

Proof of the Authenticity of the Holy Quran: Discovering the Precise Mathematical Arrangement in the Sura Fussilat

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Abstract

In this article, the numerical structure of the Holy Quran has been studied based on two main sources of the suras, namely the suras numbers and the number of verses. The objective of this study is to prove the authenticity of the Holy Quran using a unique scientific methodology, which will show, through mathematical relations and a system of equations, that the book has been correctly preserved. Firstly, the suras were divided into four groups or tables according to their even and odd numbers, which leads to discover two important properties among the Quranic parameters: the concordance relation and rule 1. The application of the proposed scientific methodology led to the discovery of a hidden element of concordance, which is Sura Fussilat, whose integration into its own Ha-Mim family allowed the discovery of other secondary concordance relations. At the same time, it reveals other important rules that will play an essential role in formulating the system of mathematical equations. Given the diversity of roles played by this sura, we consider it as a key programmed by the author of the book to achieve this amazing mathematical arrangement. The resolution of this system will prove that the total number of suras and the total number of verses have not undergone any change. Therefore, the obtained results confirm that the structure of the Quran remained consistent with the original.

1. Introduction

The Holy Quran is the word of Allah (SWT) transmitted to the Prophet and Messenger Muhammad (May the peace and blessing of Allah be upon him (SAWS)), through the archangel Gabriel (AS) incrementally over a period of about twenty-three years. It is the last of the heavenly books and the final revelation from God to humanity. Muslims believe that the Noble Quran in our possession corresponds exactly to that revealed to Muhammad (SAWS) in the years 610-632 (Esposito, 2005). The Quran book consists of 114 chapters called suras, of unequal lengths. They are not listed in chronological order, but are arranged in an appropriate arrangement. They let us know, with the help of the narrations (hadiths) of the Prophet (SAWS), that the actual arrangement of the suras was established by the Prophet himself under the recommendation of God. Therefore, they consider this as a genuine characteristic (Nassourou, 2011). More than, a quarter of the world's population considers the Noble Quran to be the main source of Islam (Lipka and Hackett, 2017). However, it is the subject of many questions and criticisms from western scientists.

Indeed, in critical-historical studies, researchers such as John Wansbrough, Joseph Schacht, Patricia Crone, Michael Cook seek to verify the different aspects of the Quran (origin, text, composition, history, etc.) (Lester, 1999). Among the most frequent criticisms are those concerning internal consistency, clarity and ethical teachings. John Wansbrough believes that the Quran was written and collected over a period of 200 years and is a redaction of parts of other sacred scriptures, in particular the Judeo-Christian scriptures (Wansbrough, 1978). Patricia Crone and Michael Cook point out in their book *Hagarism: The Making of the Islamic World*, that there is no hard evidence of the existence of the Quran in any form before the last decade of the 7th century (Crone and Cook, 1977). Gerd R-Puin, after studying the ancient manuscripts concluded that the Quran is a "cocktail" of texts, some parts of which already existed, a hundred years before Muhammad (SAWS) (Lester, 1999).

Among all these questions about Islam, there is one that summarizes them all, the problem of the veracity of the Holy Quran. It is enough to prove its authenticity to invalidate all the accusations made not only against the Quran but also against the Prophet. But how can we prove that the Quran we have today is the one revealed by God (SWT) to Muhammad (SAWS)? The answer to this question, which at first sight seems insoluble, is nevertheless conceivable for those who believe and meditate on the verses of the Noble Quran. It suffices to demonstrate that the Noble Quran has not undergone any change in its structure, including the number of suras and the number of verses. If it can be firmly proven that these two parameters have always remained the same, it means that the preaching announced by the author of the book is true. The author has promised in AL Hijr (15: 99) that the Quran will always be preserved and protected from any distortion and corruption such as adding, deleting, or transferring any of its verses from one sura to another, Verse 9: « It is certainly We who have revealed the reminder, and it is certainly We who will preserve it ».

Those who believe in this verse and in the announced promise will conclude that there would certainly be in the structure of the Noble Quran, methods based on the exact sciences (mathematics) which will allow us to know that The Noble Quran has not undergone any change. The discovery and application of these methods will therefore constitute an answer to the posed questions. During the historical period of Islam, many Quranic studies were produced, particularly in the following fields: interpretation of the Quran (AlShaari, 2014), recitation of the Quran (Khairuddin et al, 2017), vocabulary of the Quran (Emara, 2013), numerology of the Quran (Al-Kabi et Al, 2005; Arkok and Zeki, 2020; Agus, 2021; Rostam and Malin, 2021), ontology, archeology and anthropology of the Quran (Taa et al, 2013; Bin Dost, 2008), metric structure of the Quran (Rashid and Jameel, 2017), the miraculous of Quran (Khalifa, 1973; Rahman et al, 2017; Ishak et al, 2020) and the reasons for the revelation (Nassourou, 2011), etc. Regarding the use of Quranic numerology, there are many essays with various concepts such as the abjad system of numbers (Gematria) (Jarrar, 2011, Al-Fakih, 2017; Al-Fakih, 2024). This system uses the calculation of words and letters to arrive at arrangements that lead to satisfactory mathematical results. But many Muslim scholars are against the use of the concept of Gematria in Quranic numerology (Miqdadi, 2020). In the framework of this study, assumptions based on approximate theories and dubious historical arguments have not been taken into account. Only the real resources of the Noble Quran and certain mathematical operations recognized as exact will be used.

