

# Non-Linear Modelling and Control of Permanent Magnet Synchronous Machine for Actuator Applications

Phaneendra Babu Bobba<sup>1</sup>, Gaurang I Vakil<sup>2\*</sup>, Himavarsha Gajala<sup>1</sup>, B V Ravi Kumar<sup>3</sup>

<sup>1</sup> Department of Electrical and Electronics Engineering,

Gokaraju Rangaraju Institute of Engineering and Technology, Hyderabad, INDIA

<sup>2</sup> Department of Electrical and Electronics Engineering,

University of Nottingham, UNITED KINGDOM

<sup>3</sup> DRDO, Research Centre Imarat, INDIA

\*Corresponding Author: [gaurang.vakil@nottingham.ac.uk](mailto:gaurang.vakil@nottingham.ac.uk)

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## Abstract

Permanent magnet synchronous motors grabbed the attention due to their intrinsic characteristics. Aerospace applications necessitate great dependability and while reducing weight, complication, fuel intake, working expenses, and environmental effects. All these demands can be fulfilled to some extent with the PMSM motors because of their characteristics. The mathematical modelling of an Interior permanent magnet synchronous motor (IPMSM) including nonlinearities is proposed in this paper. The conventional models neglect nonlinearities such as hysteresis and eddy current losses, parameters variation with respect to rotor angle, magnetic saturation, cross-coupling effect, armature reaction, etc. The present model considers core loss and inductance varying concerning rotor position. Usually, the core loss is taken as a constant loss, but it varies with the speed. Ignoring the above parameters may deteriorate the performance of the motor in real-time compared to linear model. To validate the efficacy of the proposed model, two models are simulated in MATLAB and the results are compared.

## 1. Introduction

The inclusion of permanent magnets instead of rotor windings increased the flow of magnetic flux between the rotor and stator-made PM motors in numerous manufacturing appliances [1]. Though there are different motors including induction motor, Switched Reluctance Motor, PMSM is preferred over it. Because of their inherent features such as power density, less maintenance, small size, high energy density, and employed in applications, such as actuators, robotics, metal cutting machines, precision machining, etc. [2]. A theory is analyzed about the choice of motor for aerospace application, especially for actuators. Space appliances require high-level dependability, high convenience, and high-power density while aiming to reduce weight, complexity, fuel consumption, operational costs, and eco-friendly [3]. Innovative electric-driven systems can fulfill these needs and provide significant technical and financial improvements on already existing systems. Logical approaches are embraced to assess caged induction, reluctance, and PM motor technologies and their relative merits. The investigation recommends that the three-phase Permanent Magnet motor drive may be a preferred alternative for general aerospace applications. Though Permanent magnet machines have some disadvantages like expensive, more harmonic content in torque, etc remarkable balance, can be maintained. Rather than a Brushless dc motor

Synchronous motor of the permanent magnet have fewer ripples in torque and position control through commutation is a little easy. So, PM machines also each have their pros and cons. [3].

An actuator mechanical system could not arrange the input command signal due to Coulomb friction, and this phenomenon is known as dead-zone nonlinearity. In the current scenario, more electrical actuation systems have been adopted to minimize the effects caused by mechanical systems. The actuator is schematically represented as fig.1[4]. In actuators, electrical and mechanical non-linear factors such as the dead-zone, backlash, saturation, friction, and others can degrade the dynamic performance of the actuator, resulting in substantial degradation of attitude control effect, even causing oscillations, and jeopardizing global stability. Installation and de-bugging errors, dead-zone, gear backlash, and zero position error, among other things, exist in the actuator system due to manufacturing technology restrictions. Therefore, installation and debugging errors cannot be nullified.

Basically, the d-q axis model is considered while simulating because of quick execution compared to the time-varying model. Since PMSM has a complex mathematical model motor, for better dynamic performance the vector-controlled field Oriented Control technique is used. This enables independent control of torque and flux producing current components. The old-fashioned d-q model [7] is a classical magnetic circuit model which only considers the fundamental components of flux and inductance, views motor inductance as constant, and does not take into consideration hysteresis and eddy current loss as well [8]. Because of neglecting losses, time-varying parameters, etc the developed motor may not provide precise results and deviates from the actual motor in performance. Moreover, to nullify the variations a model must be developed which incorporates both nonlinearities.

