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# The Characterization of J-Integral for Elastic-Plastic Crack Growth Evaluation using Finite Element Analysis

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**Abstract:** J-integral is a fracture mechanic parameter that can be used to characterize elastic-plastic fracture mechanic (EPFM) behavior. The path independent characteristic in J-integral is proposed by Rice [1], and it is widely used in a lot of research. Another approach is the load-displacement approach, where the J-integral is calculated by the area under the load-displacement curve. However, the validity of the J-integral value by load-displacement approach is yet to be confirmed. This paper is aimed to investigate the effect of crack length ratio of CT specimen to J-integral value by two approaches: path-integral approach and load-displacement approach. Finite element analysis of compact tension (CT) model with crack length ratio *a*/W between 0.2 to 0.5 was carried out under displacement δ between 0.2 to 1.0 mm using ANSYS parametric design language (APDL). The J value by path integral approach, $J_{path}$ is compared to the value calculated from load-displacement approach, $J_{p\text{-}d}$. It was found that path independency occurs for J value evaluated from path integral approach. A correction factor needs to be introduced since the load-displacement approach cannot be used for shallow crack cases.

**Keywords:** J-integral, CT specimen, path-integral, load-displacement, shallow crack

## 1. Introduction

In 1968, Rice [1] developed J-integral, which is evaluated along the arbitrary contour around the crack tip. The J-integral can be used to generalize the energy release rate of cracked nonlinear materials. Besides, J-integral is a nonlinear energy release rate that is proven to be a path-independent line integral. Azmi et al. [2] found that the path independent characteristic existed in cyclic J-integral as well.

Hutchinson [3] and Rice and Rosengren [4] stated that J-integral can be viewed as stress intensity parameter by relating J-integral to crack tip stress fields in nonlinear material and the stress field is called HRR stress singularity field. Therefore, J-integral can be view as an energy parameter as well as a stress intensity parameter.

Shih and Hutchinson [5] provide a first fracture design analysis based on J-integral in 1976, and a fracture design handbook [6] was published by the Electric Power Research Institute (EPRI) based on their methodology. Landes and Begley [7], [8] calculated J-integral experimentally by plotting the load-displacement graph. The energy absorbed by the specimen is the area under the load-displacement graph. S. M. Kavale et al. [9] found that there is no effect of the plane strain and plane stress condition in the 2-dimensional model on stress intensity factor. Jian Li et al. [10] investigated the crack tip plastic deformed level to the fatigue crack propagation behavior. When a model is loading under a high loading ratio and high load amplitude, range of stress intensity factor, ΔK cannot be used to describe fatigue crack propagation as it does not consider the crack tip plasticity, other fracture parameter should be used. D. Dorribo et al. [11] test the validity of cyclic J-integral for predicting failure of spot welds in martensitic boron steel 22MnB5 and found that cyclic J-integral is better in describing the crack growth rate of the CT specimen than range of stress intensity factor, ΔK.

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Jie Wang et al. [12] use the equivalent domain integral method to compute cyclic J-integral. The numerical simulation result is compared to the experiment result, and both reach a good agreement. However, the validation of the result is yet to be confirmed as it does not compare with the ASTM standard or other researcher result. Azmi et al. [13] investigate the effect of crack length to the cyclic J-integral from small scale yielding to large scale yielding on four-point bending rectangular model by numerical method. The research found that the estimation of cyclic J-integral by load-displacement approach from Dowling and Begley [7], [8] cannot be used in large scale yielding condition for shallow crack specimen.

**Nomenclature**

| | |
|---|---|
| $a$ | crack length |
| $A_{pl}$ | plastic area under the load-displacement curve |
| $A_{el}$ | elastic area under the load-displacement curve |
| $a/W$ | crack length ratio |
| B | specimen thickness |
| b | remaining ligament of the specimen |
| ds | length increment along the contour |
| E | elastic modulus |
| J | J-integral |
| $J_{path}$ | J-integral by path-integral approach |
| $J_{p-d}$ | J-integral by load-displacement approach |
| $J_{el}$ | J-integral in elastic component |
| $J_{pl}$ | J-integral in plastic component |
| K | stress intensity factor |
| n | gradient in HRR field |
| r | distance from crack tips |
| $T_i$ | components of traction vector |
| $u_i$ | displacement vector components |
| v | Poisson's ratio |
| W | specimen width |
| w | strain energy density |