In this work, we will show that the total number of suras and the total number of verses have not undergone any change, thus prove the authenticity of the Holy Quran. The procedure chosen to solve this problem will be briefly explained. Initially, the structure of the Quran will be presented by exposing the essential tables as well as the constituent elements and the associated calculation operations. Then, the application of the methodology that will be presented and the integration of sura Fussilat into its Ha-Mim family, will allow us to discover the properties of concordance between the tables. And at the same time to define four rules, which will be needed to formulate the system of equations of the Noble Quran, whose resolution will reveal to us the proof of its authenticity.

Nomenclature

p is the even number
 i is the odd number

For the number of suras:

S_i The number of odd suras.
 S_p The number of even suras.
 $S(V_i)$ The number of suras with odd verses.
 $S(V_p)$ The number of suras with even verses.
 $S_i(V_i)$ The number of odd suras with odd verses.
 $S_p(V_p)$ The number of even suras with even verses.
 $S_i(V_p)$ The number of odd suras with even verses.
 $S_p(V_i)$ The number of even suras with odd verses.

For the sum of suras-numbers:

N_i The sum of odd suras numbers.
 N_p The sum of even suras numbers.
 $N(S(V_i))$ The sum of suras numbers with odd verses.
 $N(S(V_p))$ The sum of suras numbers with even verses.
 $N_i(S(V_i))$ The sum of odd suras numbers with odd verses.
 $N_p(S(V_p))$ The sum of even suras numbers with even verses.
 $N_i(S(V_p))$ The sum of odd suras numbers with even verses.
 $N_p(S(V_i))$ The sum of even suras numbers with odd verses.

For the sum of the numbers of verses:

V_i The sum of the odd numbers of verses.
 V_p The sum of the even numbers of verses.
 $V(S_i)$ The sum of the numbers of verses for odd suras.
 $V(S_p)$ The sum of the numbers of verses for even suras.
 $V_i(S_i)$ The sum of the odd numbers of verses for odd suras.
 $V_p(S_p)$ The sum of the even numbers of verses for even suras.
 $V_i(S_p)$ The sum of the odd numbers of verses for even suras.
 $V_p(S_i)$ The sum of even numbers of verses for odd suras.

2. Structure of the Quranic Book

2.1 Preliminary Elements

The Quran is a book made up of 114 chapters numbered from 1 to 114, called suras and 6236 sentences called verses (signs, ayas). Each sura is composed of a determined number of verses, the minimum number of verses is 3 while the maximum number of verses is 286. Indeed, each sura of the Noble Quran has a particular specificity, it is characterized by its number, its title, its number of verses, its place of revelation, its rank, etc (see **Fig. 1**). In this study and in order to avoid any ambiguity, we will consider two components: the sura number (n_i) and the number of verses (v_i). All suras will therefore be presented by two parameters n_i and v_i in the same line of the table (T_G), as presented in **Table 1**. The first component (n_i) are distinct and successive integers from 1 to n that designate the suras numbers; the second component (v_i) are any integers that designate the number of verses in each sura.

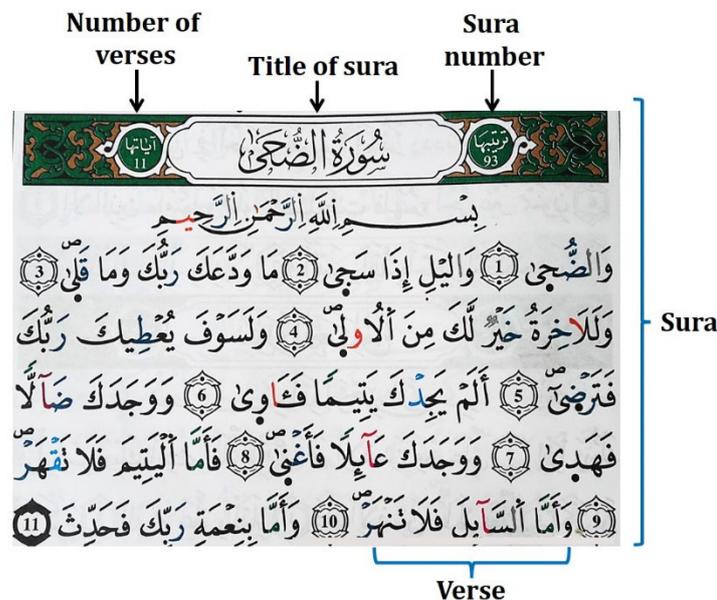


Fig. 1 Important characteristics of the suras: title, sura number and number of verses. For example, Sura Ad-Duhaa (93:11).