Several models have been developed considering iron loss and position varying parameters. The nonlinear models [8-12] used different techniques, tools like Finite Element Analysis, JMAG, and other estimation methods to analyze the varying parameters. The model presented in [13] included additional core resistance which can account for core loss in Surface-mounted PMSM in time-varying models. This paper proposed deals with IPM machines given the effect of iron loss on the machine electromagnetic behavior, spatial harmonics in phase voltages which cause torque ripples. As the IPM rotor is configured the existence of reluctance torque causes an increase in spatial harmonics owing to the naturally large variation in magnetic energy with rotor position [14,15]. Hence, Linear modelling approaches cannot identify the effects caused by the nonlinearities mentioned.

**Table 1** Non-linearities and their effects on actuators

Non-linearities	Effects
Dead zone	The correction time increases as the dead zone widens, and the overshoot gradually increases. It demonstrates the influence on mistake tracking. Future increases may cause system oscillations.
Backlash	Backlash in a mechanical system makes it harder to regulate the system with great precision. It raises the error rate between the input and output systems.
Friction	Tracking mistakes, limit cycles, and other unwanted effects can all be caused by Friction.
Saturation	Proportional-derivative (PD) controllers have seen limit cycles in their presence, which could lead to a phenomenon known as integrator windup. The control signal saturates the actuator because of this effect, therefore increasing the control signal will not result in faster system response.
Iron loss	It influences the d-axis and q-axis currents resulting in a reduction of electromagnetic torque.
Inductance variation	It may cause harmonics in current and voltage waveforms and result in spatial harmonics in the system.
Magnetic saturation	The d-axis inductance of PMSM is affected by d-axis magnetic saturation. Inductance along the q-axis of the $L_d$ The magnetic saturation of the q-axis alters $L_q$ . Meanwhile, as the q-axis current rises, the saturation of PMSM rises, resulting in a decrease in d-axis inductance; at the same time, the d-axis current affects the q-axis inductance, resulting in the so-called cross saturation effect. As a result, the motor inductance exhibits non-linear behavior.

## 2. Linear Mathematical Model of PMSM

The mathematical model of permanent magnet synchronous motor is usually composed of voltage equation (1), stator flux equation (2), electromagnetic torque equation (3), and mechanical equation of motion (4). The equations can be expressed in the d-q coordinate systems as in [3,4]. The block diagram of the system is shown in fig (1). Voltage equation is:

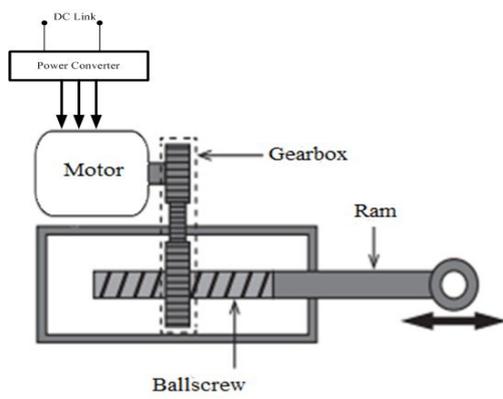
$$V_d = R_s i_d + \frac{d\Psi_d}{dt} - \omega_e \Psi_q \tag{1}$$

$$V_q = R_s i_q + \frac{d\Psi_q}{dt} + \omega_e \Psi_d \quad (2)$$

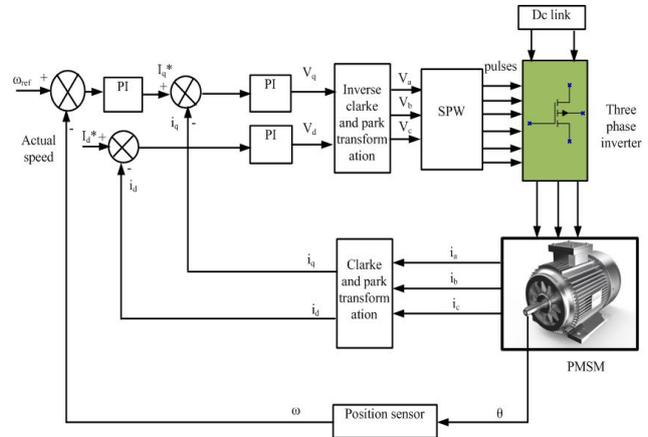
Stator equation:

