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Effect of Temperature on Free Vibration of Functionally Graded Plate with Cut-out

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Abstract: Present article performs the parametric study on vibration characteristics of functionally graded (FG) plate with central hole in thermal environment. ANSYS Parametric Design language is used in developing Finite element model and discretization of the material is done using an eight-node serendipity shell element. Displacement field of the present model is developed using first-order shear deformation theory (FOSDT) with six degrees of freedom. Frequency responses are extracted using Block Lanczo's eigenvalue extraction method. To show the accurateness of the model developed convergence study is done for various mesh sizes to obtain the suitable mesh density. Present results which are computed are compared and validated with the previously reported results. Finally, the effect of temperature on various parameters like cut-out size to thickness ratio, cut-out size to side ratio, power law index, side to thickness ratio, boundary conditions are shown through various numerical illustrations.

Keywords: Free vibration, functionally graded plate, cut-out, ANSYS APDL

1. Introduction

Functionally Graded Materials (FGM) are a new class of materials which are tailored for the desired properties by varying their composition from one surface to another surface. In general, these FGM's are blend of ceramics and metals to obtain the paramount properties of its component. These materials can overcome the inter-laminar stress which cannot be avoided in a composite. These FGM's are widely used in aerospace, biomedical, space structures and in high temperature applications. This has created interest in many researchers and scientist during the past decade to study structural stabilities of FGM'S.

Consequence of different parameters like volume fraction exponent, support conditions, edge to depth ratio and cut-out size on natural vibration of FG square plate with triangular cut-outs using ANSYS was investigated in [1]. Experimentation was conducted under tensile loading on a Glass/Epoxy composite with circular and square cut-outs and found that circular cut-outs has more load carrying capacity [2]. Finite element method (FEM) is applied in extraction of free vibration of FG plates with multiple holes [3]. Mechanical, thermal buckling and free vibration

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analysis was investigated using FEM with discrete shear gap method and Mori-Tanaka method was incorporated in determining effect material properties of FG plates [4]. Thermal buckling characteristics of an FG plate with different shapes of perforations and the effect of number of perforations on a plate is investigated using FOSDT [5]. Influence of circular and non-circular perforations on a FG straight plate, skew plate is validated using FEM and the effect of frequency is also shown by varying number of holes[6].Third order shear deformation theory is used for validating the effect of temperature on free vibration of FG rectangular plate in thermal environment and Rayleigh-Ritz method is used in determining the frequency equations[7]. FOSDT was used in investigating structural stability of FG CNT rectangular composite plates with and without cut-out [8]. Free flexural vibration is investigated using FOSDT in a FGM plate with perforations and cracks in a thermal environment and Mori-Tanaka is used for material characterization[9] and they have investigated the effect of various parameters like cut-out geometry, crack length, thermal gradient. Effect of thermal environment on FG cylindrical shells with perforation is performed for free vibration using three dimensional Chebyshev-Ritz method [10]. Gulshan Taj M. N. A., Chakrabarti A, and Prakash V has investigated the effect of skew angle on FG shell in thermal environment [11]. Four parameter power law is considered for the vibration analysis of FG spherical shell panel [12-13]. HSDT in conjunction with FEA is used in the Analysis. Effect of thermal load on Buckling analysis is performed using FSDT in conjunction with FEM on an FG rectangular plate with elliptical hole [14]. Buckling behaviour is studied on an FG plate with elliptical cut-out under combined thermal and mechanical loads [15]. Free vibration analysis is performed using higher order shear deformation theory under thermomechanical loading [16-17]. Three-dimensional theory of elasticity is used in evaluating stress and free vibration of FG plate with hole at the centre [18]. Investigation on vibrational behavior of FG conical shells and the mid-pane kinematics are derived using FOSDT [19]. 3D model is developed and validated using FEM when loaded in lateral direction of single pie [20]. Review on the effectiveness of finite element programming using ANSYS on imperfect structures is investigated by Ismail et al. [21]. Stress intensity factors are evaluated by Al-Moayed et al.[22] on an thick cylinder which consists a semi-elliptical crack on the circumference.

