

# Machinery/Service system Scheduled Replacement time determination: A combine Weighted Aggregated Sum Product Assessment, Additive Ratio Assessment and Age Replacement Model approach

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Received 24 November 2017; accepted 28 December 2017, available online 22 April 2018

**Abstract:** The productivity of manufacturing/service industries greatly depend on the safety and reliability of the machinery/service system use for production/service delivery. The safety and reliability of the system can only be guaranteed through efficient maintenance. Scheduled replacement is an integral element of maintenance strategies. The major challenge of the maintenance policy is the determination of the optimum interval for carrying out replacement of equipment item of machinery/service system. A number of approaches have been applied in the literature in addressing this problem but these techniques have one limitation or another. In this paper, an MCDM approaches which avoid these limitations is proposed. The proposed Multi-Criteria Decision Making (MCDM) techniques combined weighted Aggregated Sum Product Assessment (WASPAS) and Additive Ratio Assessment (ARAS) with Age Replacement Model (ARM) for the determination optimum scheduled replacement time interval. The ARM for cost, reliability and downtime are aggregated using WASPAS and ARAS methods in order to rank alternative time intervals. The WASPAS and ARAS methods outputs are compared with another well-known technique in literature. To demonstrate the applicability of the proposed approaches a numerical example was applied. The result of the analysis revealed that, WASPAS and ARAS produces similar ranking for alternatives when compared with that of TOPSIS technique in the literature which is more computationally intensive, thereby validating the suitability of the proposed approaches.

**Keyword:** Scheduled Replacement interval, WASPAS, ARAS, Age Replacement Model, decision criteria

## 1. Introduction

The degree of productivity of manufacturing/service industries depend on the effectiveness of the machinery/service system for production/service delivery. For optimum production/services delivery, the machinery/service system must be safe and reliable and these can only be achieved through regular and efficient maintenance of the system. British Standard define maintenance as (BS 1993) “*the combination of all technical and administrative actions, intended to retain an item in, or restore it to a state in which it can perform a required action*”. There are basically two types of maintenance techniques namely; Corrective maintenance (CM) and Preventive Maintenance (PM) [1]. The PM are classified into two; Time based Preventive Maintenance (TPM) and Condition Based Maintenance (CBM). The TPM are of two types: Scheduled Overhaul (SO) and Scheduled Replacement (SR) [2]. SR is defined as a practice that involves decision making, concerning the optimal interval to replace machinery equipment item

based on certain decision criteria in order to eliminate a sudden breakdown [3]. For some equipment items of the machinery/service system, scheduled replacement approach is most appropriate for mitigating failure. The method is typically ideal for machinery equipment that satisfy the following conditions: exposure to critical failure, large percentage of units of the equipment must survive to at least the time of replacement and the failure mode must be of major economic consequences [4].

The major challenge of the SR maintenance policy is the determination of optimum time interval to carry out replacement of machinery/service system equipment item [4]. This is due to the fact that, if the time interval is not accurately evaluated, it may either result to over-maintenance or under maintenance [5]. The over-maintenance scenario outcome is wastage of resources and man hours due to premature replacement. On the other hand, under maintenance result to catastrophic system failure which may damage company’s image irreversibly.

The study on the determination of interval for performing scheduled replacement of equipment items of

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machinery/service system have been reported in literature. However, in majority of the research, single criteria were applied in arriving at optimum solution. In this approach, either a Block Replacement Model (BRM) or Age Replacement Model (ARM) is applied whilst utilizing cost or downtime as decision criteria. The use of a single criteria may not be sufficient due to the fact that the decision problem generally involves several conflicting decision criteria such as cost, reliability, availability and risk [6].

Few authors, nevertheless have applied multi-criteria decision making (MCDM) approach in producing optimum solution. In this methodology, different decision criteria such as cost, reliability and downtime are aggregated into a single criteria using MCDM tools such as PROMETHE and TOPSIS. Cavalcante and De Almeida [7] presented a scheduled replacement interval decision model based on combination Preference Ranking Organization METHod for Enrichment Evaluations (PROMETHEE) II and Bayesian technique. The authors simultaneously aggregated two decision criteria; cost and reliability with the aid of PROMETHEE II in order to produce optimum scheduled replacement time interval. In a similar research Cavalcante et al [8] applied a combination of PROMETHEE method and ARM in a scenario of uncertainty in maintenance data. Emovon et al [6] proposed an integrated TOPSIS and ARM approach whilst considering reliability, cost and downtime as decision criteria. The approach was applied to determine optimum scheduled replacement time interval for an equipment item of a marine machinery system.