$$\Psi_d = L_d i_d + \Psi_f \quad (3)$$

$$\Psi_q = L_q i_q \quad (4)$$



**Fig. 1** Schematic diagram of an electromechanical actuator



**Fig. 2** Block diagram of conventional IPM machine model

The electromagnetic torque equation is:

$$T_e = \frac{3}{2} p [\Psi_f i_q + (L_d - L_q) i_d i_q] \quad (5)$$

The mechanical torque equation is:

$$T_e = T_L + B\omega_m + J \frac{d\omega_m}{dt} \quad (6)$$

$$\omega_m = \omega_e \frac{2}{p} \quad (7)$$

From the above equations (1-7), it implies the parameters of the motor are constant excluding core loss, armature reaction, saturation, cross-coupling effect, variable inductance, etc. This reduces the accuracy of the motor's static and dynamic performance [8]. So, there is a need to remodeling the PMSM model as of real motor's characteristic.

### 3. Proposed Model of PMSM

In this section, the modeling of PMSM is presented by considering hysteresis and eddy current losses, inductance variation concerning rotor position.

#### 3.1 Mathematical Modelling Including Hysteresis and Eddy Current Losses

To sort out the difficulties of the typical model as reviewed in section 2, a machine model is employed with the following equations (8-10)

$$V_d = R_s i_{da} + \frac{d\Psi_d}{dt} + \omega_e \Psi_q \quad (8)$$

$$V_q = R_s i_{qa} + \frac{d\Psi_q}{dt} + \omega_e \Psi_d \quad (9)$$

$$T_e = \frac{3}{2} p [\Psi_f i_{qa} + (L_d - L_q) i_{da} i_{qa}] \quad (10)$$

where  $i_{da}$ ,  $i_{qa}$  are the currents after incurring losses. Iron loss in electrical machines impacts phase currents at a given supply voltage by increasing or decreasing active power depending on whether they operate as a motor

or generator. The iron loss model given in [16, 17] is used to study the core loss influence on the machine electromagnetic behavior at different load circumstances in a computationally effective way which can be described by

$$P_{Fe\_oc} = C_h \frac{g}{2\pi} + C_e \left(\frac{g}{2\pi}\right)^2 + C_x \left(\frac{g}{2\pi}\right)^{1.5} \tag{11}$$

$$g = \frac{V_m}{\Psi_f} \tag{12}$$

$$P_{Fe\_sc} = d_h \frac{k}{2\pi} + d_e \left(\frac{k}{2\pi}\right)^2 + d_x \left(\frac{k}{2\pi}\right)^{1.5} \tag{13}$$

$$k = \frac{V_{da}}{\Psi_f} \tag{14}$$

where, the iron loss coefficients ( $C_h, C_e, C_x$ ) as well as ( $d_h, d_e, d_x$ ) are taken from [16]. The core loss of the machine at a particular operation can be computed by  $P_{Fe\_oc}$  and  $P_{Fe\_sc}$ .  $P_{Fe\_oc}$  is the open circuit iron loss and  $P_{Fe\_sc}$  is the short circuit iron loss (11-14). The voltage magnitude  $V_m$  and the voltage due to armature reaction of the d-axis current  $V_{da}$  can be calculated by (15-16).