It is evident from the above survey that most of the work is carried out under thermal free environment and limited work is available in the open source on vibrational behaviour of FG Plates with perforation. This has created an interest for the authors to work on vibrational behaviour of FG plate with circular perforations in thermal environment. In this article the effectual material characteristics are evaluated using Voigt's micromechanical model using power law distribution. ANSYS Parametric Design language is used in developing Finite element model and discretization of the material is done using an eight-node serendipity shell element. Displacement field of the present model is developed using FOSDT with movability of six. Natural vibrational responses are obtained with the process of Block Lanczos eigenvalue extraction process. The influence of various factors like edge to depth ratio, cut-out size to edge ratio, power-law index and boundary conditions are investigated.

2. Modelling of Geometry Sections

In the present article, FG plate of sides as *a, b & h* in direction along the axis x, y & z and with a circular hole of radius *r* at the canter is shown in Fig 1. Modelling and analysis of FG square with perforation is done using an ANSYS parametric design code. Discretization of the material is performed using an eight-node serendipity shell element which is defined in ANSYS library.

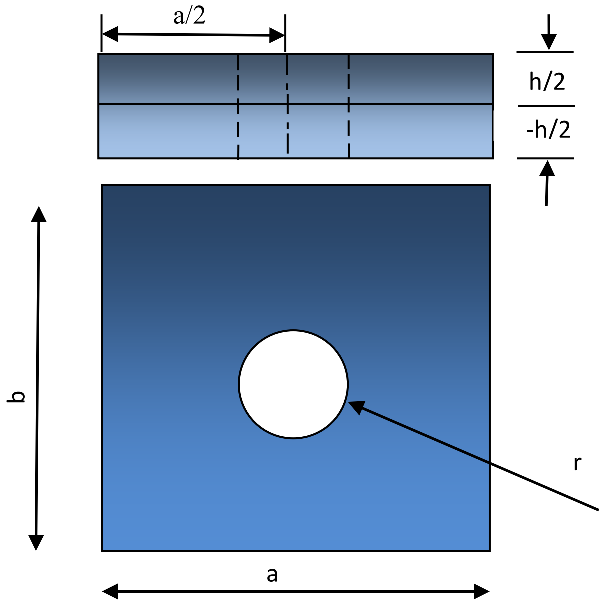

Fig. 1 - Elevation and plan of functionally graded plate with central hole

3. Properties of Functionally Graded Materials

In the present article, metal and ceramic are considered on the lower surface and top surface respectively. Voigt's micromechanical model is considered for the relevant properties of FGM square plate and are obtained using

$$P = P_m + (P_c - P_m)V_c \tag{1}$$

Where subscripts m, c refers to metal and ceramic towards the negative z axis and positive z axis respectively and V_c refers to the volume fraction of ceramic and it is obtained using the relation [23]

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^q \quad (0 <= q < \alpha) \tag{2}$$

where *q* refers to power-law exponent and the variation in composition of the ceramic material along the non-dimensional thickness is shown in the Fig 2. Temperature dependent material Properties of Metal and Ceramic are tabulated in Table 1. and evaluated using

$$P(T) = P_0(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3) \tag{3}$$

Table 1 - Temperature dependent mechanical properties [10]

	Material	E(GPa)	α (K^{-1})	ρ(Kg/m^3)	ν	*k*(W/mK)
P_{-1}	SUS304	0	0	0	0	0
	Si_3N_4	0	0	0	0	0
	Ti-6Al-4V	0	0	0	0	0
	ZrO_2	0	0	0	0	0
P_0	SUS304	201.04×10^9	12.330×10^{-6}	8166	0.3262	12.04
	Si_3N_4	348.43×10^9	5.8732×10^{-6}	2370	0.2400	9.19
	Ti-6Al-4V	122.70×10^9	7.4300×10^{-6}	4420	0.2888	6.10
	ZrO_2	132.20×10^9	13.300×10^{-6}	3657	0.3330	1.78
P_1	SUS304	3.079×10^{-4}	8.086×10^{-4}	0	-2.002×10^{-4}	0
	Si_3N_4	-3.070×10^{-4}	9.095×10^{-4}	0	0	0
	Ti-6Al-4V	-4.605×10^{-4}	7.483×10^{-4}	0	1.108×10^{-4}	0
	ZrO_2	-3.805×10^{-4}	-1.421×10^{-3}	0	0	0
P_2	SUS304	-6.534×10^{-7}	0	0	3.797×10^{-7}	0
	Si_3N_4	2.160×10^{-7}	0	0	0	0
	Ti-6Al-4V	0	-3.621×10^{-7}	0	0	0
	ZrO_2	-6.127×10^{-8}	9.549×10^{-7}	0	0	0
P_3	SUS304	0	0	0	0	0
	Si_3N_4	-8.946×10^{-11}	0	0	0	0
	Ti-6Al-4V	0	0	0	0	0
	ZrO_2	0	0	0	0	0

