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# Mathematical Formulation for Determining Lateral Displacement of Tubular Frame and Outriggers Equipped with Viscous Dampers

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**Abstract:** Viscous Dampers are mainly used to mitigate and control vibration, acceleration and lateral displacement in tall buildings. A hybrid system comprised of framed tube, shear core and outriggers belt-truss equipped with passive linear viscous dampers is investigated and the mathematical formulation is developed to analyze the hybrid system as a beam-like structure. A mathematical model, based on Euler-Bernoulli flexural beam theory is developed to build a simple, yet accurate model for calculating the lateral displacement profile of the hybrid system subjected to lateral loading patterns varying against time. The properties of the hybrid system may vary in arbitrary segments and any number of outrigger and belt-truss systems may be considered through the height of the structure in the proposed method. Kinetic and potential energies and non-conservative works due to the velocity dependent viscous dampers force and the lateral load exerted on the hybrid system are obtained. Next, Hamilton's principle is implemented, and the governing equation of motion and boundary conditions are derived. As the aforementioned partial differential equations are dependent on both time and space, the central finite difference method is chosen as a numerical method to find the answer to the equation of motion.

**Keywords:** Tall buildings, framed tube, outrigger system, viscous damper, energy concept, lateral displacement

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## 1. Introduction

A main concern during the designing procedure of a tall building is to control the maximum displacement, drifts and acceleration, which improves the occupants' comfort during wind effects. Various structural systems are suggested to overcome this issue. One of these forms is a combined system of tubular frame, central shear core and outriggers equipped with passive linear viscous dampers (PLVD's). A number of researchers have reported the efficiency of outriggers in controlling and decreasing the lateral displacement and overturning moment in the core, especially under static lateral loads [1-4].

Successful operation of outrigger system depends on the core stiffness. To withstand lateral load patterns, such as high winds, a tubular frame, as a complete exterior moment frame, contributes a lot; therefore, the size and thickness of the shear core lessens. However, sufficient rigidity and stiffness should be provided by the shear core to act as a fixed support for outriggers, to provide more opportunity for rotation restraint [5]. A simple mathematical model of a combined system of tubular frame, shear core and two outriggers through the height of the structure was suggested by Malekinejad and Rahgozar [6]. In this method, the properties of tubular frame's columns and beams were considered to be uniform through the height of the structure. According to Kwan [7] theory, tubular frame was modeled as an equivalent orthotropic membrane with uniform thickness. Furthermore, outrigger systems were considered to be the rotation springs at the level of belt trusses, which were connected rigidly to the shear core, and by using Hamilton's principle, the first natural mode shape and frequency were obtained. Another investigation was performed on the aforementioned system with geometric discontinuities through the height of the structure by Kamgar and Saadatpour [8]. They proposed a mathematical model based on Euler-Bernoulli flexural beam theory, then by implementing Hamilton's principle, the governing differential equation of motion and natural boundary conditions were obtained. By using the variable separation method, the partial differential equation of motion turned into the ordinary differential equation and was solved by the power series solution method. In this method, outriggers effects were considered to be the rotation spring at the level of the belt trusses placement. Finally, the first natural frequency and mode shape was obtained, and the accuracy of the proposed method was proved through comparing the results with 40 and 50 story finite element models.

A study on tall buildings with damped outrigger concept was implemented by Smith and Willford [9]. The dynamic response of high-rise buildings to wind loads was investigated by them. They concluded that 10- year and 100- year return periods of wind should be concerned to consider human comfort and strength design, respectively. Disadvantages of an increase in stiffness and strength and the use of tuned mass/liquid damper, to control the dynamic response of tall buildings, was discussed. In return, the use of viscous damper was studied, and mitigation of wind dynamic effects, including the response of high-rise structures, was proposed. When wind load is exerted on tall buildings, the structure should behave elastically and the dependable damped ratio should not be considered more than 2 percent [10]; therefore, the damped outrigger concept is introduced to increase the overall dependable structural damping. A simple beam-damper system was investigated using a Euler-Bernoulli beam concept by Chen et al. [11], to find the beam vibration response by an analytical solution. They considered a uniform cantilever beam with one outrigger at its mid height and two viscous dampers at the two ends of the outrigger. The governing equation of motion was derived by the use of D'Alembert's principle. The separation of variable technique was applied to the model to obtain the system's eigenvalues. Different locations of the damper were studied for the first five modes, and it was found that an optimal damper size existed, which resulted in the near maximum value of the system's modal damping. Finally, approximate equations for engineering applications were provided by them.

Takabatake et al. [12] presented simple, yet accurate mathematical models based on the flexural beam theory, to investigate the dynamic response of high-rise buildings. Earthquake ground motion was also applied to a suggested equivalent system, and internal force and lateral displacement profile of the structure were obtained. Verification of their proposed method shows that it may be used confidently during the preliminary design stages. Hanson and Soong [13] presented the first formula to design linear viscous dampers. Dampers were installed diagonally in their proposed formula, and the damping viscosity ratio was obtained based on the works done by dampers. Constantinou et al. [14] studied the use of passive energy dissipation system in analyzing, retrofitting and designing buildings. They also developed and proposed a simplified procedure to model structural systems and energy dissipation devices. Structural control criteria depend not only on the nature of dynamic loads, but also on the desired response of the structure. Decreasing the relative lateral displacement and absolute acceleration is an important purpose of structural controlling. Patel and Jangid [15], proposed a dynamic mathematical formulation for two adjacent structures connected with viscous dampers under the based harmonic excitation. They compared the lateral displacement and acceleration response for various damping ratios of connected structures.

Here, a simple mathematical formulation was developed to calculate the lateral displacement of a system comprised of tubular frame, central shear core and outrigger systems equipped with PLVD's under arbitrary lateral load pattern varying against time. The aforementioned system was suggested to control the relative lateral displacement and absolute acceleration, which are the main criteria throughout the design procedure of a tall building, by connecting the perimeter flexural frames to the central shear core with outriggers equipped with PLVD's. The hybrid system was modeled as an equivalent flexural beam, according to the classical beam theory [16-19]. The outriggers which were rigidly connected to the shear core were considered rotational springs, and their reactions were modeled as a concentrated moment which acts in the opposite direction of rotation, at the belt truss location. Kinetic and potential energies of the hybrid system were obtained, and Hamilton's principle was implemented to attain the governing equation of motion and natural boundary conditions, which are dependent on both time and space. The central finite difference method was chosen to calculate the lateral displacement of the hybrid system. Accuracy and robustness of the model were verified through comparing the results with SAP 2000 software [20] 3D fully detailed finite element models.