Table 1 The Quranic structural parameters: the suras number (n) and the number of its verses (v)

n	Sura's name	Verses	n	Sura's Name	Verses	n	Sura's name	Verses
1	Al-Fatihah	7	42	Ash-Shuraa	53	83	Al-Mutaffifin	36
2	Al-Baqarah	286	43	Az-Zukhruf	89	84	Al-Inshiqaq	25
3	Ali 'Imran	200	44	Ad-Dukhan	59	85	Al-Buruuj	22
4	An-Nisa	176	45	Al-Jathiyah	37	86	At-Tariq	17
5	Al-Ma'idah	120	46	Al-Ahqaf	35	87	Al-A'la	19
6	Al-An'am	165	47	Muhammad	38	88	Al-Ghashiyah	26
7	Al-A'raf	206	48	Al-Fath	29	89	Al-Fajr	30
8	Al-Anfal	75	49	Al-Hujurat	18	90	Al-Balad	20
9	At-Tawbah	129	50	Qaf	45	91	Ash-Shams	15
10	Yunus	109	51	Adh-Dhariyat	60	92	Al-Layl	21
11	Hud	123	52	At-Tur	49	93	Ad-Duhaa	11
12	Yusuf	111	53	An-Najm	62	94	Ash-Sharh	8
13	Ar-Ra'd	43	54	Al-Qamar	55	95	At-Tin	8
14	Ibrahim	52	55	Ar-Rahman	78	96	Al-'Alaq	19
15	Al-Hijr	99	56	Al-Waqi'ah	96	97	Al-Qadr	5
16	An-Nahl	128	57	Al-Hadid	29	98	Al-Bayyinah	8
17	Al-Isra	111	58	Al-Mujadila	22	99	Az-Zalzalah	8
18	Al-Kahf	110	59	Al-Hashr	24	100	Al-'Adiyat	11
19	Maryam	98	60	Al-Mumtahanah	13	101	Al-Qari'ah	11
20	Taha	135	61	As-Saf	14	102	At-Takathur	8
21	Al Anbya	112	62	Al-Jumu'ah	11	103	Al-'Asr	3
22	Al-Hajj	78	63	Al-Munafiqun	11	104	Al-Humazah	9
23	Al-Mu'minun	118	64	At-Taghabun	18	105	Al-Fil	5
24	An-Nur	64	65	At-Talaq	12	106	Quraysh	4
25	Al-Furqan	77	66	At-Tahrim	12	107	Al-Ma'un	7
26	Ash-Shu'ara	227	67	Al-Mulk	30	108	Al-Kawthar	3
27	An-Naml	93	68	Al-Qalam	52	109	Al-Kafirun	6
28	Al-Qasas	88	69	Al-Haqqah	52	110	An-Nasr	3
29	Al-'Ankabut	69	70	Al-Ma'arij	44	111	Al-Masad	5
30	Ar-Rum	60	71	Nuh	28	112	Al-Ikhlash	4
31	Luqman	34	72	Al-Jinn	28	113	Al-Falaq	5
32	As-Sajdah	30	73	Al-Muzzammil	20	114	An-Nas	6
33	Al-Ahzab	73	74	Al-Muddaththir	56			
34	Saba	54	75	Al-Qiyamah	40			
35	Fatir	45	76	Al-Insan	31			
36	Ya-Sin	83	77	Al-Mursalat	50			
37	As-Saffat	182	78	An-Naba	40			
38	Sad	88	79	An-Nazi'at	46			
39	Az-Zumar	75	80	'Abasa	42			
40	Ghafir	85	81	At-Takwir	29			
41	Fussilat	54	82	Al-Infitar	19			

2.2 Constitution of Tables G_1, G_2, G_3, G_4 and Tables A_1, A_2, A_3, A_4

From the **Table 1** and taking into account the even and odd numbers of the elements which compose it, we can form four groups of disjoint tables G_1, G_2, G_3 and G_4 . Table T_G and tables G_i are the main elements of this study.

For G_i tables:

- 1- Table G_1 (II) made up of odd suras numbers with odd verses,
- 2- Table G_2 (PI) made up of even suras numbers with odd verses,
- 3- Table G_3 (IP) made up of odd suras numbers with even verses.
- 4- Table G_4 (PP) made up of even suras numbers with even verses.