*Greek symbols*

| | |
|---|---|
| $\sigma$ | true stress |
| $\sigma_y$ | yield stress |
| $\sigma_{ij}$ | stress from a reference state to the current state |
| $\varepsilon_{ij}$ | strain from a reference state to the current state |
| $\eta_{pl}$ | plastic geometry factor |
| $\eta_{el}$ | elastic geometry factor |
| $\Gamma$ | an arbitrary curve around the crack tip |
| $\delta$ | displacement |
| $\Delta K$ | range of stress intensity factor |
| $\Delta P_{eff}$ | effective load from reference point to the current state |
| $\partial a$ | change of crack length |
| $\partial U$ | change of strain energy |

*Abbreviations*

| | |
|---|---|
| ASTM | American Society for Testing and Materials |
| CT | Compact Tension |
| EPRI | Electric Power Research Institute |
| FEA | Finite Element Analysis |
| LSY | Large Scale Yielding |
| SSY | Small Scale Yielding |

D. Sen and J. Chattopadhyay [14] investigate the effect of the crack length ratio on the η-factor and found that the ratio of η-factor by ASTM standard and EPRI handbook starts to deviate for the crack length ratio $a/W$ below 0.5. Eq. (1) was developed to fit into the η-factor in shallow crack condition where the $a/W$ is less than 0.5.

$$\frac{\eta_{\text{pl-FEM}}}{\eta_{\text{ASTM}}} = a\exp\left(\frac{1}{x}\right) + b\exp(1+y) + \frac{c}{x^d y^e} + \frac{gy}{x^h} + \frac{1}{x^{1.5} y^f} \tag{1}$$

where x = n (R-O strain hardening index) and y= *a*/W; and the fitting coefficients are *a* = −1.117; b = −1.294; c=3.798; d = 0.04952; e = 0.172; f = 0.8449; g = 7.698; h = −0.02912.

However, the η-factor equation is complicated to implement in the experiment because the η-factor cannot be found directly from the experiment data. Therefore, a simple equation is needed to calculate J-integral of shallow cracked specimen by experiment method. The J-integral in ASTM E1820 handbook is only valid for deep crack, therefore there will be a correction factor needed in the J-integral equation provided [15].

This paper is aimed to investigate the effect of crack length ratio of CT specimen to J-integral value by two approaches: path-integral approach and load-displacement approach.

## 2. Methodology

An elastic-plastic finite element analysis (FEA) was conducted under displacement $\delta$ = 0.2, 0.4, 0.6, 0.8 and 1.0 mm by using ANSYS APDL 15.0 under plain strain condition. Half of the specimen was modeled due to symmetry of CT specimen to reduce computation time.

## 2.1 Geometry

The dimension of the CT specimen model will follow the ASTM 1820 standard. The width of the specimen is 48mm. The model of the FEA will according to Fig.1 where standard dimension ratio is used. From A. Ortega et. al [16], there are 4 types of pin hole loading configurations show in Fig. 2. In this simulation processes, point load is chosen for loading condition in CT specimen simulation.

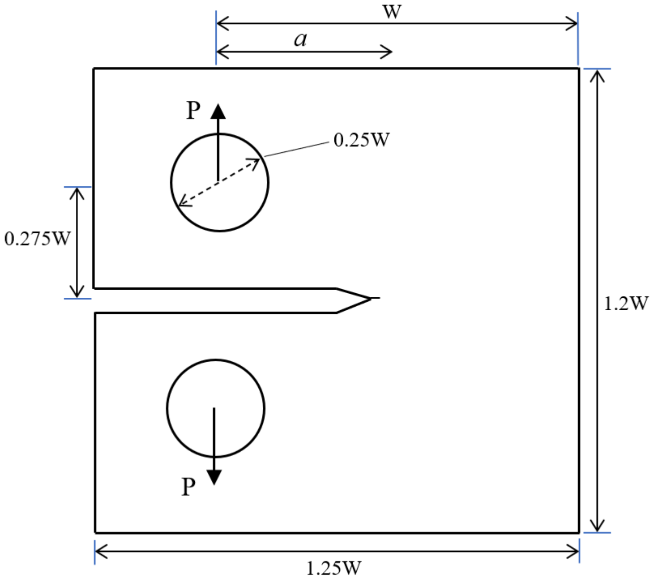



**Fig. 1 - CT specimen model**

## 2.2 Material

The material used is SPV 235 with the elastic modulus, Poisson’s ratio, yield stress and work hardening exponent of 215 GPa, 0.32, 247 MPa and 0.18, respectively [2]. The material is widely use in pressure vessel to hold liquid, vapors, or gases in extreme low or higher pressure than the ambient pressure.