$$V_m = \sqrt{\omega_e^2 \Psi_d^2 + \omega_e^2 \Psi_q^2} \tag{15}$$

$$V_{da} = -\omega_e \Psi_d + \omega_e \Psi_f \tag{16}$$

The iron losses  $P_{Fe\_d}$  and  $P_{Fe\_q}$  taken from [9] are further divided into q axis and d axis loss components as follows and calculated using the equations (17-18):

$$P_{Fe\_d} = \frac{\Psi_q^2}{\Psi_d^2 + \Psi_q^2} P_{Fe\_oc} + P_{Fe\_sc} \tag{17}$$

$$P_{Fe\_q} = \frac{\Psi_d^2}{\Psi_d^2 + \Psi_q^2} P_{Fe\_oc} \tag{18}$$

In motoring mode, the machines absorb power from the electrical source, however, in generator mode, it reduces electrical output power. The loss caused due to the inclusion of iron loss is expressed in terms of induced voltages as given in [9]:

$$i_{da} = i_d \pm \frac{P_{Fe\_d}}{V_d - R_s i_d} \tag{19}$$

$$i_{qa} = i_q \pm \frac{P_{Fe\_q}}{V_q - R_s i_q} \tag{20}$$

Whereas  $i_{da}$  and  $i_{qa}$  [9] are the currents of the d-axis and q-axis as shown in (19-20).

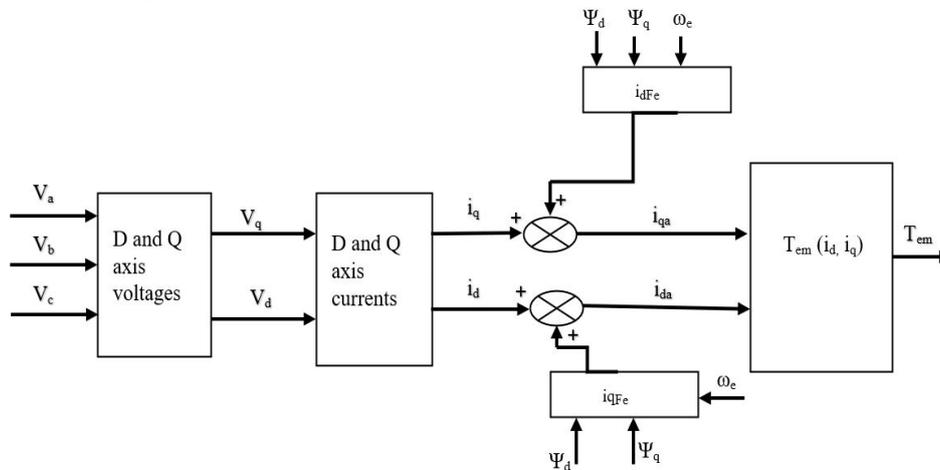
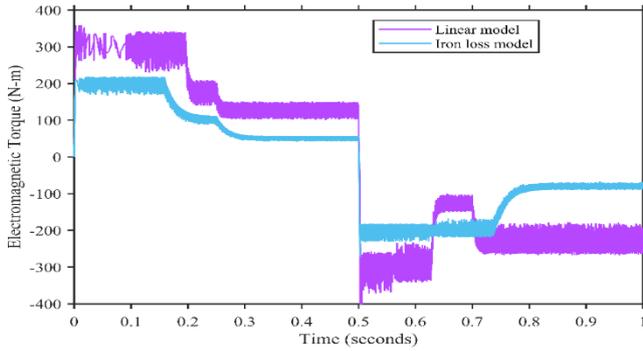


Fig. 3 Proposed IPM machine schematic with iron loss effect

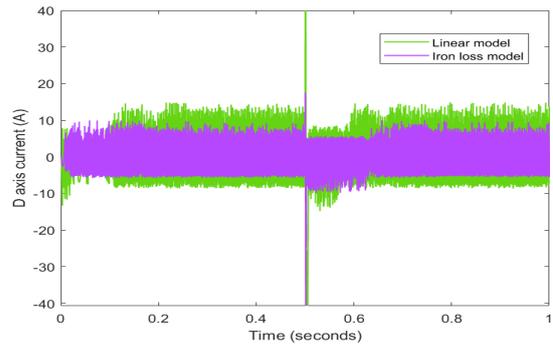
### 3.2 The Effect of Iron Loss on PMSM Motor

From the model in Fig. 2, the impact of considering hysteresis and eddy current losses on IPM machines' electromagnetic behavior can be studied. A notable difference is observed in the amplitude of the d axis and q axis current with the inclusion of losses. In the constant torque region with the current limit, the amplitudes of both d