Some models which are suitable for parametric studies of tall buildings have been investigated by researchers. Two methods based on finite element theories and a sophisticated discrete model for evaluating and determining the behavior of two tall buildings with different number of stories and different heights under seismic loads, have been investigated in detail by Tavakoli et al. [21]. Finding an optimum location for minimizing base shear and base moment, roof displacement and inter story drifts are the goals of the aforementioned paper. Tavakoli et al. [22], have been presented another model of tall buildings under dynamic loads. Nonlinearity analysis has been performed on the model in order to find the best location of the outrigger systems combined with belt trusses. The Euler-Bernoulli and Timoshenko's theories have been employed to model tall buildings as cantilever beams by Davari et al. [23]. The ability of the proposed model is to calculate the floor displacement by considering the effects of positive and negative shear lag in lateral displacement of the structure through the height of tall buildings. At last, some numerical formulas for framed tube's, modeled based on Kwan's theory as like as orthotropic membrane boxes, have been proposed. Kamgar and Rahgozar [24] investigated tall buildings with variable cross sections of beams and columns through the height of the building which are equipped with flexible outrigger systems, in order to find the best location of the outrigger systems by minimizing the total strain energy. Investigating the distribution functions of axial displacements in flange and web panels of structural systems combined of tubular frame, central shear core and outrigger systems, with higher order function forms under three kind of static load cases, has been carried out by Rahgozar et al. [25]. In the aforementioned model, the framed tube has been modeled as an orthotropic membrane box, according to Kwan's theory and outrigger systems has been modeled as rotating springs at belt truss location in order to consider the interaction between the central shear core and tubular frame. It is worth to mention that fourth and Fifth order polynomials functions have been proposed for axial displacement functions in flange and web panels of framed tube system respectively.

Dynamic analysis is a major area of interest within the field of recognition high rise structures behavior. Alavi et al. [26], proposed a parametric method to design tall buildings subjected to flexural vibration. A closed form solution has been obtained by defining an optimization problem. The optimization problem has been formulated to minimizing structural weight while maximizing structural stiffness at the same time. It is worth to mention that increasing structural stiffness resulted in increasing the structural fundamental frequency. Rahgozar et al. [27] had obtained natural frequencies and corresponding mode shapes of high-rise structures comprised of tubular frame and central shear core by proposing an approximate method. By considering shear and bending deformation, tall buildings have been modeled as cantilever beam with the same bending and shear properties of a beam with box cross section. For calculating natural frequencies and corresponding mode shapes the matrix formation of the problem has been obtained by utilizing the governing differential equation of motion and its corresponding weak form, B-spline functions. The results show the robustness and accuracy of the proposed method for using in preliminary design stages.

Malekinejad et al. [28], have been utilized Kwan's theory to model high rise structures comprised of tubular frame, central shear core and outriggers as equivalent orthotropic membrane and considered the hole system as a cantilever box beam with lumped masses at the level of each story to calculate natural frequencies and corresponding mode shapes. It is worth to mention that outrigger system has been considered as a rotating spring at the level of belt truss location. Mass matrix and stiffness matrixes of the equivalent system has been obtained in the aforementioned proposed method. Free vibration analysis of the system has been conducted by solving the eigenvalue equation. Finally, the proposed method results has been compared with the results of several finite element models of high rise buildings in order to prove the robustness and accuracy of the method.

## 2. Novelty and Assumptions of the Proposed Method

### 2.1. Novelty of the approach

Limits on lateral displacement of tall buildings should be considered in order to minimize damages to cladding and nonstructural components. One or more structural system(s) may be combined to form a high-rise building. A system comprised of tubular frame and central shear core is a flexible system for tall buildings, and its lateral displacement does not fulfill the limitations in order to control damages to nonstructural components: therefore, the outriggers equipped with PLVD's are added to the system to control drifts and maximum displacement of hybrid structure. The hybrid system is shown schematically, in 3D form, in Fig. 1, and the 2D elevation of the structure with stepwise variant properties through the height, is also depicted in Fig. 2.

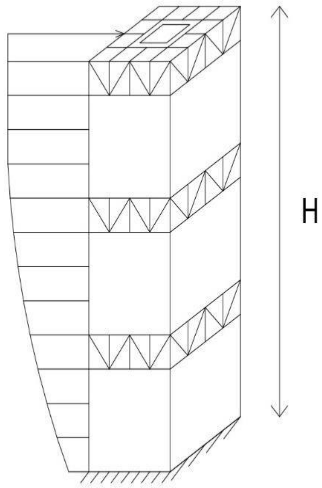


**Fig. 1 - 3D model of hybrid system**

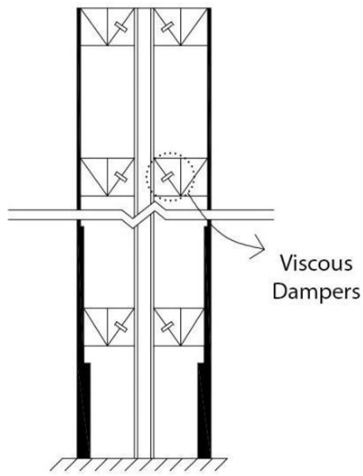


**Fig. 2 - Elevation of hybrid system with stepwisediscontinuities through the height**

In this paper, a symmetric tubular frame, central shear core and outrigger systems equipped with PLVD's are considered to form a tall building. A considerable amount of literature has been published on framed tube systems and its combination with other structures, such as shear core, outriggers and internal tubes. Previous studies were mainly focused on achieving the lateral displacement of the system based on three kind of static load cases, a point load at the top and the lateral loads distributed uniformly and triangularly through the height of the building [29], or were focused on finding frequency and mode shape characteristics for the free vibration system [30-31]. However, far too little effort was made to achieve the lateral displacement of tubular frame, shear core and outrigger systems equipped with PLVD's under arbitrary lateral load patterns. This paper attempts to develop a simple, yet accurate mathematical model to find the lateral displacement profile of the hybrid system under any arbitrary lateral load patterns varies against time. Attaining inter-story drift and maximum lateral displacement are important parameters during the design of tall buildings, therefore an accurate and simple method presentation to determine lateral displacement profile is vital during preliminary design stages, especially under wind loads, furthermore, this method provides an accurate model to calculate lateral displacement and inter story drifts, which is much less time consuming in comparison with fully detailed finite element modelling in SAP 2000.

## 2.2. Assumptions

To achieve the aforementioned goal, the following assumptions and principles are considered and adopted [29]:

1. Column spacing and height of each story is uniform through the height of the building.
2. The structure's materials are considered to be homogeneous and behave linearly elastic, accordant with Hook's law.
3. The framed tube is modeled as an orthotropic panel with equivalent elastic properties.
4. Tubular frame and shear core properties are added to each other as a result of outrigger systems operation which rigidly connect the framed tube to shear core.
5. Columns and beams sections and shear core thickness are considered to be stepwise variants through the height of the structure.
6. The structure is fixed at the base; therefore, displacement and slope are assumed to be zero at base.
7. Load carrier components of the structure are distributed symmetrically in plane.
8. Any number of outrigger systems equipped with PLVD's can be considered through the height of the hybrid system.
9. Viscous dampers are used diagonally in outrigger systems and their velocity dependent forces are taken into account as non-conservative works.
10. In plane stiffness and rigidity of the floor slabs are not considered and the floors assumed to be flexible.

According to the foregoing assumptions, a hollow box beam under lateral load pattern, which is depicted in Fig. 3, is superseded instead of the framed tube and shear core system. It should be mentioned that flange frames are two parallel frames that are perpendicular to lateral loads and web frames are two parallel panels that are parallel to lateral loads as depicted in Fig 3.

Dampers stroke and the relative velocity across the damper are effective parameters for the output force and energy dissipation during wind storms. Accordingly, double story height outriggers with diagonally installed dampers are considered. It is worth to mention that floor slabs are usually treated as rigid diaphragms in analytical structural models, but at or near outrigger floors, rigid diaphragms or master/slave node approach should not be used, because the outrigger system is artificially stiffened, and zero force in outrigger truss chord happens; therefore, in this paper, floors are considered to be flexible not rigid diaphragms. According to this assumption, the outrigger systems equipped with PLVD's are the main part which connect the perimeter flexural frames to the central shear core and makes them to interact with each other. Outrigger systems in hybrid structure are modeled as rotating springs, located at belt truss levels and the location of these outriggers are measured from the base of the structure which can be seen in Fig. 4.