For each table G_i , the 1st column consists of the suras number, the 2nd column consists of the number of verses of each sura and the last line of each table contains the sum of the elements of each column, as shown in **Table 2**. This method was chosen to provide more detail about the calculations of the sum of the elements of each column, which are performed on the last line of the tables, and are presented in gray color. Another hand, we will designate the number of elements of G_i table by $S(V)$, and we will call them the number of suras, as follows: $S_i(V_i) = 27$; $S_p(V_i) = 27$; $S_i(V_p) = 30$ and $S_p(V_p) = 30$.

Table 2 The Quranic structural parameters of each table in initial state: G_1 (II), G_2 (PI), G_3 (IP) and G_4 (PP)

G_1 (II) 27 Suras		G_2 (PI) 27 Suras		G_3 (IP) 30 Suras		G_4 (PP) 30 Suras	
n	v	n	v	n	v	n	v
1	7	6	165	3	200	2	286
9	129	8	75	5	120	4	176
11	123	10	109	7	206	14	52
13	43	12	111	19	98	16	128
15	99	20	135	21	112	18	110
17	111	26	227	23	118	22	78
25	77	36	83	31	34	24	64
27	93	40	85	37	182	28	88
29	69	42	53	41	54	30	60
33	73	44	59	47	38	32	30
35	45	46	35	49	18	34	54
39	75	48	29	51	60	38	88
43	89	50	45	53	62	56	96
45	37	52	49	55	78	58	22
57	29	54	55	59	24	64	18
63	11	60	13	61	14	66	12
81	29	62	11	65	12	68	52
87	19	76	31	67	30	70	44
91	15	82	19	69	52	72	28
93	11	84	25	71	28	74	56
97	5	86	17	73	20	78	40
101	11	92	21	75	40	80	42
103	3	96	19	77	50	88	26
105	5	100	11	79	46	90	20
107	7	104	9	83	36	94	8
111	5	108	3	85	22	98	8
113	5	110	3	89	30	102	8
				95	8	106	4
				99	8	112	4
				109	6	114	6
1551	1225	1554	1497	1698	1806	1752	1708

For A_i tables:

The tables A_i are formed by joining two tables G_i , and are presented in **Table 3**.

- 1- The table A_1 is made up of two tables $G_1(II)$ and $G_4(PP)$, $A_1 = G_1UG_4$
- 2- The table A_2 is made up of two tables $G_2(PI)$ and $G_3(IP)$, $A_2 = G_2UG_3$
- 3- The table A_3 is made up of two tables $G_3(IP)$ and $G_4(PP)$, $A_3 = G_3UG_4$
- 4- The table A_4 is made up of two tables $G_1(II)$ and $G_2(PI)$, $A_4 = G_1UG_2$.

Table 3 The Quranic structural parameters of four new tables: A_1, A_2, A_3 and A_4 , in the initial state. The last gray line of each table contains the sum of the elements of each column, and the arrow indicates the concordance between the tables

A_1		A_2		A_3		A_4	
$G_4(PP)$ and $G_1(II)$		$G_3(PI)$ and $G_2(IP)$		$G_4(PP)$ and $G_3(IP)$		$G_2(PI)$ and $G_1(II)$	
57 Suras		57 Suras		60 Suras		54 Suras	
n	v	n	v	n	v	n	v
1	7	3	200	2	286	1	7
2	286	5	120	3	200	6	165
4	176	6	165	4	176	8	75
9	129	7	206	5	120	9	129
11	123	8	75	7	206	10	109
13	43	10	109	14	52	11	123
14	52	12	111	16	128	12	111
15	99	19	98	18	110	13	43
16	128	20	135	19	98	15	99
17	111	21	112	21	112	17	111
18	110	23	118	22	78	20	135
22	78	26	227	23	118	25	77
24	64	31	34	24	64	26	227
25	77	36	83	28	88	27	93
27	93	37	182	30	60	29	69
28	88	40	85	31	34	33	73
29	69	41	54	32	30	35	45
30	60	42	53	34	54	36	83
32	30	44	59	37	182	39	75
33	73	46	35	38	88	40	85
34	54	47	38	41	54	42	53
35	45	48	29	47	38	43	89
38	88	49	18	49	18	44	59
39	75	50	45	51	60	45	37
43	89	51	60	53	62	46	35
45	37	52	49	55	78	48	29
56	96	53	62	56	96	50	45
57	29	54	55	58	22	52	49
58	22	55	78	59	24	54	55
63	11	59	24	61	14	57	29
64	18	60	13	64	18	60	13
66	12	61	14	65	12	62	11
68	52	62	11	66	12	63	11
70	44	65	12	67	30	76	31
72	28	67	30	68	52	81	29
74	56	69	52	69	52	82	19
78	40	71	28	70	44	84	25
80	42	73	20	71	28	86	17
81	29	75	40	72	28	87	19
87	19	76	31	73	20	91	15
88	26	77	50	74	56	92	21
90	20	79	46	75	40	93	11
91	15	82	19	77	50	96	19
93	11	83	36	78	40	97	5
94	8	84	25	79	46	100	11
97	5	85	22	80	42	101	11
98	8	86	17	83	36	103	3
101	11	89	30	85	22	104	9
102	8	92	21	88	26	105	5