However, the MCDM tools used by previous researchers have one limitation or another. For example, the PROMETHEE II method computational complexity increases as the number of the decision criteria increases. In the TOPSIS method, the relative distance between positive and negative ideals solutions are not put into consideration in the decision making process which negatively affect it outputs [9].

In this paper an alternative MCDM approaches which avoid these limitations are proposed. The proposed techniques are: Weighted Aggregated Sum Product Assessment (WASPAS) and Additive Ratio Assessment (ARAS). The MCDM tools are applied in turns in conjunction with the ARM to determine optimum replacement time interval. The simple ARM for cost, downtime and reliability were adopted from literature. The cost, reliability and downtime model are aggregated using WASPAS and ARAS methods in-turns to rank optimum replacement time interval. The WASPAS method was chosen because it is far less computationally intensive when compared to TOPSIS and PROMETHEE [10]. Furthermore, the technique is hardly affected by normalization approach applied in the analysis. The ARAS approach was also chosen because it even easier to implement than WASPAS approach.

The remaining part of this paper is organized as follows: In Section 2 the proposed scheduled replacement interval approach is presented. In Section 3 a numerical example is presented to demonstrate applicability of the

proposed method. Finally, the conclusions are presented in Section 4.

## 2. Method

### 2.1 Criteria modelling

In this paper three decision criteria; reliability, cost and downtime based on ARM, are applied in determining optimum time interval for replacement of a machinery/service system equipment item. The three criteria are represented as mathematical functions as follows:

**Reliability function.** The reliability function for a two weibull distribution system is represented as follows:

$$R(t_p) = \exp \left[ - \left( \frac{t_p}{\gamma} \right)^\alpha \right] \tag{1}$$

$\alpha$  is the shape parameter which indicate the nature of the distribution and  $\gamma$  is the scale parameter which influences the distribution spread.

**Cost function:** The scheduled replacement cost per unit time is given as follows [11]:

$$C(t_p) = \frac{C_x (1 - R(t_p)) + C_y R(t_p)}{\int_0^{t_p} t f(t) dt + T_x (1 - R(t_p)) + \{(T_y + t_p) R(t_p)\}} \tag{2}$$

Where:

$C_x$  is the cost of unit failure maintenance

$C_y$  is the cost of unit preventive maintenance

$t_p$  is the scheduled replacement time interval (alternatives)

**Downtime function:** The downtime per unit time of a machinery/service system can be expressed as [11]:

$$D(t_p) = \frac{T_y (1 - R(t_p)) + T_x R(t_p)}{\int_0^{t_p} t f(t) dt + T_y (1 - R(t_p)) + \{(T_x + t_p) R(t_p)\}} \tag{3}$$

Where:

$T_y$  is the time used for unit failure maintenance

$T_x$  is the time used for unit preventive maintenance

R, C and D evaluated values for each alternatives ( $t_p$ ) are then used to form a decision matrix as presented in Table 1. In Table 1, R, C and D are denoted as  $B_j$  ( $j = R, C \& D$ ) and the alternatives (scheduled replacement intervals) are indicated as  $A_i$  ( $i = 1, 2 \dots, m$ ).

Table 1 Decision matrix

Alternatives (Ai)	Decision criteria (Bj)		
	R	C	D
A <sub>1</sub>	x <sub>11</sub>	x <sub>12</sub>	x <sub>13</sub>
A <sub>2</sub>	x <sub>21</sub>	x <sub>22</sub>	x <sub>23</sub>
A <sub>3</sub>	x <sub>31</sub>	x <sub>32</sub>	x <sub>33</sub>
-	-	-	-
-	-	-	-
A <sub>m</sub>	x <sub>m1</sub>	x <sub>m2</sub>	x <sub>m3</sub>

**2.2 Decision making tools**

**2.2.1 WASPAS method**

WASPAS was developed from a systematic integration of the Weighted Sum Model (WSM) and the Weighted Product Model (WPM). The application of the technique have been reported in the literature. Chakraborty and Zavadskas [12] used the tool to solve eight manufacturing multi-criteria decision problems. Yazdani et al. [13] applied the WASPAS technique to solve material selection decision problem.