The effects of outriggers are considered to be concentrated moments acting in the opposite direction of rotation. In the mathematical proposed method, the properties of beams and columns, and shear core thickness can change in any arbitrary part through the height of the building, moreover, any number of outrigger systems, equipped with PLVD's can be defined in the height of the hybrid structure.

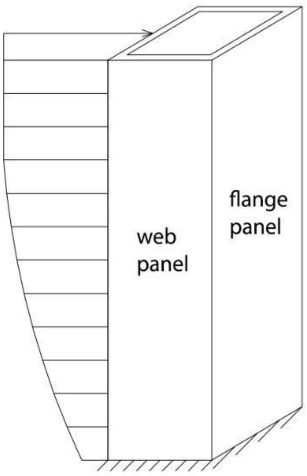


**Fig. 3 - Equivalent orthotropic membrane of tubular frame**

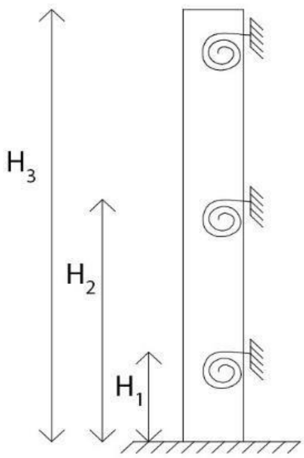


**Fig. 4 - Effects of outrigger system on tubular frame**

## 3. Mathematical Formulation

### 3.1.Attaining and Solving The Governing Equation of Motion

According to the aforementioned assumptions and definitions, an Euler-Bernoulli cantilevered flexural beam with stepwise variant properties through the height, under an arbitrary lateral load pattern, is considered. Afterwards, kinetic and potential energies and non- conservative works due to lateral forces and PLVD's forces are defined as follows:

$$\Gamma(t) = \int_0^H \frac{1}{2}\left(m + \rho A(z)\right)\left[\frac{\partial \eta(z,t)}{\partial t}\right]^2 dz \tag{1}$$

$$P(t) = \int_0^H \frac{1}{2} EI(z)\left[\frac{\partial^2 \eta(z,t)}{\partial z^2}\right]^2 dz + \int_0^H \frac{1}{2} GA(z)\left[\frac{\partial \eta(z,t)}{\partial z}\right]^2 dz + \sum_{i=1}^{N} \frac{1}{2} K_{O_i}\left[\frac{\partial \eta(H_i,t)}{\partial z}\right]^2 \tag{2}$$

$$W_{nc} = p(z,t)\eta(z,t) - \sum_{i=1}^{N} C_{e_i}\frac{\partial}{\partial t}\left[\eta(H_i,t) - \eta(H_i - 2h,t)\right] \tag{3}$$

By implementing Hamilton’s principle [29], taking variation and part by part integration, with respect to time and space,the governing equation of motion and natural boundary conditions are proved. Hamilton’s principle states that:

$$\int_{t_1}^{t_2} \delta(\Gamma - P)dt + \int_{t_1}^{t_2} \delta w_{nc} dt = 0 \tag{4}$$

Substituting equations (1) to (3) into equation (4) results in:

$$\int_{t_1}^{t_2} \{\delta \left[ \int_0^H \frac{1}{2}(m + \rho A(z)) \left[\frac{\partial \eta(z,t)}{\partial t}\right]^2 dz - \int_0^H \frac{1}{2} EI(z) \left[\frac{\partial^2 \eta(z,t)}{\partial z^2}\right]^2 dz - \int_0^H \frac{1}{2} GA(z) \left[\frac{\partial \eta(z,t)}{\partial t}\right]^2 dz - \sum_{i=1}^{N} \frac{1}{2} K_{O_i} \left[\frac{\partial \eta(H_i,t)}{\partial z}\right]^2 - \sum_{i=1}^{N} C_{e_i} \frac{\partial}{\partial t}[\eta(H_i,t) - \eta(H_i - 2h,t)] \right] + p(z,t)\delta\eta(z,t)\}dt = 0 \tag{5}$$

Geometric boundary conditions are also defined as:

$$\eta(0,t) = 0 \quad , \quad \frac{\partial \eta(0,t)}{\partial z} = 0 \tag{6}$$

Where

$m$ Floor mass per unit length

$\rho$ Structural mass per unit volume

$E$ Elastic modulus of materials

$G$ Equivalent shear modulus of materials

$K_{O_i}$ $i$th level outrigger rotational stiffness

$C_{e_i}$ $i$th level damping ratio of PLVD's

$N$ Number of outrigger systems through the height of the structure

$h$ The height of each story

$H$ Total height of the building

$(z)$ Equivalent beam cross section

$(z)$ Equivalent beam moment of inertia

$(z,t)$ Lateral displacement through the height of the beam

$(z,t)$ Lateral load pattern varies against time

### 3.1.1. Governing Equation of Motion and Boundary Conditions

By solving equation (5), the governing equation of motion and natural boundary conditions are obtained as follows:

$$-\frac{\partial^2}{\partial z^2}\left[EI(z)\frac{\partial^2 \eta(z,t)}{\partial z^2}\right] + \frac{\partial}{\partial z}\left[GA(z)\frac{\partial \eta(z,t)}{\partial z}\right] = [m + \rho A(z)] \times \left[\frac{\partial^2 \eta(z,t)}{\partial t^2}\right] - p(z,t) \tag{7}$$

$$\frac{\partial}{\partial z}\left[EI(z)\frac{\partial^2\eta(H,t)}{\partial z^2}\right]-GA(z)\left[\frac{\partial\eta(H,t)}{\partial z}\right]-\frac{\partial}{\partial t}\left\{\sum_{i=1}^{N}C_{e_i}[\eta(H_i,t)-\eta(H_i-2h,t)]\right\}=0 \tag{8}$$

$$EI(z)\left[\frac{\partial^2\eta(H,t)}{\partial z^2}\right]+\sum_{i=1}^{N}K_{O_i}\left[\frac{\partial\eta(H_i,t)}{\partial z}\right]=0 \tag{9}$$

It should be noted that Hutchinson's [32] assumptions for calculating the effective shear cross section of beams and columns are accepted, therefore, equivalent orthotropic membrane properties is calculated after effective shear cross sectional formulas are implemented. The suggested Hutchinson's [32] formulas are expressed as follows:

$$\Re=-\frac{2(1+\upsilon)}{\frac{9}{4\left(\frac{b}{2}\right)^5\left(\frac{d}{2}\right)}C_4+\upsilon\left(1-\frac{(d/2)^2}{(b/2)^2}\right)} \tag{10}$$

$$C_4=\frac{4}{45}\left(\frac{b}{2}\right)^3\left(\frac{d}{2}\right)\left[-12\left(\frac{b}{2}\right)^2-15\upsilon\left(\frac{b}{2}\right)^2+5\upsilon\left(\frac{d}{2}\right)^2\right]+\sum_{n=1}^{\infty}\frac{16\upsilon^2\left(\frac{d}{2}\right)^5}{(n\pi)^5(1+\upsilon)}\left[n\pi\left(\frac{b}{2}\right)-\left(\frac{d}{2}\right)tgh\left(\frac{n\pi\left(\frac{b}{2}\right)}{\left(\frac{d}{2}\right)}\right)\right] \tag{11}$$

"$b$" *and* "$d$" are respectively columns and beams depth parallel to web and flange panels, whichever is considered. The parameters "$b$" *and* "$d$" are depicted in Fig. 5.