## 2.3 Mesh

Before modelling, element characteristics of model need to be defined. The CT specimen was meshed according to iso-parametric 8 node in quadrilateral shape element due to the 2-dimensional model. For J-integral analysis around the crack tip, a more finite meshing was use at the crack tip. A total of 6128 nodes and 1864 elements on average is found in each model. Fig. 3 shows finite element mesh of 2-dimensional model for half region of CT specimen with crack length ratio, *a*/W = 0.5. Total three geometries of CT specimens were modeled with the crack length ratio, *a*/W = 0.2, 0.4 and 0.5.

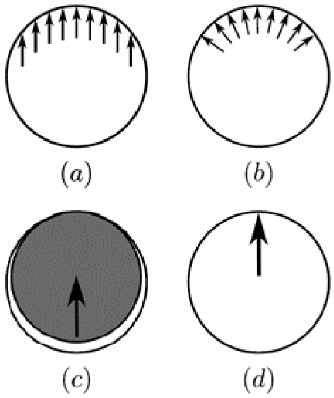
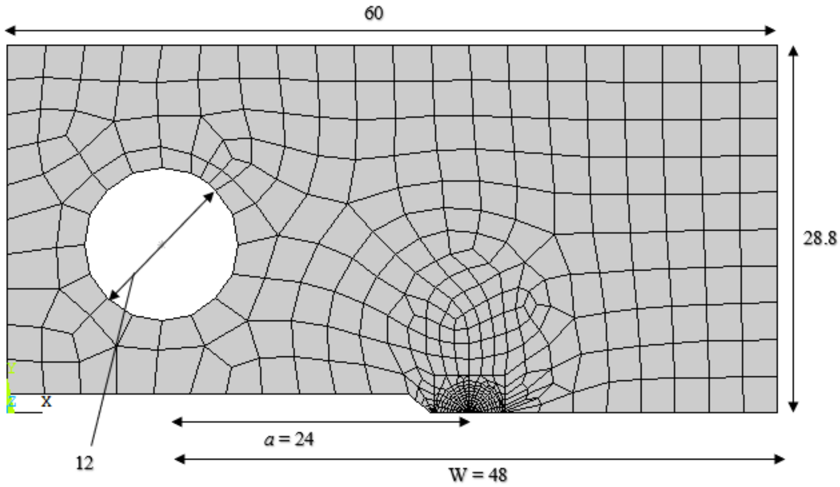
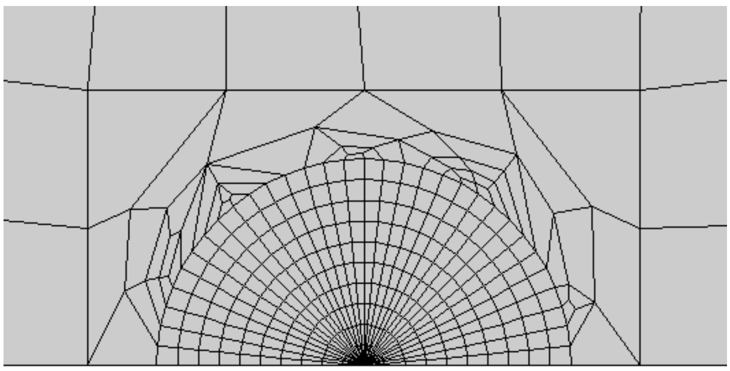



**Fig. 2 - Pin hole loading configurations (a) uniformly distributed; (b) uniform radial stress; (c) contact interaction between pin and hole; (d) point load [16]**




(a)



(b)

**Fig. 3 - Boundary condition of meshed CT specimen with *a*/W =0.5 (a) Half specimen model; (b) Zoom view of crack tip**

## 2.4 Analytical Procedure

Two approaches of J-integral estimation have been used in the FEA, path integral approach and the load-displacement approach. J-integral from path-integral approach is estimate by using the equation proposed by Rice [1] while the J-integral from load-displacement approach is calculated by using the equation provided by the ASTM standard. The relationship of J-integral from both approaches will be analyzed and discussed.