103	3	95	8	89	30	107	7
105	5	96	19	90	20	108	3
106	4	99	8	94	8	110	3
107	7	100	11	95	8	111	5
111	5	104	9	98	8	113	5
112	4	108	3	99	8		
113	5	109	6	102	8		
114	6	110	3	106	4		
				109	6		
				112	4		
				114	6		
3303	2933	3252	3303	3450	3514	3105	2722

As can be seen in **Table 3**, after adding the elements in the tables A_1 and A_2 , we discover a remarkable equality (3303) between the sum of the suras numbers of the of the 1st column of A_1 and the sum of the numbers of verses of the 2nd column of A_2 . This equality has been previously noted by this work (Jalghoum, 2018). In contrast, there is no equality observed between the G_i tables, as shown in **Table 2**.

2.3 Description of G_i and A_i Tables and the Designation of Their Sum Parameters

The calculations that will perform at the level of the tables G_i or A_i concern only the sum of the suras numbers in the first column and the sum of the numbers of the verses in the second column. Each sum is designated by a term whose exact expression will be determined later for each table.

Table 4 Description of two proposed tables G_u and G_m and the calculation operations used

Table G_u		Table G_m	
Suras number	Number of verses	Suras number	Number of verses
u_1	v_{u1}	m_1	v_{m1}
u_2	v_{u2}	m_2	v_{m2}
...
u_i	v_{ui}	m_i	v_{mi}
N_u	V_u	N_m	V_m

For the proposed tables G_u and G_m , Once the sum operation is performed for all the columns of these tables (see **Table 4**), the obtained result is considered as a single element and will be called the sum parameter. The sum parameters of each table G will then be presented in a one-dimensional array $G_u s$ or $G_m s$ with two components in the form of a row matrix whose first component is the sum parameter of the first column N_u or N_m and the second component is the sum parameter of the second column V_u or V_m .

- For a single table (example the table G_u), we will perform the sum of the elements of the first column which will be designated by N_u and also the elements of the second column which will be designated by V_u . To simultaneously express the sums of the two columns for each table or group, we will represent them in order by the following format of matrix: $G_u s = (N_u \ V_u)$, which we will call the sum parameters of the table G_u . Likewise for the table G_m , the sum parameters are given by $G_m s = (N_m \ V_m)$.
- For two tables (example between the tables G_u and G_m), we will perform the sum of the first two columns which will be designated by $N_u + N_m$ and also the sum of the second two columns which will be designated by $V_u + V_m$. To show the sums of the first two columns and the second two columns of the two tables at the same time, we will represent them in order by the following format of matrix: $A_s = G_u s + G_m s = (N_u + N_m \ V_u + V_m)$, which we will call the sum parameters of the combined tables G_u and G_m .

In the following, we will perform the calculations in the same way as described above at the level of the Quranic tables G_1, G_2, G_3, G_4 and those formed by their combination A_1, A_2, A_3, A_4 . Therefore, the designation of all sum parameters of the tables G_i and A_i , in the initial state, is summarized in the **Table 5**.

Table 5 The sum parameters of G_i tables and their values in the initial state: the sum of suras numbers (N) and the sum of the number of verses (V)

	Sum parameters	N	V
A_1S	G_4S (PP)	$N_p(S(V_p)) = 1752$	$V_p(S_p) = 1708$
	G_1S (II)	$N_i(S(V_i)) = 1551$	$V_i(S_i) = 1225$
A_2S	G_2S (PI)	$N_p(S(V_i)) = 1554$	$V_i(S_p) = 1497$
	G_3S (IP)	$N_i(S(V_p)) = 1698$	$V_p(S_i) = 1806$

The expression of the sum parameters as a function of suras numbers and number of verses is given below for each table G_i .

- The elements of the groups G_u and G_m defined in Table 4, namely: (u_1, u_2, \dots, u_i) , $(v_{u1}, v_{u2}, \dots, v_{ui})$ and (m_1, m_2, \dots, m_i) , $(v_{m1}, v_{m2}, \dots, v_{mi})$ will be applied respectively for the groups G_1 and G_2 .
- For the groups G_3 and G_4 , their elements are respectively: (p_1, p_2, \dots, p_j) , $(v_{p1}, v_{p2}, \dots, v_{pj})$ and (q_1, q_2, \dots, q_j) , $(v_{q1}, v_{q2}, \dots, v_{qj})$.