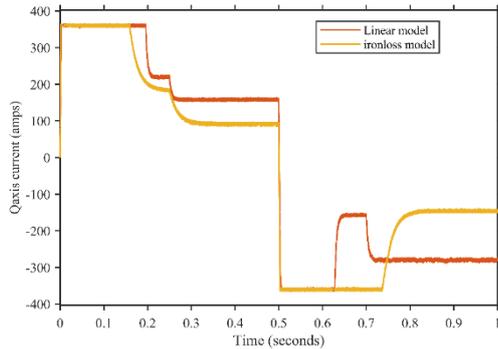
and q-axis currents,  $i_{da}$ , and  $i_{qa}$ , associated with electromechanical conversion are slightly reduced [9]. As a result, the output power of the motor is also reduced.



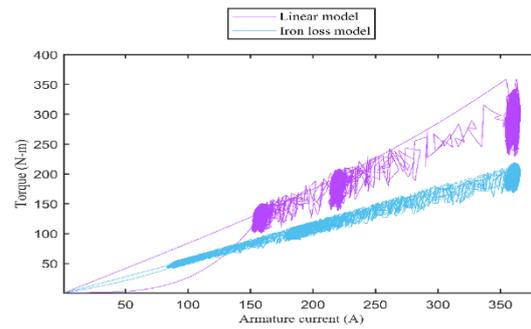
**Fig. 4** Comparison of electromagnetic torque with and without iron loss



**Fig. 5** Comparison of d axis current with and without iron loss



**Fig. 6** Comparison of q axis current with and without iron loss



**Fig. 7** Comparison of Torque versus armature current with and without loss

### 3.3 PMSM Modelling with Inductance Function of Rotor Angle

The PMSM is modelled with inductance variation using the following equations (21-28):

$$\Psi_d = L_{dd}i_d + L_{dq}i_q + \Psi_f \cos \theta \quad (21)$$

$$\Psi_q = L_{qq}i_q + L_{dq}i_d + \Psi_f \sin \theta \quad (22)$$

$$L_{dd} = L_1 - L_2 \cos \theta \quad (23)$$

$$L_{qq} = L_1 + L_2 \cos(2\theta) \quad (24)$$

$$L_{qd} = -L_1 \sin \theta \quad (25)$$

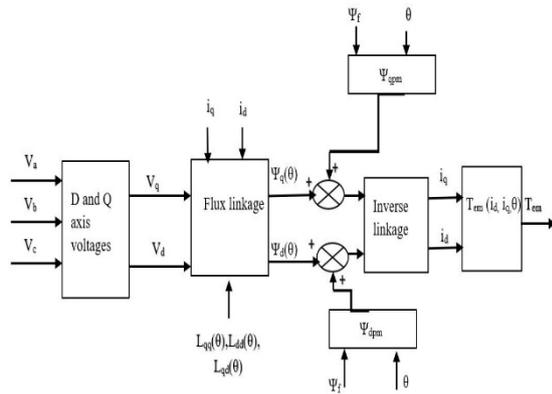
$$L_1 = \frac{1}{2}(L_q + L_d) \quad (26)$$

$$L_2 = \frac{1}{2}(L_q - L_d) \quad (27)$$

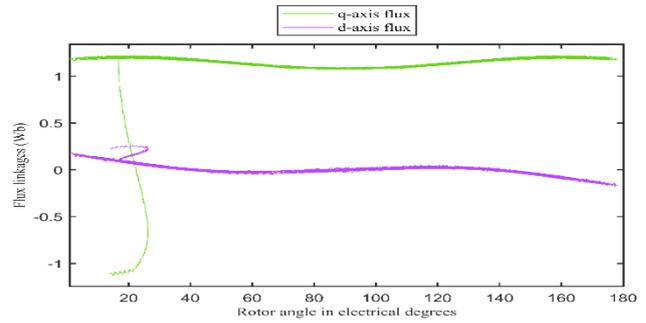
$$T_e = \frac{3}{2} p [(L_{dd} - L_{qq})i_d i_q + (\Psi_{dpm} i_q - \Psi_{qpm} i_d) + (L_{dq} i_q^2 - L_{dq} i_d^2)] \quad (28)$$