4. Finite Element Formulation

In the present article, FG plate considered with sides as *a,b & h* in direction of x,y & *h* respectively and with a circular hole of radius *r* at the centre is considered. Modelling is governed by FSDT. The displacement field are given [24]

$$\left.\begin{aligned} u &= u_0 + z\theta_x \\ v &= v_0 + z\theta_y \\ w &= w_0 + z\theta_z \end{aligned}\right\} \tag{4}$$

Where u_0, v_0, w_0 are the midplane displacements and θ_x, θ_y, θ_z are the shear rotation terms.

Present model of FG flat plate is discretised using an eight noded serendipity shell 281 elements with forty eighty degrees-of-freedom. The mid-plane displacement vector can be written in nodal form as

$$\{\delta_0\} = \sum_{i=1}^{8} N_i \{\delta_{0_i}\} \tag{5}$$

Where, $\{\delta_{0_i}\}$ and N_i are the nodal displacement vector and the approximation function at i^{th} node.

The generalized stress-strain relation in connection to its reference plane is represented as

$$\{\sigma\} = [Q]\{\varepsilon - \varepsilon_{th}\} \tag{6}$$

Where, $\{\sigma\} = \{\sigma_x \quad \sigma_y \quad \sigma_z \quad \tau_{xy} \quad \tau_{yz} \quad \tau_{xz}\}^T$ and $\{\varepsilon\} = \{\varepsilon_x \quad \varepsilon_y \quad \varepsilon_z \quad \gamma_{xy} \quad \gamma_{yz} \quad \gamma_{xz}\}^T$ are the linear stress and strain vector, respectively and [*Q*] is the reduced stiffness matrix.

The final form of equilibrium equation of thermally vibrated FGM flat panel can be obtained using Hamilton's principal and expressed as:

$$\left([K - K_G] - \omega^2 [M]\right)\Delta = 0 \tag{7}$$

Where, [*K*], [K_G] and [*M*] are the global stiffness, geometric stiffness and mass matrices, respectively. The frequencies (ω) and modes (Δ) are extracted through Block Lanczos method.

5. Results and Discussion

Free vibration behaviour is evaluated for FG flat panels in thermal environment using ANSYS parametric design language in combination with Block Lanczos method. An eight-node serendipity element is used in discritization of the element as defined in ANSYS. The support conditions used in this section are shown in Table 2.

Table 2 - Support conditions of FG plate

SSSS	At $x = 0, a$: $v_0 = w_0 = \theta_y = \theta_z = 0$ At $y = 0, b$: $u_0 = w_0 = \theta_x = \theta_z = 0$
CSCS	At $x = 0, a$: $u_0 = v_0 = w_0 = \theta_x = \theta_y = \theta_z = 0$ At $y = 0, b$: $u_0 = w_0 = \theta_x = \theta_z = 0$
CCSS	At $x = 0$, and $y=0$: $u_0 = v_0 = w_0 = \theta_x = \theta_y = \theta_z = 0$ At $x = a$, and $y=b$: $u_0 = w_0 = \theta_x = \theta_z = 0$
CFFF	At $x = 0$: $u_0 = v_0 = w_0 = \theta_x = \theta_y = \theta_z = 0$

5.1 Convergence and Authentication

With a focus on accurateness of the current model, it is essential to validate the obtained frequency responses. A simply supported FG plate (SUS304/Si_3N_4) is considered at ambient temperature (T_c=T_m=300K) with a=0.2, h=0.025, *q*=0.2. Convergence test for various mesh sizes from 10x10 to 20x20 is performed. Non dimensional frequency parameter is computed using $\overline{\omega} = \omega(a^2/h)\sqrt{(1-v_m^2)\rho_m/E_0}$. Table 3 conveys that the model developed in the present investigation is converging at mesh density of 14x14.