Table G_1 : The sum parameters are given by: $G_1S = (N_i(S(V_i)) \quad V_i(S_i))$; $\begin{cases} N_i(S(V_i)) = u_1 + u_2 + \dots + u_i \\ V_i(S_i) = v_{u1} + v_{u2} + \dots + v_{ui} \end{cases}$

Table G_2 : The sum parameters are given by: $G_2S = (N_p(S(V_i)) \quad V_i(S_p))$; $\begin{cases} N_p(S(V_i)) = m_1 + m_2 + \dots + m_i \\ V_i(S_p) = v_{m1} + v_{m2} + \dots + v_{mi} \end{cases}$

Table G_3 : The sum parameters are given by: $G_3S = (N_i(S(V_p)) \quad V_p(S_i))$; $\begin{cases} N_i(S(V_p)) = p_1 + p_2 + \dots + p_j \\ V_p(S_i) = v_{p1} + v_{p2} + \dots + v_{pj} \end{cases}$

Table G_4 : The sum parameters are given by: $G_4S = (N_p(S(V_p)) \quad V_p(S_p))$; $\begin{cases} N_p(S(V_p)) = q_1 + q_2 + \dots + q_i \\ V_p(S_p) = v_{q1} + v_{q2} + \dots + v_{qj} \end{cases}$

We will also give below for each table A_i the expression of the sum parameters.

Table A_1 : The sum parameters are given by: $A_1S = G_4S + G_1S = (N_p(S(V_p)) + N_i(S(V_i)) \quad V_p(S_p) + V_i(S_i))$

Table A_2 : The sum parameters are given by: $A_2S = G_3S + G_2S = (N_p(S(V_i)) + N_i(S(V_p)) \quad V_i(S_p) + V_p(S_i))$

Table A_3 : The sum parameters are given by: $A_3S = G_4S + G_3S = (N_p(S(V_p)) + N_i(S(V_p)) \quad V_p(S_p) + V_p(S_i))$

Table A_4 : The sum parameters are given by: $A_4S = G_2S + G_1S = (N_p(S(V_i)) + N_i(S(V_i)) \quad V_i(S_p) + V_i(S_i))$

2.4 Discovering Two Apparent Properties in the Structure of the Quran

This study was made possible by the discovery of two important properties: the concordance relation and rule 1.

a) Concordance relation

The analysis of the **Table 3**, has revealed important property. The two tables A_1 and A_2 are in concordance if and only if, the value of the sum of the elements of the first column of A_1 (sum of the suras numbers) is equal to the value of the sum of the elements of the second column of A_2 (sum of the number of verses). Based on the expression of the sum parameters and the observed equality (3303), the first mathematical relation can be modelled, under the name of the concordance relation as follows:

$$N_i(S(V_i)) + N_p(S(V_p)) = V_p(S_i) + V_i(S_p) \tag{1}$$

This relation is an important mathematical condition in the organization of the structure of the Quran. The order of the values of the sum parameters of the concordance relation and the order of the tables G_i must satisfy the following condition:

$$V_p(S_i) > N_p(S(V_p)) > N_i(S(V_i)) > V_i(S_p)$$

b) Rule 1

The ratio between the sum of suras numbers with odd verses and the sum of suras numbers with even verses is equal to the ratio between the number of suras with odd verses $S(V_i)$ and the number of suras with even verses $S(V_p)$ which is equal to 9/10. This rule will be explained during the presentation of the four rules.

2.5 Concordance Between the G_i Tables

Knowing that A_1 and A_2 are each composed of two tables G_i , it is necessary to indicate their order which is determined by the application of the concordance. The first column of A_1 composed by suras numbers represents the first columns of G_1 and G_4 , while, the second column of A_2 composed by number of verses represents the second columns of G_2 and G_3 . In the case of existence of a concordance between these four tables, we will have in this order: G_1 in concordance with G_2 or G_3 and G_4 in concordance with G_2 or G_3 . The tables which will verify this property will be designated by the expression of concordant tables. In view of this result, it seemed logical to us to investigate whether there is a concordance between the tables G_1, G_2, G_3 and G_4 which compose A_1 and A_2 , although we know that there is currently no apparent concordance between them. In fact, we thought that the Author of the book would surely have followed up on this relation (Eq.1) by introducing a veiled concordance between the tables, perhaps by transferring an element from a given table to another table, this concordance will appear. The goal is therefore to find this element and to which group does it belong? It will then be called concordance element or concordance key, denoted K having as components $(X_O F)$.

