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Robust Cooperative Controller for Output Regulation in Multi-Robot Systems

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Abstract: In this paper a robust cooperative multi-agent controller is designed for a linear uncertain multi-robot system with a nonlinear exosystem. Multi-robot system is divided into two subsystems; the first group of subsystems can be associated with the leader while the second group has no leader. Assuming that all agents are similar and have the same dynamics and decentralized control, state and output feedback control rules are proposed. Multi-robot system with nonlinear exosystem is simulated in MATLAB and tracking performances with state and output feedback controllers are investigated.

Keywords: Robust Control, Parameter Uncertainty, Decentralized Control, Multi-Agent Systems

1. Introduction

Generally a linear uncertain multi-agent system is defined as follows:

$$\dot{x}_i = A_i x_i + B_i u_i + E_i \boldsymbol{v}\upsilon$$

$$y_i = c_i x_i \quad , \quad i=1,\ldots.N \tag{1}$$

where $x_i \in \mathbb{R}^n$, $y_i \in \mathbb{R}^p$, $u_i \in \mathbb{R}^m$ are the state, measurement output and control input of the ith subsystem, E_i is input reference of each agent and $\upsilon \in \mathbb{R}^q$ is the exogenous signal representing the reference input to be tracked or the disturbance to be rejected and is assumed to be generated by a so called exosystem as follows:

$$\dot{v}_1 = \upsilon v_1^2 + v\upsilon_2 - v\upsilon_1^3$$

$$\dot{v}_2 = -v\upsilon_1 - 10v\upsilon_2^3 + v\upsilon_3^3 \tag{2}$$

$$\dot{v}_3 = -2v\upsilon_1^3 - 5v\upsilon_2^3 - 3v\upsilon_3^3$$

Equations 2 show that the exosystem is non-linear, that applies a nonlinear disturbance signal from the external environment to a multi-agent system [1], [2]. This signal has three components those are $\dot{v}_1$, $\dot{v}_2$, $\dot{v}_3$ and:

$$\dot{\boldsymbol{v}} = \begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ \dot{v}_3 \end{bmatrix} \tag{3}$$

The exosystem does not have a control input. The regulated outputs for the subsystems are defined as:

$$e_i = y_i - y_o \quad , \quad i=1,\ldots, N \tag{4}$$

In [3], [4] and [5] cooperative output regulation problem of linear multi-agent system has also been studied by the feedforward control method. This method is quite different from our internal model based control method, and cannot handle the plant uncertainty. In the context of cooperative output regulation (see [6] for example), system (1) and exosystem (3) together are considered as a multi-agent system of $N+1$ agents with the exosystem as the leader and all N subsystems of system (1) as the followers, and the communication topology of this multi-agent system is described by a diagraph ψ.

2. System description

In Figure 1 we have node 0 associated with the exosystem and the other N nodes associated with the N subsystems of system (1) and for $i=1,\ldots, N$, $j=0,1,\ldots,$

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N , $i \neq j$, $(j, i) \in \bar{\mathcal{E}}$; If and only if the control u_i of the subsystem i can access the state x_j or the output y_i of subsystem j, and, for $j=1,\ldots, N$, $(j, 0) \notin \bar{\mathcal{E}}$ because the system has no control input.

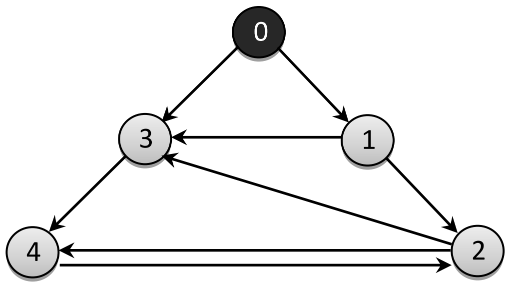

Fig. 1 Network topology ψ .

As a result, the subsystems of system (1) can be divided into two groups. The first group consists of those subsystems whose controls can access y_o , while the second group consists of those subsystems whose controls cannot access y_o. The first group of subsystems is composed of agents 1 and 3 those an access y_o, and the second group is composed of agents 2 and 4 those cannot access y_o. A matrix ψ is assumed to be a weighted adjacency matrix of a digraph $\bar{\psi}$ for $i=1, \ldots, N$ and $j=1, \ldots, N$. The components of ψ is defined as:

$$\begin{cases} a_{ii} = 0 \\ a_{ij} = 1 \quad ; (j, i) \in \mathcal{E} \end{cases}$$

Then for graph of Fig 1 one can write:

$$\psi = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Due to complexity and large scale of multi-agent systems, it is desired that their controllers is designed and implemented in a distributed manner. That is each controller takes feedback from its local states or outputs rather than taking feedback from all states or outputs of the system [7]. Fig. 2 shows a block diagram of a closed-loop multi-robot control system with state feedback and output feedback by the centralized control method based on the robust control [8]. We will consider the class of distributed control laws described as follows:

$$u_i = k_1\Big(\sum_{j \in N_i} a_{ij}\big(x_i - x_j\big) + a_{i0} x_i\Big) + k_2 z_i$$

$$\dot{z}_i = G_1 z_i + G_2 e_{\upsilon i} \quad , \quad i=1,\ldots,N \qquad (5)$$

where G_1 , G_2 are linear functions and k_1 , k_2 are constant matrices.

