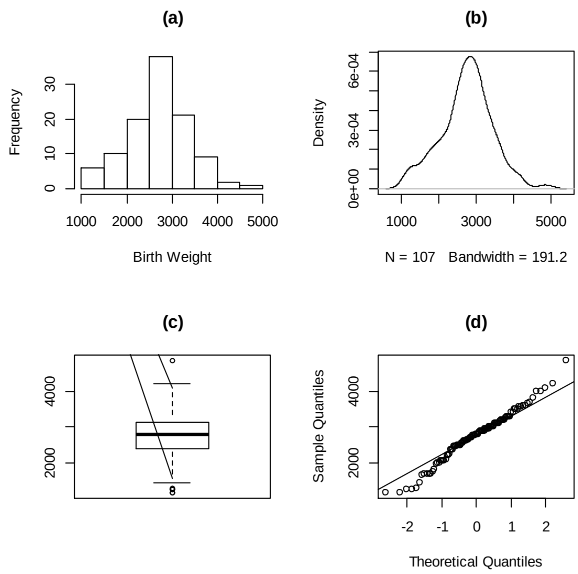

Fig. 2 - (a) Histogram, (b) density plot, (c) Box-and-Whisker plot and (d) normal quantile-quantile plot of the birth weight of babies in the second data-set.

```
Shapiro-Wilk normality test
data:  BW1
W = 0.99247, p-value = 0.4383

Shapiro-Wilk normality test
data:  BW2
W = 0.98412, p-value = 0.2329
```

Fig. 3 - Shapiro-Wilk normality test for normality of the birth weight data of babies from the first and the second data-sets.

Table 5 – 95% confidence intervals and corresponding widths using both intervals of the difference between the SDs

(n,m)		CI_{SRS}	CI_{RSS}
(25,25)	Confidence interval	(-105.27, 517.63)	(-463.32, 101.21)
	Width	622.90	564.53
(50,50)	Confidence interval	(-102.11, 339.73)	(-162.03, 247.10)
	Width	441.84	409.13

7. Conclusions

In this study, the confidence interval for the difference between the SDs of normal distributions was derived using MOVER. It was improved using the estimators of the population SDs via RSS instead of SRS. Since a theoretical comparison is not possible, a simulation study was conducted to compare the performance of these confidence intervals. The empirical coverage probabilities and the average lengths of the confidence intervals based on RSS and SRS were compared as the criteria to find the best confidence interval. Based on the simulation results, the proposed confidence interval based on RSS performed more efficient than the existing one based on SRS in terms of these criteria. In addition, the empirical coverage probabilities of both confidence intervals are independent of the values of the sample sizes and the SD. On the other hand, the average lengths of both confidence intervals are dependent on both of these. In addition, simulation results indicate that the average length of both confidence intervals increased as the difference between the SDs increased but decreased as the sample size increased. Overall, our proposed method based on RSS instead of SRS improved the coverage probability and provided shorter width confidence intervals.