2.6 Concordance Key Location

To determine exactly which table the concordance element $K : (X_O F)$ should belong to, we will analyze the different transfer cases by applying the following conditions:

- 1- $S_i(V_i) = S_p(V_i)$ and $S_i(V_p) = S_p(V_p)$,
- 2- $V_p(S_i) > N_p(S(V_p)) > N_i(S(V_i)) > V_i(S_p)$,
- 3- $N_p(S(V_p)) + N_i(S(V_i)) = V_i(S_p) + V_p(S_i)$
- 4- The eight sum parameters that represent the sums of the columns of the G_i tables are all natural integers (**Table 5**).
- 5- The transfer of the concordance element between the tables A_1 and A_2 cannot be performed because of the concordance relation. The only transfers that can be performed are between $(G_4$ and $G_1)$ or $(G_3$ and $G_2)$.
- 6- If the element $K : (X_O F)$ exists, the order of the tables in concordance is determined by the above conditions. We will have G_1 in concordance with G_2' or G_3' as well as G_4 in concordance with G_2' or G_3' , where G_2' and G_3' are called a derived tables or transformed tables after the displacement of the key.

Case (I): The transfer of element between G_1 and G_4 : ($V_i(S_p)$ and $V_p(S_i)$ are constant)

- Transfer from G_4 to G_1 : the concordance element $K : (X_O F)$ is (Even Even)
 $N_p(S(V_p))$ remains even, greater than $V_i(S_p)$ and less than $V_p(S_i) \Rightarrow$ the derivative of G_4 cannot be in concordance with G_3 or G_2 .
- Transfer from G_1 to G_4 : the concordance element $K : (X_O F)$ is (Odd Odd)
 $N_i(S(V_i))$ becomes even, remains less than $V_p(S_i)$ and $V_i(S_p)$ is odd, \Rightarrow the derivative of G_1 cannot be in concordance with G_3 or G_2 .
We deduce that the key $(X_O F)$ cannot belong to A_1 (G_4 or G_1).

Case (II): The transfer of the concordance element between G_2 and G_3 : ($N_p(S(V_p))$ and $N_i(S(V_i))$ are constants)

- Transfer from G_2 to G_3 : the concordance element $K : (X_O F)$ is (Even Odd)
 $V_p(S_i)$ becomes odd, greater than $N_p(S(V_p))$ and $N_i(S(V_i))$, $V_i(S_p)$ becomes even and strictly less than $N_p(S(V_p))$ and $N_i(S(V_i)) \Rightarrow G_4$ and G_1 cannot be in concordance with G_3' and G_2' .
- Transfer from G_3 to G_2 : the concordance element $K : (X_O F)$ is (Odd Even)
 $V_p(S_i)$ remains even, can be equal to $N_p(S(V_p))$, $N_i(S(V_i))$ remains odd, $V_i(S_p)$ remains odd and can be equal to $N_i(S(V_i))$, $N_p(S(V_p))$ remains even $\Rightarrow G_4$ and G_1 can be in concordance with G_3' and G_2' .

It follows that, if the concordance element $K : (X_O F)$ exists, it must belong to the group $G_3(IP)$, which consists of odd suras and even verses. In this case the displacement of the key (element) will be from G_3 to G_2 . We will then have: G_1 in concordance with G_2' or G_4 in concordance with G_3' . The table G_3' is the table G_3 without the element K and G_2' is the table G_2 with the element K .

3. Methodology

3.1 Methodology Steps

The discovery of the main concordance relation led us to think about the possibility of the existence other hidden secondary concordance relations. After the above analysis we thought that there might be a hidden element belonging to the G_3 table that would act as a key to achieve the required concordance. This prompts us to check whether the structure of the Noble Quran provides such a key and to try to find it, to solve the problem of its authenticity, by adopting a methodology based on the following steps:

Step 1: We will try to find the concordance element in the structure of the Quran. By transferring this element from table G_3 to table G_2 , we determine the two secondary concordance relations, the transformation relations, and we create a concordance between tables (G_1, G'_2) and (G_4, G'_3) . G'_2 and G'_3 are derived tables of G_2 and G_3 . The sum parameters before and after transferring the concordance element between the tables G_3 and G_2 will be defined in two different states: the initial state (original plan) and the final state (transformed plan). The initial state includes the tables G_i with their sum parameters before the transfer. The final state includes the new tables G'_i with their new sum parameters after the transfer. All the sum parameters in both states are therefore presented in **Table 6**.

Step 2: After that, we will try to establish relations between all the parameters namely the new secondary parameters and the other sum parameters. From these, we will identify four fundamental rules used to structure the Noble Quran

Step 3: Mathematical equations will be formulated for all the sum parameters whose values are assumed to be unknown. We determine them in order to prove that the structure of the book is consistent with the original. To do this, we will define a rigorous mathematical procedure using the concordance relations, transformation relations and the four rules. In the final state, the four rules and the two equations relating to the sum of odd suras and the sum of even suras were used to form a system of equations with six unknowns. Solving this system allows obtaining the equations of the sum parameters. Then, the sum parameters of the initial state will be determined using the sum parameters in the final state, the relations of concordance and transformation. The mathematical equations in both the initial and final states are expressed as a function of three Quranic quantities: the number of suras (n) and the components of the concordance element (X_o, F).