where  $L_{dd}$  is self-inductance of d-axis,  $L_{qq}$  is self-inductance of q-axis,  $L_{qd}$  is mutual inductance of stator,  $i_d$  and  $i_q$  are the d-axis and q-axis current. Generally, flux, torque, inductance, permanent magnet flux, etc parameters are taken as constant but they are dependent on rotor position  $\theta$  as shown in figures below and above equations (21-28) taken from [21]. The spatial harmonics are neglected in the traditional model as parameters are considered independent of rotor angle. The distortions in voltage, ripples in torque are caused due to spatial harmonics. Ignoring the rotor dependency, makes the model deviate from the actual performance of the machine. From

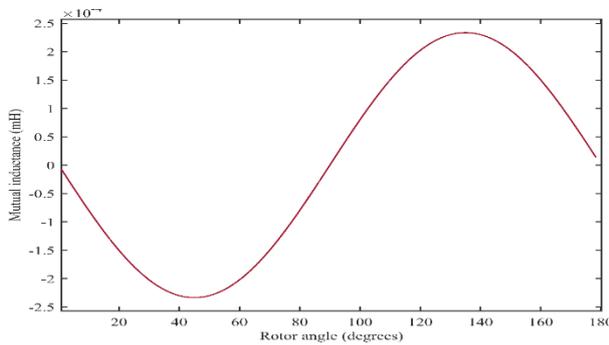
figures (9), (10), (11), it is evident that flux, inductance, and torque vary with rotor position. As from the fig.11, torque is constant in linear models whereas it varies with the rotor in real-time.



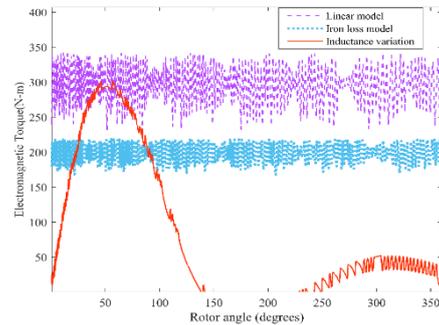
**Fig. 8** Proposed schematic diagram of IPMSM with inductance function of rotor position.



**Fig.9** Flux linkages variation with rotor position in electrical degrees



**Fig. 9** Mutual inductance variation with rotor position angle in electrical degrees



**Fig. 10** Comparison of electromagnetic torque for rotor position

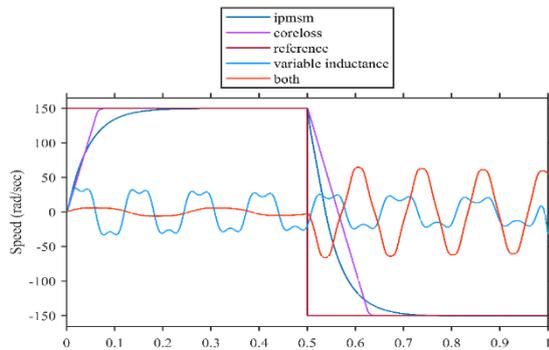
**Table 2** Specifications of motor

S.no	Parameters	Unit	Value
1.	Poles	--	4
2.	Rated speed	rpm	1500
3.	Rated torque	N.m	400
4.	Dc link voltage	V	550
5.	Moment of Inertia	Kg.m <sup>2</sup>	0.09
6.	Friction coefficient	Nm-s	0.002
7.	Flux linkage	V.s	0.185
8.	Resistance	Ω	6.5e-3
9.	D-axis Inductance	mH	1.597e-3
10.	Q-axis Inductance	mH	2.057e-3