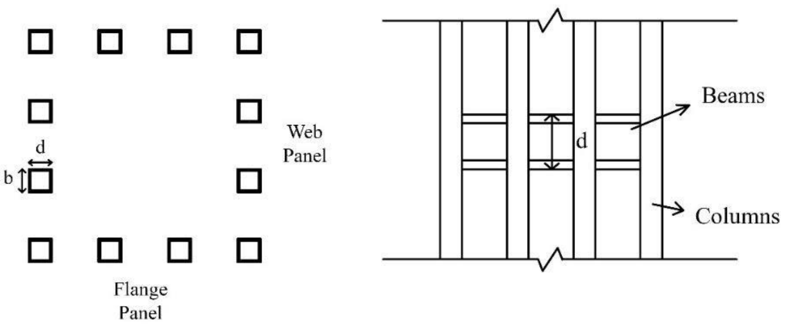


**Fig. 5 - Tubular frame plan and side view for defining "$b$" *and* "$d$"**

### 3.1.2. Solving the Partial Differential Equations

As equations (7) to (9) are dependent on both time and space, the central finite difference method is chosen to solve these partial differential equations. The first to forth order derivatives, with respect to space, in the central finite difference method are expressed as:

$$\eta_z'(z_i,t_j)=\frac{\eta(z_i+h,t_j)-\eta(z_i-h,t_j)}{2h} \tag{12}$$

$$\eta_z''(z_i,t_j)=\frac{\eta(z_i+h,t_j)-2\eta(z_i,t_j)+\eta(z_i-h,t_j)}{h^2} \tag{13}$$

$$\eta_z^{(3)}(z_i,t_j)=\frac{\eta(z_i+2h,t_j)-2\eta(z_i+h,t_j)+2\eta(z_i-h,t_j)-\eta(z_i-2h,t_j)}{2h^3} \tag{14}$$

$$\eta_z^{(4)}(z_i,t_j)=\frac{\eta(z_i+2h,t_j)-4\eta(z_i+h,t_j)+6\eta(z_i,t_j)-4\eta(z_i-h,t_j)+\eta(z_i-2h,t_j)}{h^4} \tag{15}$$

The first and second derivatives of the lateral displacement, with respect to time, in central finite difference method is also defined as:

$$\dot{\eta}_z(z_i,t_j) = \frac{\eta(z_i,t_j+dt)+\eta(z_i,t_j-dt)}{2dt} \qquad (16)$$

$$\ddot{\eta}_z(z_i,t_j) = \frac{\eta(z_i,t_j+dt)-2\eta(z,t_j)+\eta(z_i,t_j-dt)}{dt^2} \qquad (17)$$

The length of the equivalent Euler-Bernoulli flexural beam is divided into "$n$" equal parts, and the vibration duration of the beam "$T$" is also divided into "$m$" equal parts, therefore, "$h$" $and$ "$dt$" are defined as follows:

$$h = H/n \quad ,dt = T/m \qquad (18)$$

Takabatake [12] derived that assuming "$h$" equal to the floor height makes no problem in the convergence of solution, therefore, "$h$" was taken as the story height. Discrete points in the finite difference method are shown in Fig. 6. Substituting Eqs. (13), (15) and (17) into Eq. (7) yields:

$$\begin{aligned} &a_i\eta(z_i-2h,t_j)+b_i\eta(z_i-h,t_j)+C_i\eta(z_i,t_j)+d_i\eta(z_i+h,t_j) \\ &+e_i\eta(z_i+2h,t_j)=f_i\eta(z_i,t_j-dt)-g_i\eta(z_i,t_j)+f_i\eta(z_i,t_j+dt) \\ &-p(z_i,t_j) \end{aligned} \qquad (19)$$

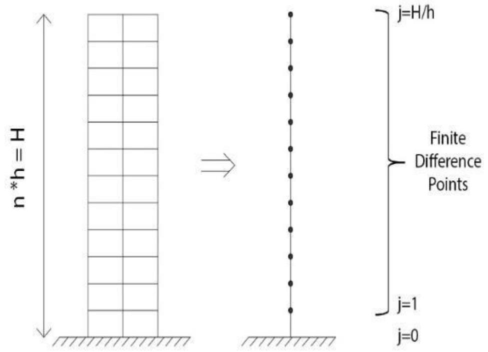


**Fig. 6 - Discrete points in finite difference model**

Moreover, by substituting Eqs. (12) to (14) and (16) into Eqs. (8) and (9), these partial differential equations change into algebraic form, as shown in Eqs. (20) and (21).

$$\begin{aligned} &R_j\eta(z_{H-2},t_j)+\Gamma_j\eta(z_{H-1},t_j)+S_j\eta(z_H,t_j)+Q_j\eta(z_{H+1},t_j) \\ &+M_j\eta(z_{H+2},t_j)=\sum_{i=1}^{N}L_i[\eta(H_i,t_j-dt)-\eta(H_i-h,t_j-dt)] \end{aligned} \qquad (20)$$

$$\begin{aligned} &U_j\eta(z_{H-1},t_j)-2U_j\eta(z_H,t_j)+U_j\eta(z_{H+1},t_j)= \\ &\sum_{i=1}^{N}W_i[\eta(H_i+h,t_j)-\eta(H_i-h,t_j)] \end{aligned} \qquad (21)$$

Eqs. (19), (20) and (21) are the algebraic form of partial differential equations, which are defined in Eqs. (7) to (9), and can be expressed in matrix form. The natural boundary conditions related to the outriggers and PLVD's are appeared in the two last rows and also they stand in proper column of the characteristic matrix of the hybrid structure, according to the outrigger systems location, throw the height of the building.

## 3.2. Presentation of algebraic equation in matrix form

The matrix formation of algebraic equations is depicted in Fig. 7. The parameters in the characteristic matrix of the hybrid structure are defined as follows:

$$a_i = -(EI_i)/h^4 \quad (22)$$
$$b_i = [-(4EI_i)/h^4] + [(G_iA_i)/h^2] \quad (23)$$
$$c_i = [-(6EI_i)/h^4] - [(2G_iA_i)/h^2] \quad (24)$$
$$d_i = [(4EI_i)/h^4] + [(G_iA_i)/h^2] \quad (25)$$
$$e_i = (EI_i)/h^4 \quad (26)$$
$$f_i = (mh + \rho A_i)/dt^2 \quad (27)$$
$$g_i = [2(mh + \rho A_i)]/dt^2 \quad (28)$$
$$\kappa_i = -f_i(i, dt-1) + g_i(i, dt) - f_i(i, dt+1) \quad (29)$$
$$Q_j = \{-[EI(H)]/h^3\} - \{[G(H)A(H)]/(2h)\} \quad (30)$$
$$S_j = -[2EI(H)]/h^2 \quad (31)$$
$$U_j = [E(H)]/h^2 \quad (32)$$
$$M_j = -[EI(H)]/(2h^3) \quad (33)$$
$$R_j = [EI(H)]/2h^3 - K_{O_i}/(2h) \quad (34)$$
$$W_i = K_{O_i}/(2h) \quad (35)$$
$$L_i = -C_{e_i}/(2dt) \quad (36)$$
$$\Omega_1 = \{[EI(H)]/h^3\} + \{[G(H)A(H)]/(2h)\} - L(H-1, dt-1) + L(H-1, dt+1) \quad (37)$$
$$\Omega_i = L(H_i, dt-1) - L(H_i, dt+1) \quad (38)$$