### 2.4.1 Path Integral Approach

Following the path-integral approach [1], the J-integral value for monotonic loading was defined by Eq. (2).

$$J = \int_{\Gamma} \left( wn_x - T_i \frac{\partial u_i}{\partial x} \right) ds \quad (2)$$

where $\Gamma$ is an arbitrary curve around the crack tip; w is the strain energy density; $T_i$ is the components of traction vector; $u_i$ is the displacement vector components; ds is the length increment along the contour; and x and y are the rectangular coordinates with the y direction taken normal to the crack line and the origin at the crack tips.The strain energy density, w is defined as

$$w = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij} \quad (3)$$

where $\sigma_{ij}$ is stress from a reference state to the current state and $\varepsilon_{ij}$ is strain from a reference state to the current state

### 2.4.2 Load-Displacement Approach

Landes and Begley [7], [8] are the first who calculate J-integral experimentally by plotting the load-displacement graph. On the other hand, Azmi et al. [2] compute the cyclic J-integral by using the load-displacement approach by calculating the area under the load-displacement graph as shown in Fig. 4 where $A_{pl}$ refer to corresponding plastic work and $\Delta P_{eff}$ show the effective load from reference point to the current state. Using load displacement approach, J integral value was calculated using the equations below.

$$J = -\frac{1}{B}\left(\frac{\partial U}{\partial a}\right)_{\Delta} \quad (4)$$

where B is the thickness of the specimen; $\partial U$ is the change of strain energy; and $\partial a$ is the change of crack length.

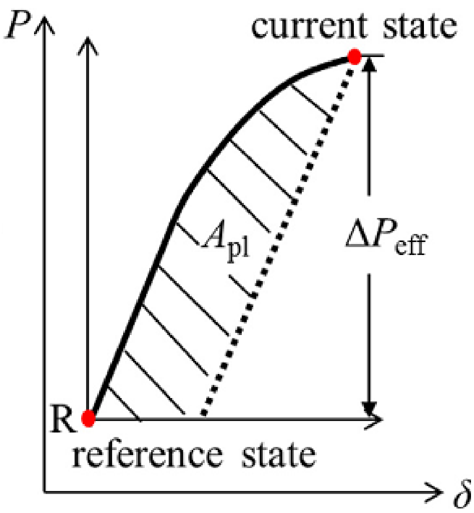



**Fig. 4 - Load-displacement graph [2]**

According to Rice [1], the J-integral can be simplified in elastic and plastic components as stated in Eq. (5) and (6).

$$J = J_{el} + J_{pl} \quad (5)$$

$$J = \frac{\eta_{el} A_{el}}{Bb} + \frac{\eta_p A_{pl}}{Bb} \quad (6)$$

Elastic J-integral is equal to G in linear elastic condition; therefore Eq. (6) can be derived into Eq. (7) as follows,

$$J = \frac{K^2}{E'} + \frac{\eta_p A_{pl}}{Bb} \quad (7)$$

$$E' = \frac{E}{(1-v^2)} \quad (8)$$

$$J = \frac{K^2(1-v^2)}{E} + \frac{\eta_{pl} A_{pl}}{B\, b} \quad (9)$$

where K is stress intensity factor; v is Poisson's ratio; E is elastic modulus; $\eta_{pl}$ is plastic geometry factor; $A_{pl}$ is the plastic area under the load-displacement curve; B is specimen thickness; and b is the remaining ligament of the specimen.

## 3. Result and Discussion

Fig. 5 shows a typical load-displacement graph for CT specimen with *a*/W = 0.5 subjected to 1.0 mm displacement, and the shaded area of $A_{pl}$ shows the plastic work component obtained from the chart. The path independence of $J_{path}$ calculated from Eq. (2), and its comparison with load-displacement approach, $J_{p\text{-}d}$ is demonstrated in Fig. 6. It was found that J values calculated by path-integral approach, $J_{path}$ and load-displacement approach, $J_{p\text{-}d}$ show a good agreement. Furthermore, the path independent characteristic is hold, and the value is independent to the paths around the crack tip.