The steps of the WASPAS methods, are as follows [13]:

Step 1: Normalization of the beneficial criteria and non-benefit criteria in Table 1. The beneficial criteria is normalised as follow:

$$Q_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}} \tag{4}$$

The non-benefit criteria is normalized in two stages. The first stage is to find the reciprocal of the alternative with respect to the decision criteria as follow:

$$x_{ij}^r = \frac{1}{x_{ij}} \tag{5}$$

The second stage is the application of the linear normalisation approach as follow:

$$Q_{ij}^r = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}^r} \tag{6}$$

Step 2. The evaluation of alternatives performance based on WSM and WPM is carried out as follows:

For WSM, the performance of alternatives is expressed as

$$sG_i = \sum_{j=1}^n Q_{ij} \cdot w_j \tag{7}$$

For WPM, performance of alternative is expressed as:

$$pG_i = \prod_{j=1}^n (Q_{ij})^{w_j} \tag{8}$$

Step 3. Aggregation of Equations 7 and 8 to obtained a single performance index for the ranking of alternatives as follows:

$$WP = \lambda sG_i + (1 - \lambda) pG_i = \lambda \sum_{j=1}^n Q_{ij} \cdot w_j + (1 - \lambda) \prod_{j=1}^n (Q_{ij})^{w_j} \tag{9}$$

$\lambda$  takes value from 0.1 to 1 but generally set at 0.5.

Based on the performance index, WP, the alternatives are ranked and the best alternative is the one with the highest value of WP.

**2.2.2 ARAS method**

ARAS is an acronym for Additive Ratio Assessment and technique was developed by Zavadskas and Turksis. The optimum solution is determined by comparing alternatives scores with the ideal alternative. The application of the method have been reported in the literature. Zavadskas et al. [14] utilised the approach to assessed project managers for construction work. Chatterjee and Chakraborty [15] applied ARAS for gear selection problem. Nguyen et al. [16] used the technique to address problem of conveyor equipment selection problem under uncertainty.

The ARAS methodological steps are as follows [15]:

Step 1. The decision matrix in Table 1 is normalised in this paper using normalisation techniques applied for WASPAS method.

Step 2. Evaluation of the weighted normalised matrix using the following expression:

$$P_{ij} = Q_{ij} \cdot w_j \tag{10}$$

Where  $w_j$  is the weight of  $j$ th criterion. The decision criteria weights have been evaluated with different approaches in the literature. Emovon and Samuel [17] applied entropy method in evaluating decision weights. However, in this paper Analytical Hierarchy Process (AHP) is applied.

Step 3. Determination of the optimality function value for each alternative is performed with the following Equation:

$$S_i = \sum_{j=1}^n P_{ij} \tag{11}$$

The best and the worst alternative are the ones with the highest and lowest values of  $S_i$  respectively.

Step 4. The performance index values of each alternative is evaluated as follows:

$$U_i = \frac{S_i}{S_o} \tag{12}$$

The performance index values ranges from 0 % to 100 % and the alternative with the highest value is the best alternative.

### 3. Numerical Example

The connecting rod scheduled replacement interval selection problem used to demonstrate the applicability and suitability of the proposed methods was taken from the

work of Emovon et al. [6]. The connecting rod is one of the key components of the marine diesel engine. Scheduled replacement had been identified as the optimum maintenance strategy for mitigating it failure effect in the literature [18].

Having known that, scheduled replacement is optimal maintenance strategy. Emovon, et al [6] obtained data from multiple sources which they use as input into Eq. 1 to 3 to produces values for R, C and D for each alternative. The result were used to form a decision matrix presented in Table 2 which the authors solved using combination of AHP and TOPSIS methods. However, in this paper WASPAS and ARAS methods are use as viable options to TOPSIS technique.

Table 2 Decision matrix for connecting rod [6]

Alternatives	tp(hrs)	Rtp	Ctp(£)	Dtp(hrs)
1	5000	0.998234	0.402036	0.000604
2	6000	0.996702	0.336712	0.000507
3	7000	0.994408	0.290747	0.000439
4	8000	0.991171	0.257035	0.000389
5	9000	0.986803	0.231631	0.000352
6	10000	0.981108	0.212175	0.000324
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-
29	33000	0.317263	0.228948	0.000420
30	34000	0.280303	0.234641	0.000432
<b>Criteria type</b>		Max	Min	Min
<b>Criteria Weights (w<sub>j</sub>)</b>		0.6989	0.1673	0.1338

### 4. Application of WASPAS and ARAS for Ranking of Alternatives

#### 4.1 WASPAS analysis

Having known the decision matrix, the next step in the WASPAS and ARAS analysis steps is the normalization of the matrix. The benefit criterion; R was normalized with Eq. 4 while the non-benefit criteria; C and D was normalized with Eq. 5 and 6. The normalized decision matrix is presented in Table 3. The performance of each alternative based on WSM and WPM is then evaluated using Eq. 7 and 8 respectively. Finally, the overall performance index is evaluated using Eq. 9 and the results together with the corresponding alternatives ranking are presented in Table 4 and Figure 1.