$$\begin{bmatrix}
a_1+c_1+\kappa_1 & d_1 & e_1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
b_1 & c_1+\kappa_1 & d_1 & e_1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
a_1 & b_1 & c_1+\kappa_1 & d_1 & e_1 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
0 & a_1 & b_1 & c_1+\kappa_1 & d_1 & e_1 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & a_1 & b_1 & c_1+\kappa_1 & d_1 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a_1 & b_1 & c_1+k_1 & \cdots & e_1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a_2 & b_2 & \cdots\cdots\cdots & d_2 & e_2 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a_2 & \cdots & c_2+\kappa_2 & d_2 & e_2 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\cdots\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & a_i & b_i & c_i+\kappa_i & d_i & e_i & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & a_i & b_i & c_i+\kappa_i & d_i & e_i & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & a_i & b_i & c_i+\kappa_i & d_i & e_i & \cdots & 0 & 0 & 0 & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\cdots\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & a_H & b_H & c_H+\kappa_H & \cdots & e_H & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & a_H & b_H & \cdots\cdots\cdots & d_H & e_H & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & a_H & \cdots & c_H+\kappa_H & d_H & e_H & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & b_H & c_H+\kappa_H & d_H & e_H & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & a_H & b_H & c_H+\kappa_H & d_H & e_H \\
0 & 0 & 0 & -W_i & 0 & W_i & \cdots & 0 & 0 & 0 & -W_i & 0 & W_i & 0 & \cdots & 0 & R_j & S_j & U_j & 0 \\
0 & 0 & 0 & 0 & \Omega_i & 0 & \cdots & 0 & 0 & 0 & 0 & \Omega_i & 0 & 0 & \cdots & M_j & \Omega_1 & \Omega_i & Q_j & -M_j
\end{bmatrix}$$

**Fig. 7 - Characteristic matrix of hybrid structure**

It's worth to mention that the lateral load exerted to the equivalent flexural beam is considered as a vector on the right side of the matrix equation, therefore, any kind of lateral load distribution, varying against time can be defined and exerted on the proposed model. In numerical examples, the lateral load distribution through the height of the structures are considered to be similar to the wind force exerted on tall buildings, suggested by ASCE 07-2016 [33].

## 3.3. Outrigger Systems Stiffness

The outriggers effect on the proposed model is considered a concentrated moment which acts in the opposite direction of the hybrid system rotation at belt trusses locations. The equivalent concentrated moment directly depends on the outriggers' stiffness, defined by Smith and Coull [2], Ding et al [4], Malekinejad and Rahgozar [6] and Kamgar and Saadatpour [8] referred to the process of outrigger equivalent stiffness $(k_{O_i})$ calculation, and defined the $k_{O_i}$ formulationas follows:

$$K_{O_i} = \left[(2H_i)/(l^2 EA_{co}) + l/\left(12EI_{ou_e}\right) + (1/h_{ou}\, G\ A_{ou})\right]^{-1} \quad (39)$$

In the above equation, $H_i$ is the location of outriggers measured from the base of the structure, $l$ is the center to center distance of exterior columns of the framed tube. $EA_{CO}$ is the axial rigidity of perimeter columns, transverse the direction of lateral load exertion, $EI_{ou_e}$ is the flexural rigidity of outriggers, which is calculated based on the actual flexural rigidity of each outrigger $(EI_{ou})$, as shown in equation (40), $EI_{ou}$ is calculated based on the parallel axes theory, $h_{ou}$ represents the outriggers' height, and $GA_{ou}$ is the racking shear stiffness of outriggers as defined by Stafford Smith and Coull [2].

$$EI_{ou_e} = EI_{ou}\{1 + (L_{sc}/2)/[(L_{ou} - L_{sc})/2]\}^3 \quad (40)$$

$L_{sc}$ and $L_{ou}$ are the lengths of shear core and outrigger trusses, respectively.

## 4. Numerical Examples

### 4.1. Lateral Load Distribution

A lateral load pattern similar to wind load distribution, is exerted through the height of the hybrid structure which is depicted in Fig 8. This lateral force distribution is similar to the method proposed by ASCE 07-2016 [33] for determining wind loads on dynamically sensitive tall buildings located in New York City in exposure D. The gradient height is equal to 213.36 meters, and the wind load is distributed uniformly above this level. Abrupt wind velocity changing is called gustiness. The ratio of the duration, which makes that wind gust develop to its maximum value, to the first period of the hybrid structure is an effective parameter to take the wind loads into account as dynamic or static load case [34]. According to the 3 second gustiness definition, wind load on tall buildings is considered to be a dynamic load case, in this section; therefore, an arbitrary diagram of lateral load distribution which varies against time is considered, and exertedto the proposed model as depicted in Fig. 9.

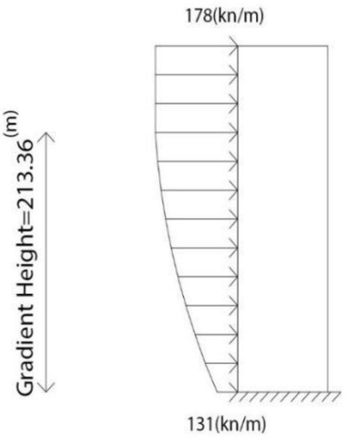


**Fig. 8 - Wind load exerted on hybrid System**

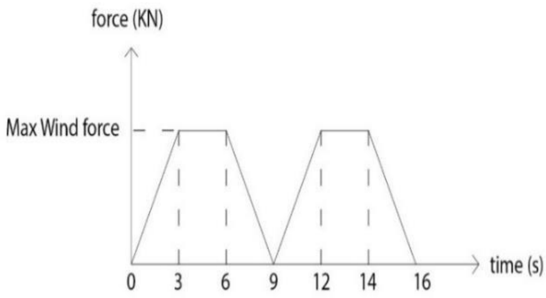


**Fig. 9 - Lateral load distribution change against time**

### 4.2. Finite Element Models

In order to prove the correctness and to verify the robustness of the mathematical formulation development, 3D fully detailed finite element models were analyzed with SAP 2000 software [20]. According to the specifications listed in table 1, the lateral displacement and the inter story drifts profile, under fully exerted lateral load pattern, was compared with the results of the proposed model. A wind returns period of 100 years for buildings, which are dynamically sensitive, was suggested to check and control drifts. A load combination of D+0.5L+W was also considered to control the lateral displacement. D, L and W refer to dead, live and wind loads, respectively. Afterwards two 60 and 70 story tall buildings with a hybrid system comprised of tubular frame, central shear core and 3 outrigger systems equipped with PLVD's through the height of the structure which is deformed under lateral load pattern, as schematically depicted in Fig. 10, are considered, modeled and designed according to the elastic properties listed in tables 2 and 3 and tables 4 and 5, respectively. Finite element model of 70 story hybrid system is schematically depicted in Fig. 11.