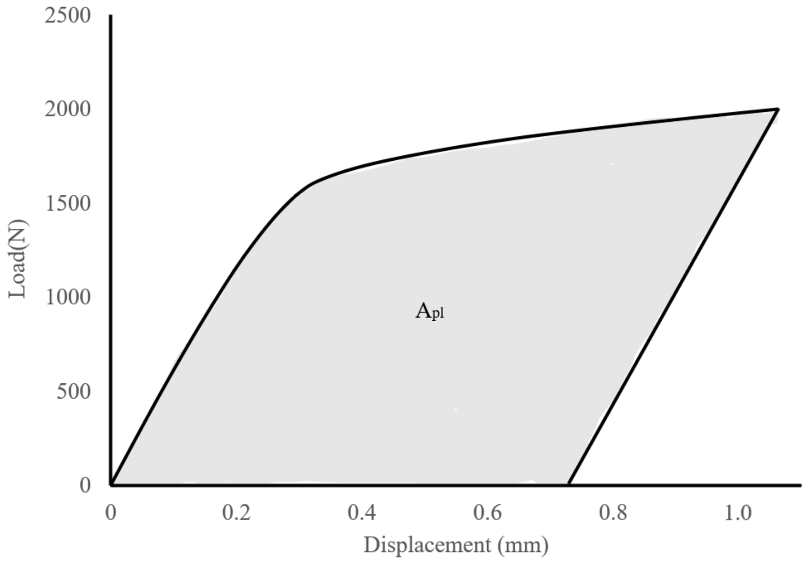



**Fig. 5 - Load-displacement graph for CT specimen with *a*/w=0.5 under displacement of 1.0 mm**

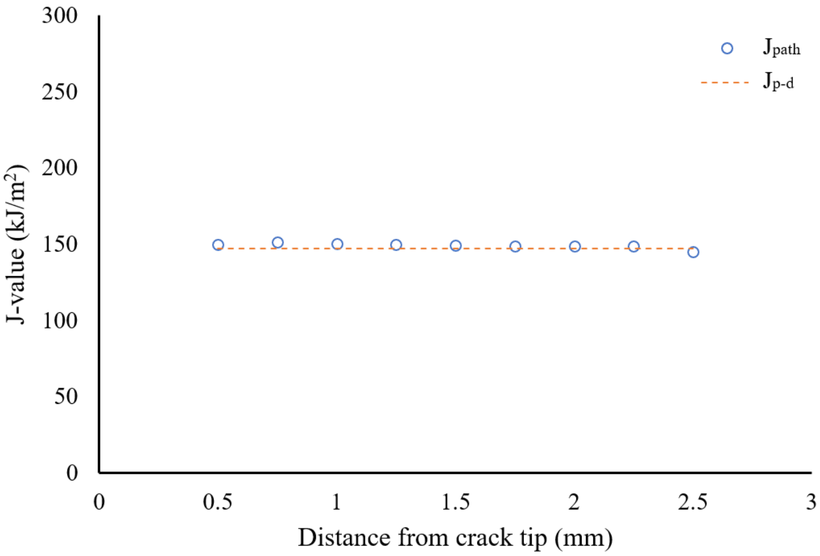



**Fig. 6 - J-integral path independent characteristic**

Fig. 7 shows the effect of crack length *a*/W to the ratio of J values ($J_{p\text{-}d}/J_{path}$). The $J_{p\text{-}d}$ starts deviating from $J_{path}$ for shallow crack cases, *a*/W = 0.4 and *a*/W = 0.2 as the load-displacement equation no longer supports the J-integral by load-displacement approach for the specimen with shallow crack. This result proves that the load-displacement equation from ASTM standard [15] is only valid for CT specimens with a crack length ratio, *a*/W ranging from 0.45-0.55.