The aim of controller design is to find a control law of the form (5), which is called a distributed dynamic state feedback control law, in such a way that:

i. Closed-loop system stability can be demonstrated using the Hurwitz case.
ii. Parameters $x_i(0)$, $z_i(0)$, $\upsilon(0)$ have initial conditions so that the regulated output, $\lim_{t \to \infty} e_i(t) = 0$, $i=1,\ldots, N$.

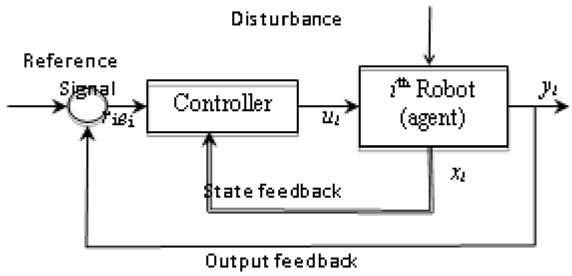

Fig. 2. Control block diagram of ith agent in multi-robot system shown in Figure 1. The distributed control approach uses state feedback or output feedback (only one of them is selcted).

Accordingly, our distributed dynamic output feedback control law is defined as:

$$u_i = k_1\Big(\sum_{j \in N_i} a_{ij}\big(\xi_i - \xi_j\big) + a_{i0}\xi_i\Big) + k_2 z_i$$

$$\dot{\xi}_i = A\xi_i + Bu_i - LC\Big(\sum_{j \in N_i} a_{ij}(\xi_i - \xi_j) a_{i0} x_i\Big) + L e_{\upsilon\upsilon i}$$

$$\dot{z}_i = g_1 z_i + g_2 e_{\upsilon i} \quad , \quad i=1,\ldots, N \qquad (6)$$

where ξ_i and z_i are dynamic modes stabilize the nominal closed loop system [7] and k_1, k_2, L are some constant matrices to be designed.

Assumption 1: The following conditions must be established to robust cooperative output regulation to perform centralized control method in a multi-agent system.

- Parameter v has no eigenvalues with negative real parts
- The pair (A, B) is stabilizable and the pair (C, A) is detectable.

3. Controller Design

A multi-robot system consists of five subsystems that one of the subsystems is assumed to be leader as shown in Figure 1. Consider the following state-space model:

$$\dot{x}_{1i} = x_{2i}$$

$$\dot{x}_{2i} = \delta_{1i}x_{1i} + \delta_{2i}x_{2i} + u_i + \mu_i v_3$$

$$y_i = x_{1i}$$

$$e_i = x_{1i} - (\upsilon v_1 + v\upsilon_2),\ \text{i=1,2,3,4} \qquad (7)$$

where

- **δ** is Parameter uncertainty factor of each subsystem and is equal to δ_{ij} *=0.05×j×i* , *i=1,2,3,4* , *j=1,2* ;
- **μ** is disturbance factor to each of the subsystems and it is equal to $\mu_i = i$, i=1,2,3,4 ;
- $\boldsymbol{v}$ is nonlinear exogenous signal is generated by the exosystem of form (2).

At the sequel, we introduce two control strategies for dynamic multi-robot system. One of them based on state feedback and the other one based on output feedback.

3.1 State feedback control

Our distributed dynamic state feedback control law is described as:

$$u_i = k_1\xi_i + k_2 z_i$$

$$\dot{z}_i = G_1 z_i + G_2 e_{\upsilon v i}$$

$$e_{\upsilon i} = Cx + Fv \qquad (8)$$

where z_i state feedback design for and k_1,k_2 are some constant matrices to be designed.

3.2 Output feedback control

Our distributed dynamic output feedback control law is defined as:

$$u_{i=} k_1\xi_i + k_2 z_i$$

$$\dot{z}_i = G_1 z_i + G_2 e_{\upsilon v i}$$

$$\dot{\xi}_i = A\xi_i + Bu_i - LC\xi_i + Le_{\upsilon v i}$$

$$e_{\upsilon i} = Cx + Fv \qquad (9)$$

where z_i and ξ_i is output feedback of system for each agent and k_1 , k_2 , G_1 , G_2 and L are some constant matrices to be designed for the robust cooperative output regulation aim.