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Appendix A: The R code

```
sigma.ci <- function(sdx,n,alpha){
   k1 <- qchisq(alpha/2,df=n-1)
   k2 <- qchisq(1-alpha/2,df=n-1)
        s2 <- sdx^2
   lower <- sqrt(((n-1)*s2)/k2)
   upper <- sqrt(((n-1)*s2)/k1)
   output <- cbind(lower,upper)
   return(output) }

diff.sigma.ci.1 <- function(sdx,sdy,n,m,alpha){
   cn <- sqrt(2/(n-1))*(gamma(n/2)/gamma((n-1)/2))
   cm <- sqrt(2/(m-1))*(gamma(m/2)/gamma((m-1)/2))
   theta1.hat <- cn*sdx
   theta2.hat <- cm*sdy
   sigma.ci.1 <- sigma.ci(sdx,n,alpha)
   sigma.ci.2 <- sigma.ci(sdy,m,alpha)
   l1 <- sigma.ci.1[1]
   u1 <- sigma.ci.1[2]
   l2 <- sigma.ci.2[1]
   u2 <- sigma.ci.2[2]
   L <- (theta1.hat-theta2.hat)-sqrt(((theta1.hat-l1)^2)+((u2-theta2.hat)^2))
   U <- (theta1.hat-theta2.hat)+sqrt(((u1-theta1.hat)^2)+((theta2.hat-l2)^2))
   output <- cbind(L,U)
   return(output) }

diff.sigma.ci.2 <- function(sdx,sdy,n,m,alpha) {
   theta1.hat <- sdx
   theta2.hat <- sdy
   sigma.ci.1 <- sigma.ci(sdx,n,alpha)
   sigma.ci.2 <- sigma.ci(sdy,m,alpha)
   l1 <- sigma.ci.1[1]
   u1 <- sigma.ci.1[2]
   l2 <- sigma.ci.2[1]
   u2 <- sigma.ci.2[2]
   L <- (theta1.hat-theta2.hat)-sqrt(((theta1.hat-l1)^2)+((u2-theta2.hat)^2))
   U <- (theta1.hat-theta2.hat)+sqrt(((u1-theta1.hat)^2)+((theta2.hat-l2)^2))
   output <- cbind(L,U)
```

```
   return(output) }

SRS.sampling <- function(X,k,nc) {
   SRSsamp <- sample(X,(k*nc),replace=TRUE)
   SRSstdev <- sd(SRSsamp)
   return(SRSstdev) }

RSS.sampling <- function(X,k,nc) {
   n <- k*nc
   samp    <- vector(length=k)
   ssamp   <- vector(length=k)
   rsobs   <- vector(length=k)
   rscycle <- matrix(nrow=nc,ncol=k)

   for(j in c(1:nc)){
   for(i in c(1:k)){
   samp <- sample(X,k,replace=TRUE)
   ssamp <- sort(samp)
   rsobs[i] <- ssamp[i] }
   rscycle[j,] <- rsobs
   }
   ID <- seq(1,n,by=1)
   RSdata <- data.frame(ID)
   a <- 1
   for (i in c(1:nc)){
   for (j in c(1:k)){
   RSdata$factor[a] <- i
   RSdata$data[a] <- rscycle[i,j]
   a <- a+1
   }}
   # Create a linear model, run a one-way ANOVA, and pull the MST and MSE values
   LinModel <-lm(RSdata$data~RSdata$factor)
   MSE <-anova(LinModel)["Residuals","Mean Sq"]
   MST <-anova(LinModel)["RSdata$factor","Mean Sq"]
   # Calculate Ranked Set Sample Variance and Standard Deviation
   RSSstdev <- sqrt(((k-1)*MST+(k*nc-k+1)*MSE)/(k*nc))
        return(RSSstdev) }

main.diff.sigma <- function(M,alpha) {
   n <- c(10,20,30,50,100)
   m <- n
   mux <- 10
   muy <- 10
   sigmax <- c(1,3,5)
   sigmay <- c(1,3,5)
   k <- 5
   temp1 <- rep(0,M)
   temp2 <- rep(0,M)
   len1 <- rep(0,M)
   len2 <- rep(0,M)
   cat("n",'\t',"m",'\t',"sigmaX",'\t',"sigmaY",'\t',"CP1",'\t',"CP2",'\t',"AW1",'\t',"AW2",'\n')
        for (p in 1:length(n)) {
               nc <- n[p]/k
        for (j in 1:length(sigmax)) {
        for (h in 1:length(sigmay)) {
               D <- sigmax[j]-sigmay[h]
        for (i in 1:M) {
               X <- rnorm(5000,mux,sigmax[j])
               Y <- rnorm(5000,muy,sigmay[h])
               sd.x.srs  <- SRS.sampling(X,k,nc)
               sd.y.srs  <- SRS.sampling(Y,k,nc)
               sd.x.rss  <- RSS.sampling(X,k,nc)
               sd.y.rss  <- RSS.sampling(Y,k,nc)

# compute CI based on SRS
               ci.1 <- diff.sigma.ci.1(sd.x.srs,sd.y.srs,n[p],m[p],alpha)
# compute CI based RSS
               ci.2 <- diff.sigma.ci.2(sd.x.rss,sd.y.rss,n[p],m[p],alpha)
               low.ci1 <- ci.1[1]
               up.ci1  <- ci.1[2]
               low.ci2 <- ci.2[1]
               up.ci2  <- ci.2[2]
               if ((D>=low.ci1) & (D<=up.ci1)) {temp1[i] <- 1}
               else { temp1[i] <- 0}
               if ((D>=low.ci2) & (D<=up.ci2)) {temp2[i] <- 1}
               else { temp2[i] <- 0}
```

```
                len1[i] <- up.ci1-low.ci1
                len2[i] <- up.ci2-low.ci2
        }
   cov1 <- mean(temp1)
   cov2 <- mean(temp2)
   avlen1 <- mean(len1)
   avlen2 <- mean(len2)
   cat(n[p],'\t',m[p],'\t',sigmax[j],'\t',sigmay[h],'\t',cov1,'\t',cov2,'\t',avlen1,'\t',avlen2,
'\n')
        }
        }
        }
}
# This is an example that will run the function
main.diff.sigma(10000,0.05)
```

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