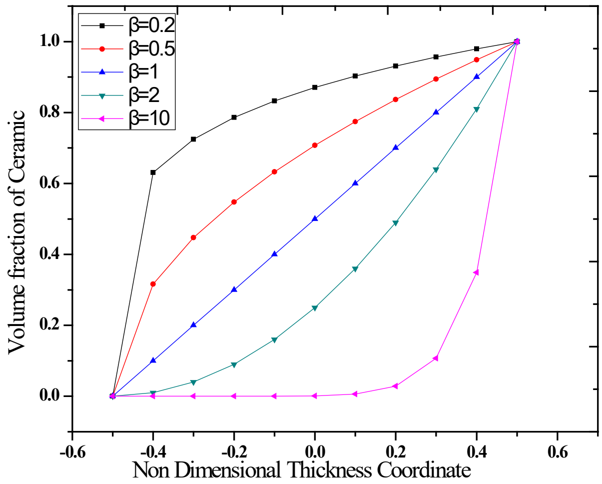

Fig 2 - Volume fraction of ceramic along the non-dimensional thickness

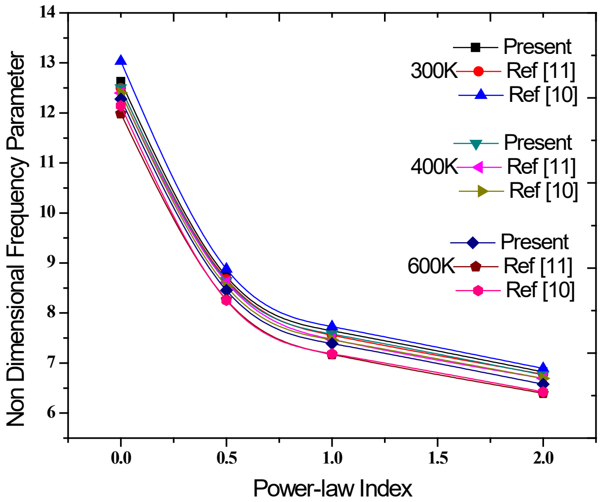

Fig 3 - Non-Dimensional Frequency of SSSS FG square Flat panel under various temperatures

Table 3: Convergence study of SSSS FG square plate

Mesh Size	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
10x10	1.0161	2.5428	2.8798	4.2733	4.3001
12x12	1.0161	2.5426	2.8794	4.2733	4.2995
14x14	1.0161	2.5425	2.8793	4.2733	4.2992
16x16	1.0161	2.5425	2.8793	4.2733	4.2992
18x18	1.0161	2.5424	2.8792	4.2733	4.2989
20x20	1.0160	2.5424	2.8791	4.2733	4.2989

Validation is extended for different temperatures (T_c=300K, 400K and 600K and T_m=300K) of SSSS FG square plate b/h=8. The non-dimensional frequency parameters are computed with a multiplication factor of 8/5 and are compared with Huang and Shen, Gulshan Taj [10,11]. The research findings of the present model are closely aligned to the published results.

5.2 New Results of FG Square Plate with Perforations

Example 1. In an effort to study the consequence of temperature on dimensionless responses of the frequency, FG (Ti-6Al-4V/ZrO_2) square plate is considered under CFFF boundary condition. Properties of ceramic and metals used in the present model used are tabulated in Table 1. The effect of temperature (T_c=400K, 500K and 700K) on various edge to thickness ratios (*b/h*= 4, 5, 10, 20, 100) power-law exponent (*q*=2) is shown in Fig 4 to Fig 6 shows. It can be observed that as the ratio between edge to depth of the plate is increased the frequency parameter is decreasing. It is also evident from the figures that as the temperature escalated from 400K to 700K the responses of frequencies are diminished. It is known that thin plates are less stiffer compared to thick plates. Therefore, diminishing trend is observed for the vibrational responses as the plate edge to thickness ratio is enhanced.