Table 3 Normalized decision matrix

Alternatives	R	C	D
1	0.0443	0.0163	0.0181
2	0.0442	0.0195	0.0215
3	0.0441	0.0225	0.0249
4	0.0439	0.0255	0.0281
5	0.0438	0.0283	0.0310
6	0.0435	0.0309	0.0337
-	-	-	-
-	-	-	-
-	-	-	-
29	0.0141	0.0286	0.0260
30	0.0124	0.0279	0.0253

Table 4 Performance index value and corresponding rank

Alternatives	WP	Rank
1	0.02536	17
2	0.02623	15
3	0.02701	13
4	0.02770	11
5	0.02830	9
6	0.02879	7
-	-	-
-	-	-
-	-	-
29	0.01284	29
30	0.01182	30

From Table 4 and Figure 1, the optimum alternative is 9. The implication of this alternative is that for every 13000hrs the connecting rod of the marine diesel engine should be replaced. In real life application, this may not be realistic and as such the input data into the analysis may not be real life data. However, if quality data is inputted into the methodology a realistic result can be obtained.

**4.2 ARAS analysis**

The weighted normalized matrix is firstly determined by applying Eq. 10 on data in Table 3 and the results generated are shown in Table 6. Next, is the evaluation of the optimality function values for each alternatives by applying Eq. 11 on data in Table 6. Finally, performance of each alternative is evaluated using Eq. 12 and the results produced together with the corresponding ranking is presented in Table 7 and Figure 2.

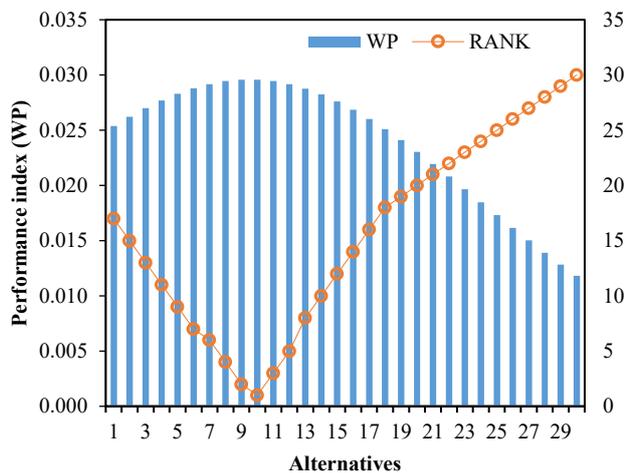


Fig. 1 WASPAS Performance index and corresponding rank

Table 6 Weighted normalized matrix

Alternatives	R	C	D
1	0.0309	0.0027	0.0024
2	0.0309	0.0033	0.0029
3	0.0308	0.0038	0.0033
4	0.0307	0.0043	0.0038
5	0.0306	0.0047	0.0041
6	0.0304	0.0052	0.0045
-	-	-	-
-	-	-	-
-	-	-	-
-	-	-	-
29	0.0098	0.0048	0.0035
30	0.0087	0.0047	0.0034

From Table 7 and Figure 2, the optimum alternative using the ARAS method is 10. Alternative 10 denotes 14000hrs. For every 14000 hrs. of operation, the connecting rod of a marine diesel engine should be replaced. However, the result is based on the data inputted into the methodology. The data is mainly for demonstration purpose and if real life quality data is inputted a more realistic result will be obtained.

**4.3 Comparison of WASPAS and ARAS with TOPSIS**

To validate WASPAS and ARAS both techniques are compared with TOPSIS previously applied by Emovon et al. [6]. The results of the comparative analysis are presented in Table 8 and Figure 3.