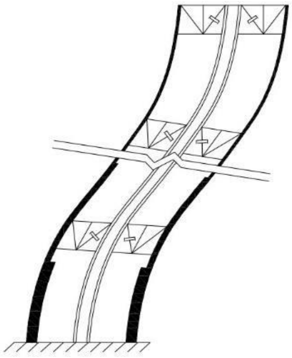

**Fig. 10 - Deformed shaped of the hybrid system under lateral load**

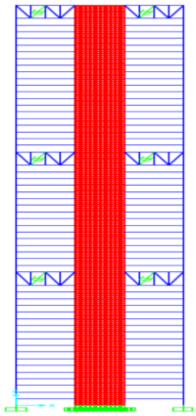

**Fig. 11 - Finite element elevation of hybrid system**

**Table 1 - Specification of the 60 and 70 story hybrid system**

| Plan dimensions | | Height of story | Core dimensions | | Space of spans(n) | |
|---|---|---|---|---|---|---|
| $L_w(m)$ | $L_f(m)$ | $h(m)$ | $l_w$ | $l_f$ | $s_w$ | $s_f$ |
| 40 | 40 | 4 | 12 | 12 | 2 | 2 |

**Table 2 - Columns, beams and shear core properties of the 60 story hybrid system**

| Story | Columns | | | Beams | | | Shear core |
|---|---|---|---|---|---|---|---|
| | Material Properties $(kg/cm^2)$ E=2 ×10$^6$ G=7.7 × 10$^5$ | | | Material Properties $(kg/cm^2)$ E=2 ×10$^6$ G=7.7 × 10$^5$ | | | Material Properties $(kg/cm^2)$ E=2 ×10$^5$ G=8.34 × 10$^4$ |
| | dim $(cm)$ $h \times b \times t$ | $A(m^2)$ | $I(m^4)$ | dim $(cm)$ $h \times b_f \times t_f \times t_w$ | $A(m^2)$ | $I(m^4)$ | Thickness $(cm)$ |
| 1-10 | 60×60×5 | 0.11 | 0.0056 | 90×40×2×1.5 | 0.0289 | 0.0039 | 80 |
| 11-20 | 60 × 60 ×4 | 0.0896 | 0.00471 | 85×40×2×1.5 | 0.0282 | 0.0034 | 80 |
| 21-30 | 50 ×50×5 | 0.09 | 0.00308 | 85×30×2×1.5 | 0.0242 | 0.0027 | 60 |
| 31-40 | 50 × 50×4 | 0.0736 | 0.00252 | 80×30×2×1.5 | 0.0234 | 0.0024 | 60 |
| 41-50 | 40×40×3 | 0.0444 | 0.00102 | 70×25×2×1.5 | 0.0199 | 0.00152 | 40 |
| 51-60 | 30×30×2 | 0.0224 | 0.000294 | 50×20×2×1.5 | 0.0149 | 0.00058 | 40 |

**Table 3 - Equivalent orthotropic membrane and outriggers properties of the 60 story hybrid system**

| Story | Equivalent orthotropic membrane | | Outriggers | | | | | | Total viscosity |
|---|---|---|---|---|---|---|---|---|---|
| | Material Properties $(kg/cm^2)$ E=2 ×10⁶ | | Material Properties $(kg/cm^2)$ E=2 ×10⁶ G=7.7 × 10⁵ | | | | | | $(\frac{kg.sec}{m})$ |
| | Shear modulus $(\frac{kg}{Cm^2})$ | Thickness $(cm)$ | Located Level | Vertical Members $A(m^2)$ | Horizontal Members $A(m^2)$ | Horizontal Members $I(m^4)$ | Chords $A(m^2)$ | | |
| 1-10 | $6.26 \times 10^4$ | 5.5 | | | | | | | |
| 11-20 | $5.69 \times 10^4$ | 4.48 | 19th – 20th | 0.11 | 0.15 | 0.014 | 0.11 | | 1138032 |
| 21-30 | $4.81 \times 10^4$ | 4.5 | | | | | | | |
| 31-40 | $4.56 \times 10^4$ | 4.5 | 39th – 40th | 0.11 | 0.15 | 0.014 | 0.11 | | 1138032 |
| 41-50 | $3.845 \times 10^4$ | 2.22 | | | | | | | |
| 51-60 | $2.284 \times 10^4$ | 1.12 | 59th – 60th | 0.11 | 0.15 | 0.014 | 0.11 | | 1138032 |

**Table 4 - Columns, beams and shear core properties of the 70 story hybrid system**

| Story | Columns | | | Beams | | | Shear core |
|---|---|---|---|---|---|---|---|
| | Material Properties $(kg/cm^2)$ E=2 ×10⁶ G=7.7 × 10⁵ | | | Material Properties $(kg/cm^2)$ E=2 ×10⁶ G=7.7 × 10⁵ | | | Material Properties $(kg/cm^2)$ E=2 ×10⁵ G=8.34 × 10⁴ |
| | dim $(cm)$ $h \times b \times t$ | $A(m^2)$ | $I(m^4)$ | dim $(cm)$ $h \times b_f \times t_f \times t_w$ | $A(m^2)$ | $I(m^4)$ | Thickness $(cm)$ |
| 1-10 | 60×60×5 | 0.11 | 0.0056 | 90×40×2×1.5 | 0.0289 | 0.0039 | 80 |
| 11-20 | 60 × 60 ×4 | 0.0896 | 0.00471 | 85×40×2×1.5 | 0.0282 | 0.0034 | 80 |
| 21-30 | 50 ×50×5 | 0.09 | 0.00308 | 85×30×2×1.5 | 0.0242 | 0.0027 | 60 |
| 31-40 | 50 × 50×4 | 0.0736 | 0.00252 | 80×30×2×1.5 | 0.0234 | 0.0024 | 60 |
| 41-50 | 40×40×3 | 0.0444 | 0.00102 | 70×25×2×1.5 | 0.0199 | 0.00152 | 40 |
| 51-60 | 30×30×2 | 0.0224 | 0.000294 | 50×20×2×1.5 | 0.0149 | 0.00058 | 40 |
| 61-70 | 30×30×2 | 0.0224 | 0.000294 | 50×20×2×1.5 | 0.0149 | 0.00058 | 40 |

**Table 5 - Equivalent orthotropic membrane and outriggers properties of the 70 story hybrid system**

| Story | Equivalent orthotropic membrane | | Outriggers | | | | | Total viscosity |
|---|---|---|---|---|---|---|---|---|
| | Material Properties $(kg/cm^2)$ E=2 ×10⁶ | | Material Properties $(kg/cm^2)$ E=2 ×10⁶ G=7.7 × 10⁵ | | | | | $(\frac{kg.sec}{m})$ |
| | Shear modulus $(\frac{kg}{Cm^2})$ | Thickness $(cm)$ | Located Level | Vertical Members $A(m^2)$ | Horizontal Members $A(m^2)$ | Horizontal Members $I(m^4)$ | Chor $A(m^2)$ | |
| 1-10 | $6.26 \times 10^4$ | 5.5 | | | | | | |
| 11-20 | $5.69 \times 10^4$ | 4.48 | 19th – 20th | 0.11 | 0.15 | 0.014 | 0.11 | 1138032 |
| 21-30 | $4.81 \times 10^4$ | 4.5 | | | | | | |
| 31-40 | $4.56 \times 10^4$ | 4.5 | 39th – 40th | 0.11 | 0.15 | 0.014 | 0.11 | 1138032 |
| 41-50 | $3.845 \times 10^4$ | 2.22 | | | | | | |
| 51-60 | $2.284 \times 10^4$ | 1.12 | | | | | | |
| 61-70 | $2.284 \times 10^4$ | 1.12 | 69th – 70th | 0.11 | 0.15 | 0.014 | 0.11 | 1138032 |

## 5. Results and Discussion

### 5.1. Verification and Robustness of The Proposed Method

The lateral displacement profile of the hybrid systems modeled in SAP 2000 software [20] are compared with the results of the proposed method. The diagrams which compare the lateral displacement profiles are shown in Figs. 12 and 13. The results in table 6 prove that the maximum potential errors of the proposed method are 10 and 5.6 percent in 60 and 70 story buildings, respectively.