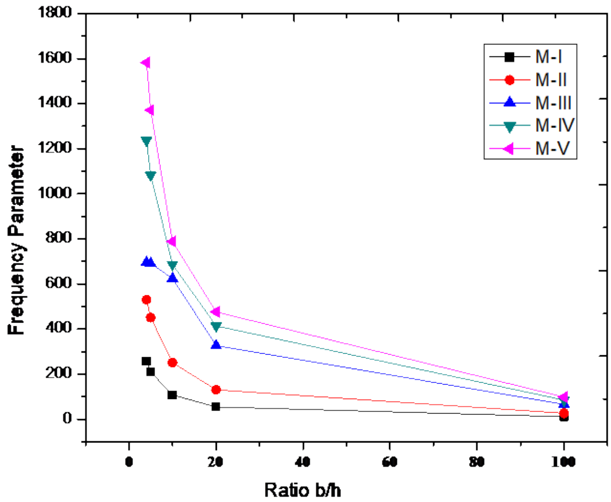

Fig 4 - Frequency parameter of first 5modes of CFFF FG square plate (Ti-6Al-4V/ZrO_2) with central hole r=0.1, *q*=2 T_c=400K

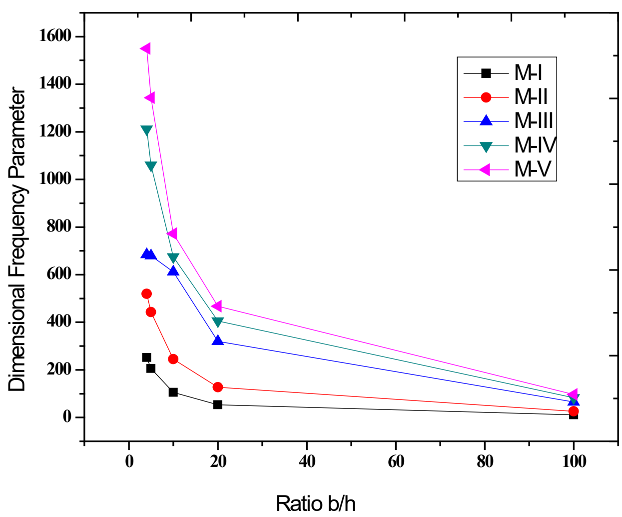

Fig 5 - Vibration responses of first 5modes of CFFF FG square plate (Ti-6Al-4V/ZrO_2) with central hole r=0.1, *q*=2 T_c=500K

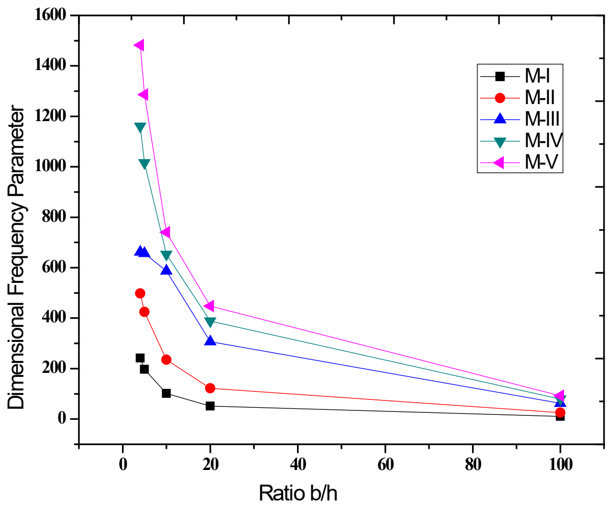

Fig 6 - Frequency parameter of first 5modes of CFFF FG square plate (Ti-6Al-4V/ZrO_2) with central hole r=0.1, *q*=2 T_c=700K

Example 2. Effect of temperature FG square plate (Ti-6Al-4V/ZrO_2) with hole *r*=0.1 at the centre are shown in Fig 7 to Fig 9. A variety of boundary conditions (SSSS, CSCS, CCSS) are adopted for FG plate with side to thickness ratio b/h=5 and power law index *q*=1. Here the temperature of the ceramic is increased from 400K to 700K and the material properties (E, ν, α) are considered under the temperature condition. It is observed from the figures as the temperatures is enhanced the frequency is decreased. The reason behind the trend is as the material property of the FG plate considered tends to degrade with the increase in temperature. This degradation in material properties effects the frequency and the values are decreased and the same is observed.