4. Simulations

System (7) can be written in the form (1) where:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C = [1 \quad 0],\ \delta_i A = \begin{bmatrix} 0 & 1 \\ \delta_{1i} & \delta_{2i} \end{bmatrix},$$

$$E_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & i \end{bmatrix},\ G_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix},\ G_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

$$F = [-1 \quad -1 \quad 0],\ K_1 = [-12.8844 \quad -12.3868],$$

$$K_2 = [-3.0000 \quad +1.8948 \quad -7.8384],$$

$$L = [6.9284 \quad 4.0000]^T ;$$

Controllers (8) and (9) are applied with the following initial conditions:

$$x_{11}(0) = 1,\ x_{12}(0) = 4,\ x_{13}(0) = 3,\ x_{14}(0) = 2$$

$$x_{21}(0) = 1,\ x_{22}(0) = 3,\ x_{23}(0) = 2,\ x_{24}(0) = 1$$

$$z_1(0) = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix},\ z_2(0) = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix},\ z_3(0) = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix},\ z_4(0) = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$\xi_1(0) = \begin{bmatrix} 1 \\ 4 \end{bmatrix},\ \xi_2(0) = \begin{bmatrix} 4 \\ 3 \end{bmatrix},\ \xi_3(0) = \begin{bmatrix} 3 \\ 2 \end{bmatrix},\ \xi_4(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

A challange with existance of the nonlinear exosystem in multi-robot system is to control a system that strongly depends on initial conditions. The dependence is such that with increase or decrease of initial values, the system can be unstable or choppy. For this reason we define a limit for the initial conditions of exosystem as the following:

$$-1 \le v\upsilon_i(0) \le 1 \qquad (10)$$

Accordingaly we set $\upsilon v_1(1) = -1$, $\upsilon v_2(1) = 0.9$, $\upsilon v_3(1) = 1$;

Remark1: Our design method is different than that of [5] in one aspect. That is the system used here is nonlinear which reduces the instability in the system.

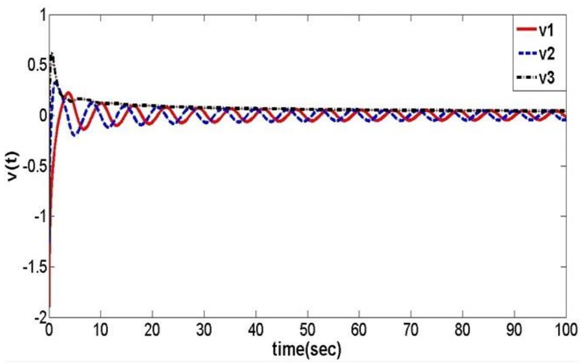

Fig. 3 exogenous signal produce by nonlinear exosystem

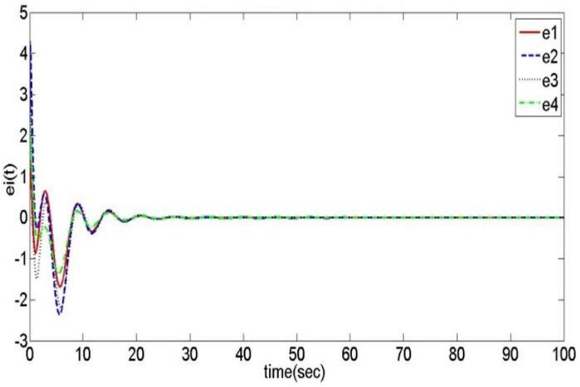

Fig. 4 Tracking performance under the dynamic state feedback control rule.

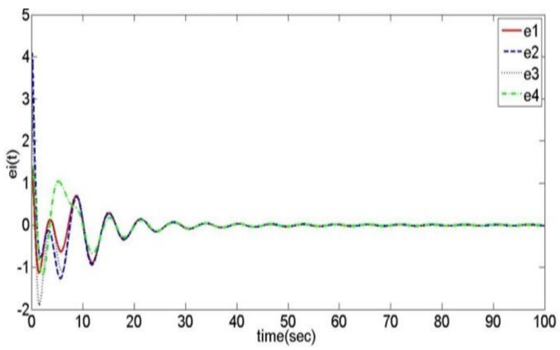

Fig. 5 Tracking performance under the dynamic output feedback control rule.

After a simulation multi-robot system in MATLAB, with above initial conditions [7], following results have been obtained. Figure 3 shows the exogenous signal that prodused by non-linear exosystem (2). Figures 4 and 5 show result of tracking performance of robots by applying state feedback and output feedbach control rules respectively.

5. Conclusions

A robust cooperative multi-agent controller was designed for a linear uncertain multi-robot system with a nonlinear exosystem. Simulation results showed that the state feedback control method is better than the output feedback control. Figure (3) and (4) imply that the system error converges to zero in about 25 seconds for state feedback rule and in more than 80 seconds for output feedback rule. Morover transient response for ouput feedback rule is more oscilatory than that for stste feedback rule. This is because the second group of substems, consists of those subsystems whose controls can not access y_o. Therefore the system communication between the sub-systems is not complete.

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