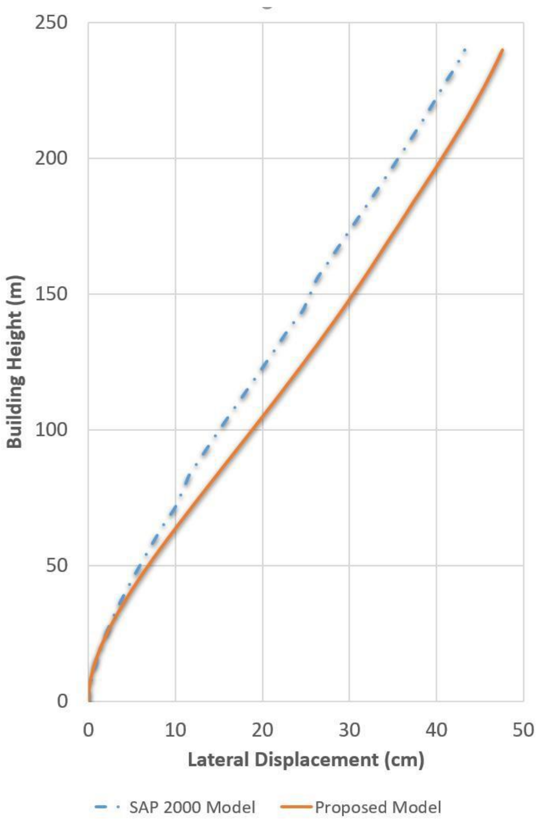


**Fig. 12 - Lateral displacement profile of 60 storyhybrid system**

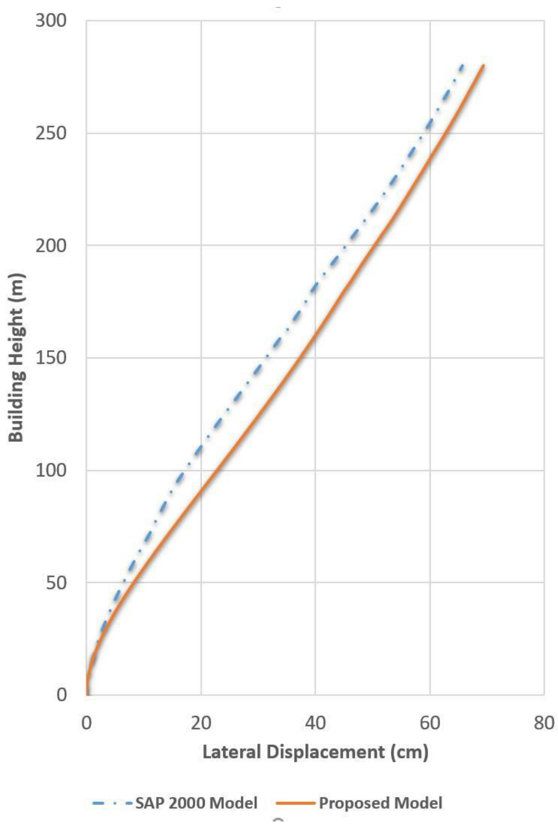


**Fig. 13 - Lateral displacement profile of 70 storyhybrid system**

**Table 6 - Comparison of displacement between the results of the proposed method and the finite element model in SAP 2000 at the top of hybrid structures**

| 60 story structure | | | 70 story structure | | |
|---|---|---|---|---|---|
| Proposed method $(cm)$ | SAP 2000 finite element model $(cm)$ | Percentage of error | Proposed method $(cm)$ | SAP 2000 finite element model $(cm)$ | Percentage of error |
| 47.60 | 43.22 | 10 | 69.35 | 65.68 | 5.6 |

The first three natural frequency of the 60 and 70 story hybrid structures are listed in table 7. The inter story drifts profile of the 3D fully detailed finite element models of hybrid systems, are also compared with the results of the mathematical proposed method. The diagrams which compare the inter story drifts are shown in Figs. 14 and 15. The diagrams depicted inter story drifts reveal good agreements between the results of the proposed model and finite element models; therefore, the method can be confidently used during the preliminary design stages with much less time consumed, in comparison with 3D fully detailed finite element models.

**Table 7 - The first three natural frequency of the 60 and 70 story hybrid structures**

| 60 story structure | | | 70 story structure | | |
|---|---|---|---|---|---|
| First natural frequency ($Hz$) | Second natural frequency ($Hz$) | Third natural frequency ($Hz$) | First natural frequency ($Hz$) | Second natural frequency ($Hz$) | Third natural frequency ($Hz$) |
| 0.178 | 0.562 | 1.161 | 0.134 | 0.417 | 0.887 |