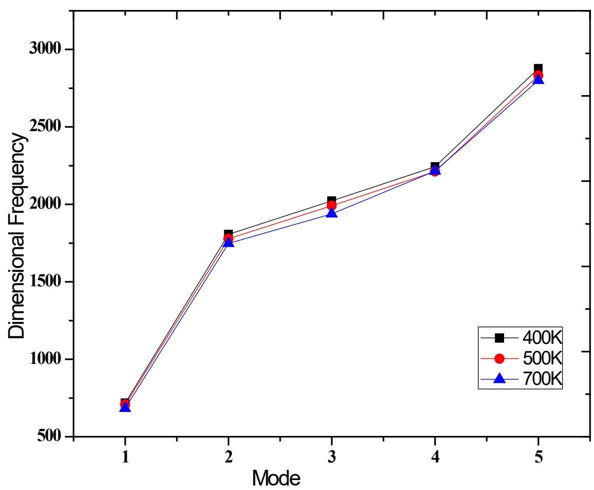

Fig 7 - Frequency parameter of first 5modes of simply supported FG square plate (Ti-6Al-4V/ZrO_2) with central cutout *r*=0.1, *b/h*=5 *q*=1

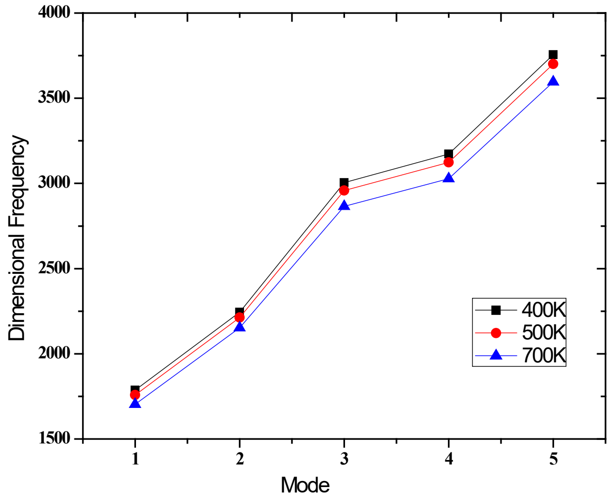

Fig 8 - Frequency parameter of first 5modes of CSCS FG square plate (Ti-6Al-4V/ZrO_2) with central cutout *r*=0.1, *b/h*=5 *q*=1

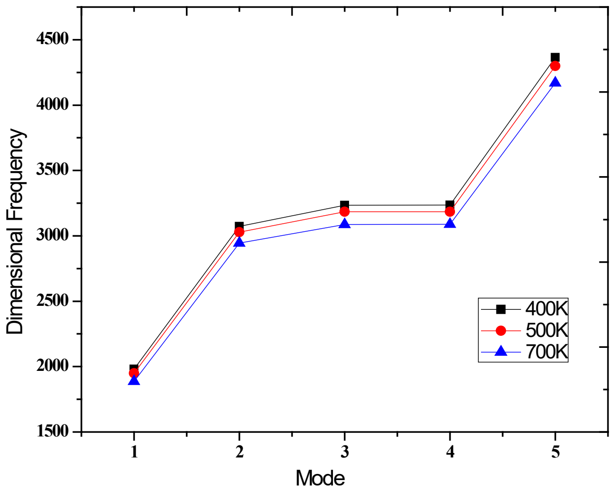

Fig 9 - Frequency parameter of first 5modes of CCSS FG square plate (Ti-6Al-4V/ZrO_2) with central cutout *r*=0.1, *b/h*=5 *q*=1

Example 3. Variation in frequency for FG square plate (Ti-6Al-4V/Si_3N_4) is depicted in Fig 10-12. Power-law exponent *p*(0.5,1,2,5,10) is increased keeping the boundary condition CSCS and the ratio of edge of plate to thickness(b/h) as 2 and the hole is located at the center with radius *r*=0.1.