Table 7 ARAS Performance index and corresponding rank

Alternatives	U	Rank
1	0.87867	17
2	0.90167	15
3	0.92334	13
4	0.94336	11
5	0.96107	9
6	0.97607	7
-	-	-
-	-	-
-	-	-
29	0.44076	29
30	0.40768	30

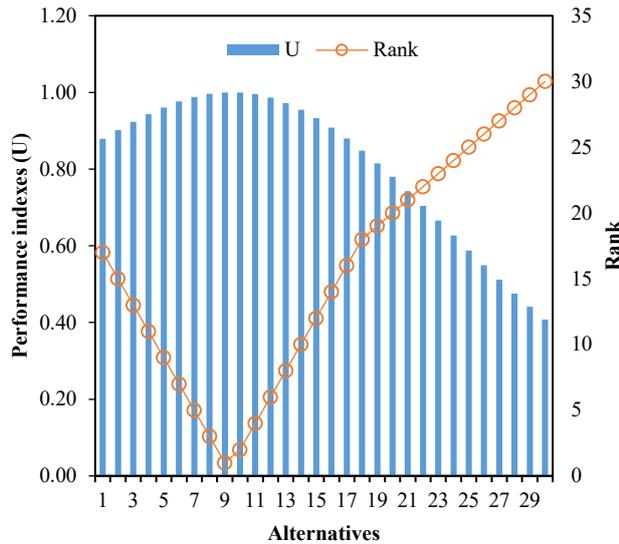


Fig. 2 ARAS Performance index and corresponding rank

From Table 8 and Figure 3, alternative 18 to 30 have the same ranking for WASPAS, ARAS and TOPSIS. For other alternatives WASPAS and ARAS produced the same ranking with the exception of alternatives 6, 7, 8, 9, 10, 11 and 12 that have a difference of one rank in between but results deviates slightly further from that of TOPSIS. The optimum solution obtained for TOPSIS, WASPAS and ARAS are alternatives 8, 9 and 10 denoting 12000hrs, 13000hrs. and 14000hrs. of operation before replacement respectively. The optimum solution obtained from the three methods are similar. The slight deviation of the ranking of TOPSIS for alternative 1 to 17 from that of WASPAS and ARAS may be connected to non-consideration of the relative distance between positive and negative ideals by TOPSIS which negatively affect the outputs [9].

Table 8 Comparison of WASPAS and ARAS with TOPSIS

Alternatives	WASPAS	ARAS	TOPSIS
1	17	17	16
2	15	15	14
3	13	13	11
4	11	11	9
5	9	9	6
6	7	7	4
-	-	-	-
-	-	-	-
-	-	-	-
29	29	29	29
30	30	30	30

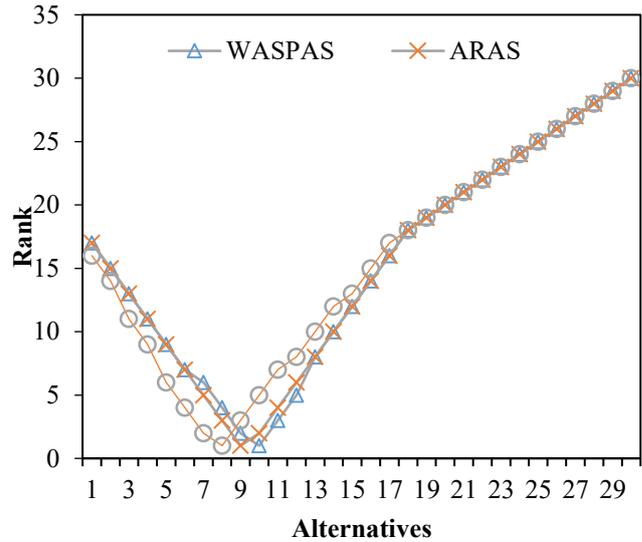


Fig. 3 WASPAS and ARAS comparison with TOPSIS

### 5. Conclusion

Scheduled replacement is an integral element of maintenance strategies and its major challenge is how to determine the optimum time for performing equipment item replacement. This paper presented an integrated WASPAS, ARAS and ARM for determining the appropriate interval for replacing equipment item of a machinery/service system. Three decision criteria; R C and D modelled with ARM was aggregated with WASPAS and ARAS and alternative scheduled replacement time intervals were ranked based on the WASPAS and ARAS indexes. The output of the WASPAS and ARAS were compared with a well-known approach (TOPSIS) in the literature. The optimum replacement interval from the comparative analysis were found to be 12000hrs, 13000hrs and 14000hrs respectively for TOPSIS, WASPAS and ARAS methods. The analysis, therefore validate the proposed techniques. The proposed techniques are simpler in-terms of application than the TOPSIS approach and should be more attractive to maintenance managers in the manufacturing/service industries.

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