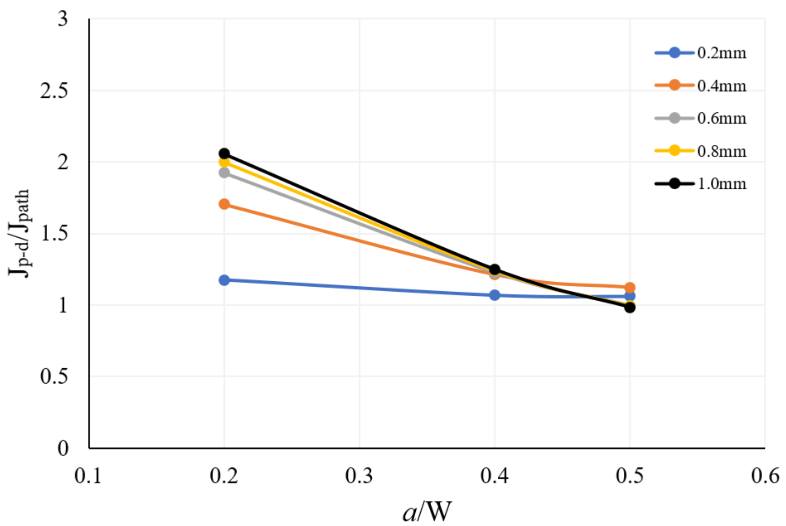



**Fig. 7 - Effect of crack length ratio *a*/W to $J_{p-d}/J_{path}$**

The J-integral evaluation consist of two components: elastic component and plastic component. From Fig. 8, the elastic component, $J_{el}$ has a major contribution to the J values when the CT specimen is subjected to a displacement of 0.2 mm. When the displacement is over 0.4 mm, the plastic component, $J_{pl}$ occupied large portion of the total values. Azmi et al. [2] stated that the condition where the elastic component has a major contribution is called small-scale yielding (SSY) condition, while when plastic component gives major contribution, the condition is defined as large-scale yielding (LSY) condition.

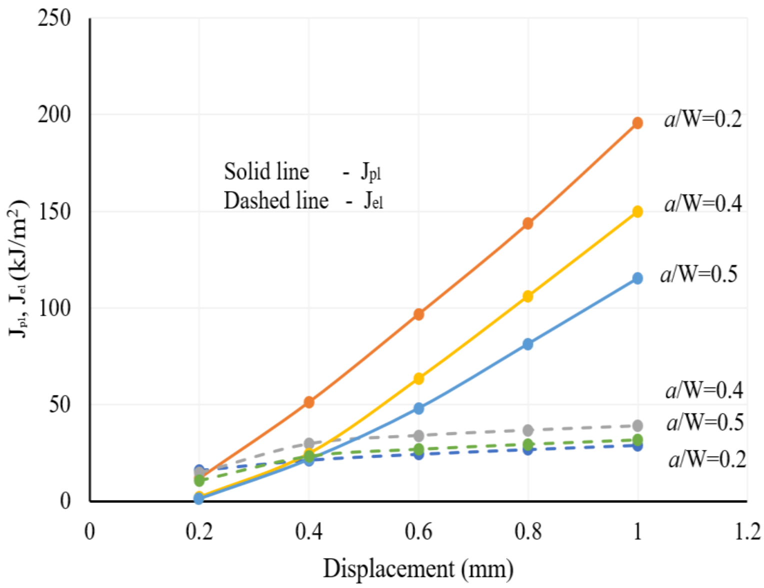
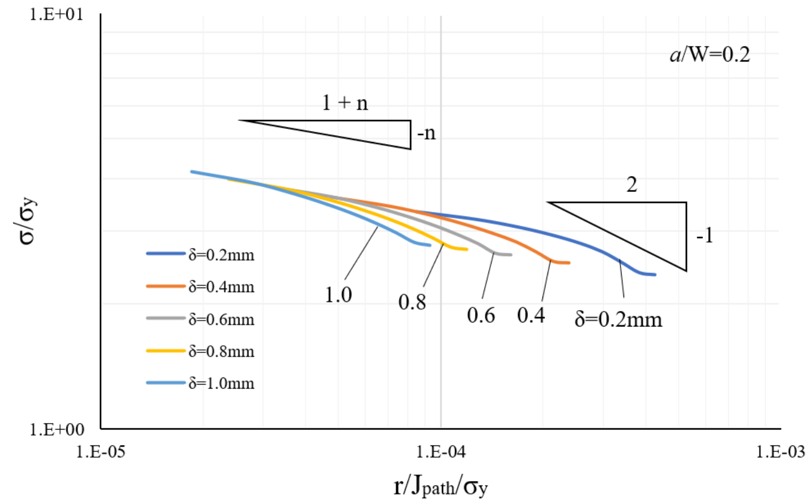
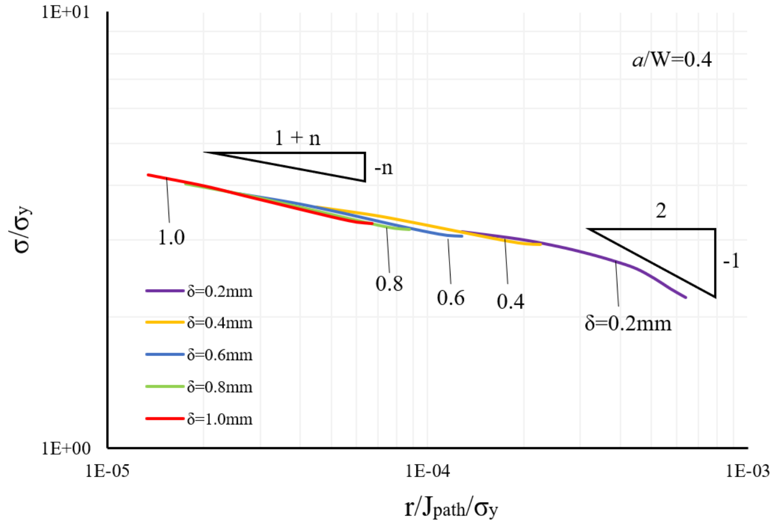