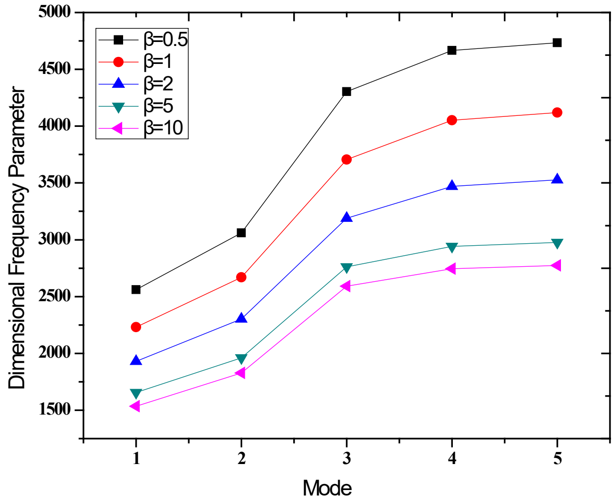

Fig 10 - Frequency parameter of first 5modes of CSCS FG square plate (Ti-6Al-4V/ZrO_2) with central hole r=0.1, *b/h*=2 T_c=400K

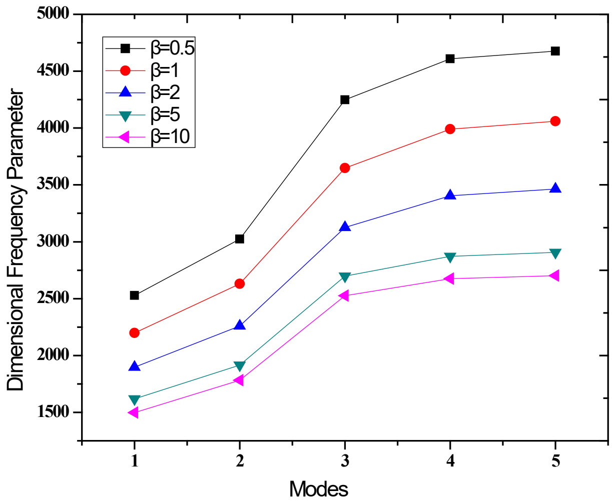

Fig 11 - Frequency parameter of first 5modes of CSCS FG square plate (Ti-6Al-4V/ZrO_2) with central hole r=0.1, *b/h*=2 T_c=500K

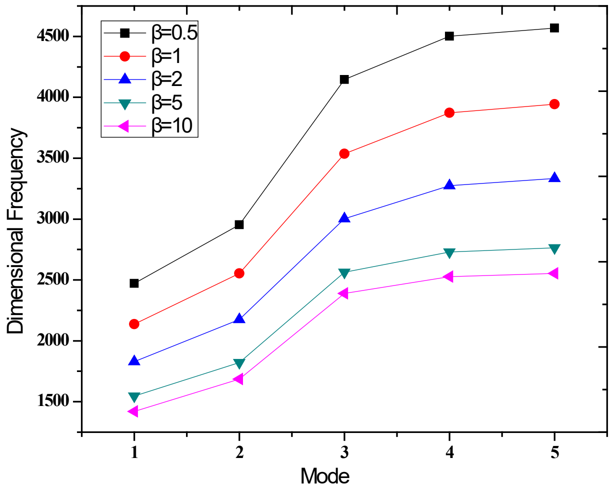

Fig 12 - Frequency parameter of first 5modes of CSCS FG square plate (Ti-6Al-4V/ZrO_2) with central hole r=0.1, *b/h*=2 T_c=700K

Three observations are made from the figures 10-12. First, the frequency parameter is increased as the modes are increased in all the three cases shown. Second, frequency parameter is reduced as the power law exponent is increased. This is expected as the FG panels become stiffer as the power-law indices are reduced. Finally, as the temperature is amplified the modulus of elasticity of the material diminish, which effects the frequency parameter of the plate.

6. Conclusions

In the present article, free vibration of the FG square plate with central hole is investigated. Voigt's rule of mixtures and simple power law is used in evaluating the effective material properties. ANSYS commercial software is used for convergence and validation of the problem is done by comparing with the published literature. Influence of Temperature is seen on the frequency for varying thickness ratio, boundary conditions and power-law indices. From the parametric study the following conclusion remarks are furnished in the succeeding lines

- As side to thickness ratio is increased for an FG square plate with central hole and clamped on one side the frequency responses are decreasing and similar trend is observed when the temperature is varied.
- Natural vibration of an FG panel with simply supported boundary condition less compared to CSCS and CCSS boundary conditions and as the increase in temperature natural vibration responses are decreasing.
- Ceramic rich FG material has responded with higher frequency values compared to metal rich FG material. Frequency responses are decreased as the temperature of the material is increased.

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