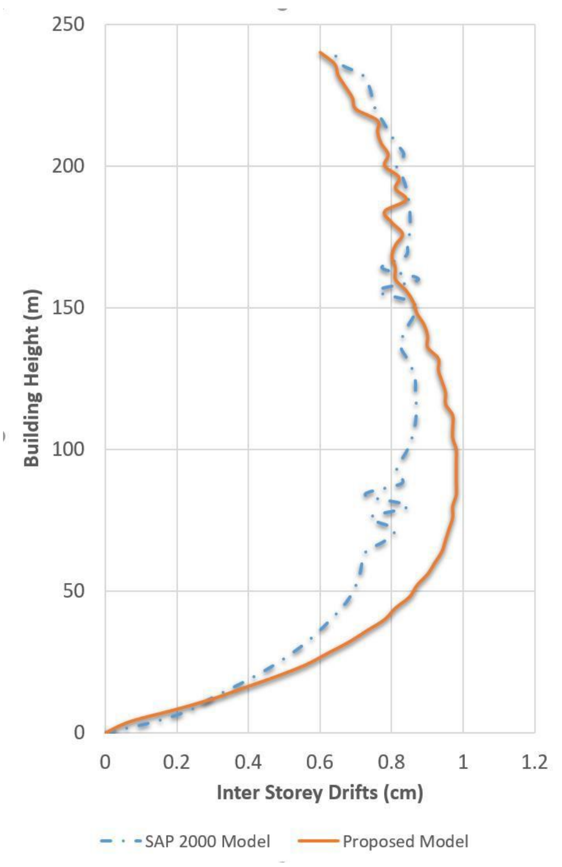


**Fig. 14 - Inter story drift profile of 60 story hybrid system**

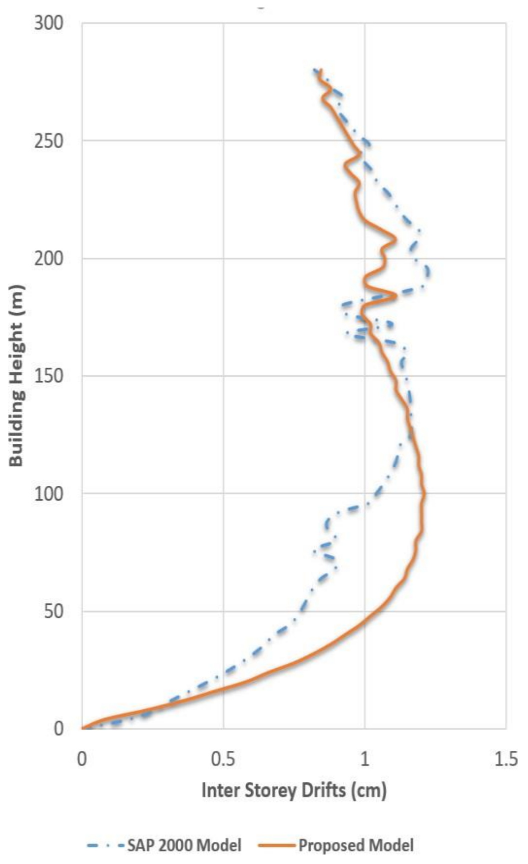


**Fig. 15 - Inter story drift profile of 70 story hybrid system**

## 5.2. The Effect of Outrigger Systems on Decreasing the Lateral Displacement

CTBUH outrigger working group [5] suggested that the floor slabs at or near the outrigger systems should be considered as flexible slabs in order to achieve real values for the forces in chord elements of the outriggers, therefore the outrigger systems equipped with PLVD's are the main part in the hybrid system which connect the perimeter flexural frames to the central shear core and makes the tubular frame and shear core to interact with each other and decrease the lateral displacement. In order to study the effects of the outrigger systems on the decrease of lateral displacement, another comparison is made between the results of the proposed model of the hybrid system and the finite element model of a system combined of only tubular frame and central shear core and depicted in Figs. 16 and 17 for 60 and 70 story buildings respectively. The results depicted in Figs. 16 and 17 reveal the effectiveness of the hybrid system in decreasing the lateral displacement through the height of the building, in order to satisfy the limitation of the lateral displacement and drifts under wind loads according to ASCE 07 [33].

The results in table 8 show that the maximum displacement reductions are about 41 percent for both 60 and 70 story buildings.

**Table 8 - Displacement comparison between the results of the new hybrid system and the system comprised oftubular frame and shear core**

| 60 story structure | | | 70 story structure | | |
|---|---|---|---|---|---|
| New hybrid system, proposed method $(cm)$ | Tubular frame and shear core, SAP 2000 $(cm)$ | Percentage of reduction | New hybrid system, proposed method $(cm)$ | Tubular frame and shear core, SAP 2000 $(cm)$ | Percentage of reduction |
| 47.60 | 81.36 | 41 | 69.35 | 117.69 | 41 |

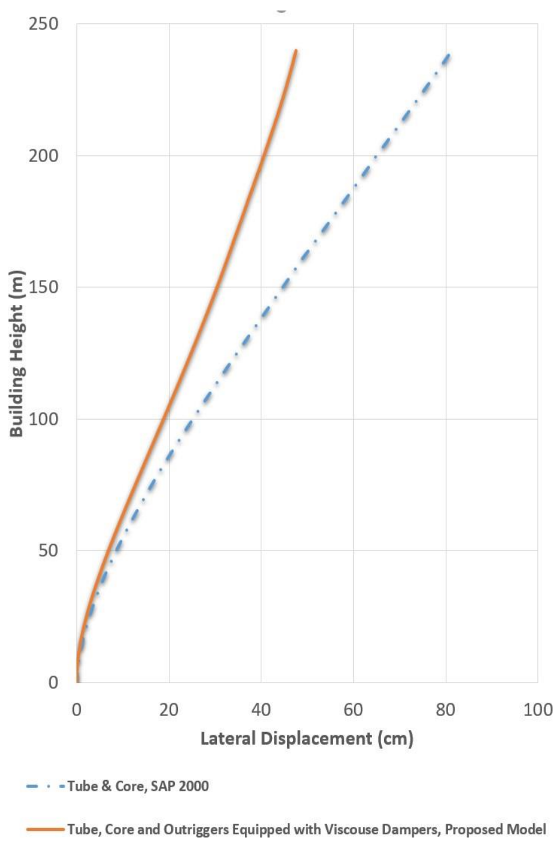


**Fig. 16 - Lateral displacement comparison of 60 story hybrid system with tubular and shear core system**

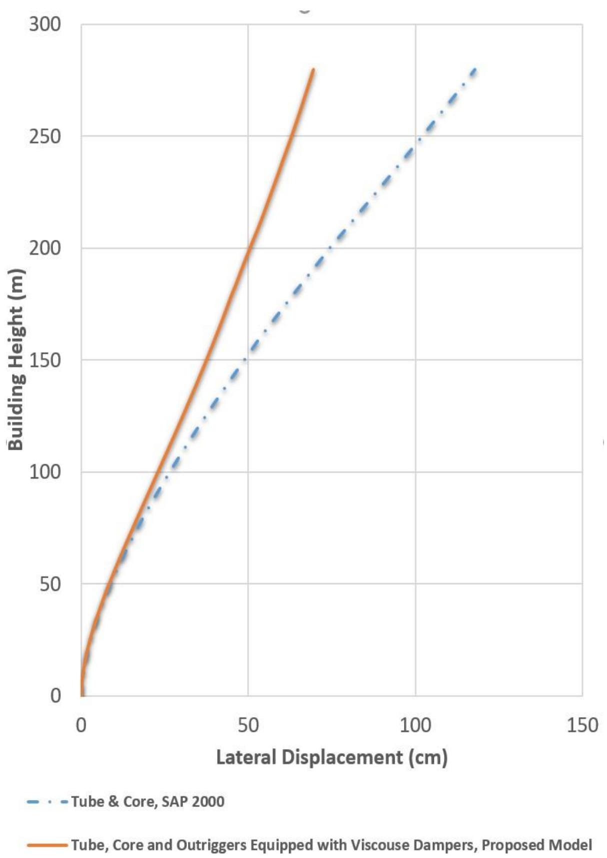


**Fig. 17 - Lateral displacement comparison of 70 storyhybrid system with tubular and shear core system**

## 6. Conclusion

A new hybrid system comprised of framed tube, central shear core and outriggers equipped with PLVD's under lateral load pattern was introduced, and a mathematical formulation for an equivalent cantilevered flexural hollow box beam was developed. The properties of the equivalent Euler-Bernoulli beam changed step by step through the height of beam. The ability of exerting any arbitrary lateral load pattern varying against time on a new hybrid system with outriggers equipped with PLVD's are novelties of this approach. These novelties present an accurate yet simple method to calculate lateral displacement profile of the hybrid system, which is much less time consuming in comparison with 3D fully detailed finite element modelling. By considering the aforementioned beam, kinetic and potential energies and non-conservative works due to lateral load pattern and the force of PLVD's were calculated. After implementing Hamilton's principle, the governing equation of motion and natural boundary conditions were derived. The central finite difference method was chosen as a numerical method to convert partial differential equations to three algebraic equations in matrix form, and solve some simultaneous linear equations for a lateral load pattern similar to wind load distribution through the height of tall buildings. The accuracy, correctness and simplicity of the proposed method was assessed through some numerical examples.

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