**Fig. 8 - Effect of displacement to $J_{p-d}$ in elastic and plastic component**

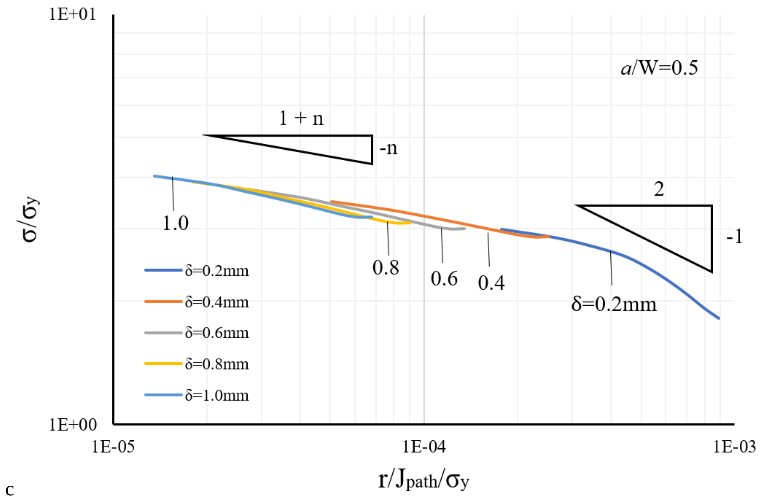
Fig. 9 shows the relation of stress distribution along the line ahead of crack tip versus the distance normalized by yield strength of the material and value of $J_{path}$ in a log-log diagram. The gradient of -1/2 show elastic singular stress field while -n/(1+n) show elastic-plastic singular stress field which both defined as the HRR field. The elastic singular stress field present in shallow crack case (a/W = 0.2) regardless of displacement applied. For other cases (a/W = 0.4 and 0.5), the elastic stress field only occur in the SSY condition where displacement is 0.2mm. On the other hand, the material was subjected to an elastic-plastic singular stress field for LSY condition where displacement is 0.4mm to 1.0mm.




(a)




(b)




(c)

**Fig. 9 - Stress range distribution ahead of a crack tip with different crack depth ratio (a) *a*/W=0.2; (b) *a*/W=0.4; (c) *a*/W=0.5**

## 4. Conclusion

In this research, CT specimen of different crack length ratios is analyzed using the ANSYS APDL 15.0 software. Throughout the study, the main conclusions can be obtained from all the above research,

- The $J_{path}$ and $J_{p-d}$ meet an excellent agreement for *a*/W = 0.5 but not in the specimen with shallow crack, *a*/W = 0.2 and *a*/W = 0.4. The result matched with the D. Sen and J. Chattopadhyay [14] finding and ASTM standard [15] where the J-integral by load-displacement approach is only valid for deep crack.
- The elastic component, $J_{el}$ has a significant contribution to the J values when the CT specimen is subjected to a displacement of 0.2 mm. However, when the displacement is over 0.4 mm, the plastic component, $J_{pl}$ occupied a large portion of the total values.
- The elastic singular stress field present in shallow crack case (*a*/W = 0.2) regardless of displacement applied. For other cases (*a*/W = 0.4 and 0.5), the elastic stress field only occur in the SSY condition (displacement = 0.2mm). On the other hand, the material was subjected to an elastic-plastic singular stress field for LSY condition (displacement = 0.4mm-1.0mm).