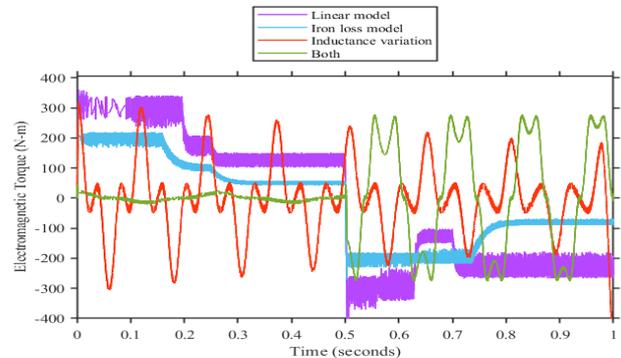
#### 4. Simulation Results

An Interior Permanent Magnet Synchronous Machine is modelled by field-oriented control technique as shown in fig. 2 with parameters in Table 2 As it is advantageous compared to dynamic modelling. From fig (5-15) comparative analysis is done on the performance of motor with and without nonlinearities. The motor is operated

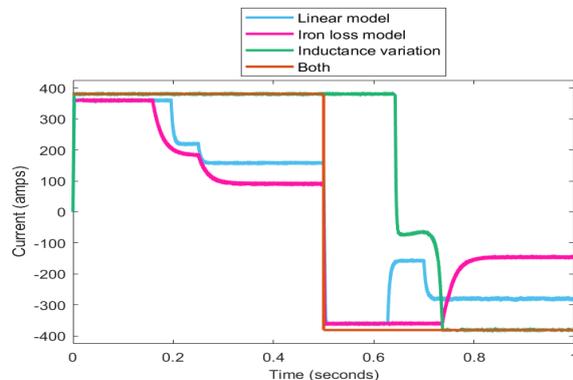
forward and reverse direction with 0.5s step time. In fig (6) it is evident that when the iron loss is considered, the magnitude of currents, is decreased and power output is reduced. As a result, the electromagnetic torque will be reduced slightly. When variation of parameters according to rotor angle is considered, there are oscillations and harmonic content exhibited in torque, current, etc. From fig (9-11) the flux, stator current, torque, and inductance are a function of rotor position. If various non-linearities are taken into consideration, we may see the change in parameters from the above figures (12), (13), (14). The traditional mode, however, shows no distortion in the expected voltages, currents, or flux connections. Position-dependent d- and q-axis flux connections harmonics are visible in torque and current waveforms, unlike the typical conventional model that ignores spatial harmonics. Ignoring the iron loss effect in the simulation cannot accurately replicate real-world machine behavior, especially at high speeds where the iron loss influence is substantial and important.



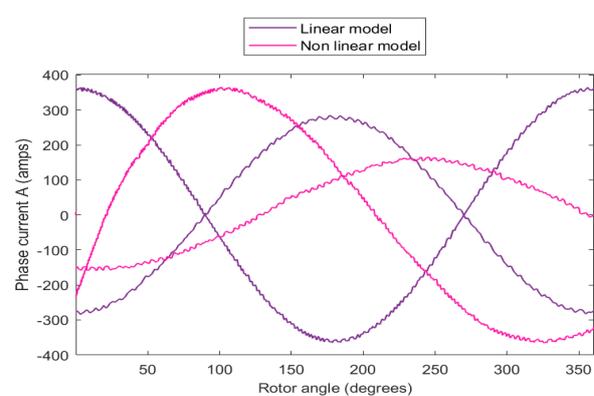
**Fig. 11** Comparison of the speed of the motor with the conventional and proposed model



**Fig. 12** Comparison of electromagnetic torque with the conventional and proposed model



**Fig. 13** Comparison of armature current with the conventional and proposed model



**Fig. 15** Comparison of phase current with the conventional and proposed model

## 5. Conclusion

In aerospace applications as guided missiles need extremely accurate control systems. This is because the missile must reach the target without fail. So, in Electromechanical actuators, motor control plays a vital role. The inclusion of non-linearities is complex than the simplified model but it yields better performance. The proposed mathematical model includes iron loss and varying inductance impact on the machine. To build the best control technique for precise controls, it is critical to reflect these negative impacts in drive system simulations. The results of the developed and simplified model are compared and presented. This model is more accurate when compared with the conventional model. The efficacy of the proposed model is done through a comparison of models. The simulation of the motor close to the real motor provides satisfaction to the user. It does not give false assurance about the performance of the motor.

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## Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

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