Further research is needed to solve the problem where a correction factor is needed to compute for shallow crack CT specimen in calculating the J-integral by load-displacement approach.

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## References

[1] Rice, J. R. (1968). A path independent integral and the approximate analysis of strain concentration by notches and cracks. Journal of Applied Mechanics, 35, 379

[2] Azmi, M. S. M., Fujii, T., Tohgo, K. & Shimamura, Y. (2017). On the ∆J -integral to characterize elastic-plastic fatigue crack growth. Engineering Fracture Mechanics, 176, 300–307

[3] Hutchinson, J. W. (1968). Singular behaviour at the end of a tensile crack in a hardening material. Journal of the Mechanics and Physics of Solids, 16, 13–31

[4] Rice, J. R. & Rosengren, G. F. (1968). Plane strain deformation near a crack tip in a power-law hardening material. Journal of the Mechanics and Physics of Solid, 16, 1–12

[5] Shih, C. F. & Hutchinson, J. W. (1976). Fully plastic solutions and large-scale yielding estimates for plane stress crack problems. Journal of Engineering Materials and Technology, 98, 289–295

[6] Kumar, V., German, M. D. & Shih, C. F. (1981). An Engineering Approach for Elastic-Plastic Fracture Analysis. EPRI Report NP-1931. Electric Power Research Institute, Palo Alto, CA

[7] Landes, J. A. & Begley, J. D. (1972). The effect of specimen geometry on JIc. ASTM STP 514. American Society for Testing and Materials, Philadelphia, PA

[8] Herrera, R. & Landes, J. (1988). A direct J –R curve analysis of fracture toughness tests. Journal of Testing and Evaluation

[9] Kavale, S. M., Kodancha, K. G. & Ekabote, N. (2018). Effect of Poisson's ratio on KI, T11 and T33 for SENB and CT specimen - A FE study. Procedia Structural Integrity, 14, 584–596

[10] Li, J., Su, C. Y., Lu, L., Zhang, P., Chang, L., Miao, X. T., Zhou, B. B., He, X. H. & Zhou, C. Y. (2019). Investigation on fatigue crack growth behavior for commercial pure titanium at different crack tip plastic deformed levels. Theoretical and Applied Fracture Mechanic, 100, 1–13

[11] Dorribo, D., Greve, L., Díez, P., Arias, I. & Larráyoz-Izcara, X. (2018). Numerical estimation of the bearing capacity of resistance spot welds in martensitic boron steels using a J-integral fracture criterion. Theoretical and Applied Fracture Mechanics, 96, 497–508

[12] Wang, J., Jiang, W., Li, Y., Wang, Q. & Xu, Q. (2018). Numerical assessment of cyclic J-integral ∆J for predicting fatigue crack growth rate. Engineering Fracture Mechanics, 205, 455–469

[13] Azmi, M. S. M., Fujii, T., Tohgo, K., Hashim, M. S. M., Ismail, A. H. & Razlan, Z. M. (2019). Evaluation of Δj-integral for a shallow crack in steel for pressure vessels under large scale yielding (LSY) condition. IOP Conference Series: Materials Science and Engineering, 670

[14] Sen, D. & Chattopadhyay, J. (2018). New η-factor equation for evaluation of J-integral of shallow cracked CT specimen considering R-O material strain hardening. Theoretical and Applied Fracture Mechanics, 97, 98–107

[15] ASTM E1820-13 (2014). Standard Test Method for Measurement of Fracture Toughness. American Society for Testing and Materials, Philadelphia

[16] Ortega, A., Maimí, P., González, E. V. & Ripoll, L. (2014). Compact tension specimen for orthotropic materials. Composites Part A: Applied Science and Manufacturing, 63, 85–93