Step 4: The number of suras 114 will be determined after the key transfer. As for the number of verses 6236, it will be determined by the four sum parameters of verses.

Table 6 The sum parameters of the tables: G_1, G_2, G_3 and G_4 in the initial state and in the final state (transformed), which are assumed to be unknown

Sum parameters	Initial state		Sum parameters	Final state	
	N	V		N	V
$G_1s(II)$	$N_i(S(V_i))$	$V_i(S_i)$	$G_1s(II)$	$N_i(S(V_i))$	$V_i(S_i)$
$G_2s(PI)$	$N_p(S(V_i))$	$V_i(S_p)$	$G'_2s(PI)$	$N'_p(S(V_i))$	$V'_i(S_p)$
$G_4s(PP)$	$N_p(S(V_p))$	$V_p(S_p)$	$G_4s(PP)$	$N_p(S(V_p))$	$V_p(S_p)$
$G_3s(IP)$	$N_i(S(V_p))$	$V_p(S_i)$	$G'_3s(IP)$	$N'_i(S(V_p))$	$V'_p(S_i)$

3.2 Cases of the Concordance Effect

Before proceeding to study the conditions for implementing the key and its consequences on the structure of the Quran, it seems necessary to clearly highlight the property of concordance in order to understand its purpose. To do this, we will study three practical cases that all verify only this important property (A_1 in concordance with A_2). For each of these cases which are arbitrary, since they do not verify the Quranic criteria, but which allow us to understand the effect of concordance, we will take as an example a table composed of ten elements.

The first column is made up of the first ten integers (from 1 to 10), the second column is made up of natural integers chosen so that the distribution of the four sub-tables is approximately similar to that of the Quranic sub-tables G_i and only respects two conditions: the concordance between A_1 and A_2 ; and the number of even verses must be greater than odd verses. The same names of the Quranic groups have been kept to facilitate the

understanding of the cases. We would like to point out that the cases mentioned below cannot be interpreted as part of the structure of the Holy Quran. Because the Quran presents an infinitely complex structure whose constituent elements are linked to an inaccessible number of properties.

Case I: G_1 in concordance with G_2 or G_4 in concordance with G_3 .

Consider a general table T_G of ten elements (from 1 to 10). According to the parity of the number of its components n and v , we form four tables $G_i : G_1(II), G_2(PI), G_3(IP)$ and $G_4(PP)$. From these four tables, we again form two new tables $A_1 (G_1, G_4)$ and $A_2 (G_2, G_3)$. All the tables are presented together in one **Table 7**. This first case is the most elementary example. If G_1 is in concordance with G_2 or G_4 is in concordance with G_3 , we will have A_1 in concordance with A_2 .

Table 7 G_1 in concordance with G_2 or G_4 in concordance with G_3 . The gray color of the last line of each column includes the sum parameters and the arrows indicate the concordance between them

T_G		$G_1(II)$	$G_2(PI)$	$G_4(PP)$	$G_3(IP)$
n	v	n v	n v	n v	n v
1	4	5 9	6 7	2 4	1 4
2	4	7 11	8 5	4 8	3 6
3	6			10 6	9 6
4	8	12 20	14 12	16 18	13 16
5	9				
6	7				
7	11				
8	5	$A_1(II + PP)$		$A_2(IP + PI)$	
9	6	n v	n v	n v	n v
10	6	2 4	4 8	3 6	6 7
		5 9	7 11	8 5	8 5
		10 6	10 6	9 6	9 6
55	66	28	38	27	28

Case (II): G_1 and G_4 are not in concordance with G_2 and G_3

Table 8 G_1 and G_4 are not in concordance with G_2 and G_3 . The gray color of the last line of each column represents the sum parameters and the arrow indicates the concordance between them.

T_G		$G_1(II)$	$G_2(PI)$	$G_4(PP)$	$G_3(IP)$
n	v	n v	n v	n v	n v
1	2	5 9	6 3	2 4	1 2
2	4	7 11	8 5	4 8	3 12
3	12			10 6	9 6
4	8	12 20	14 8	16 18	13 22
5	9				
6	3				
7	11				
8	5	$A_1(II + PP)$		$A_2(IP + PI)$	
9	6	n v	n v	n v	n v
10	6	2 4	4 8	3 6	12 3
		5 9	7 11	8 5	8 5
		10 6	10 6	9 6	9 6
55	66	28	38	27	28

Consider a general table T_G of ten elements (from 1 to 10). In the same way as the previous example, we will form, from the tables G_i , two tables A_1 and A_2 . All the tables are presented together in the **Table 8**. In this case, the tables G_1, G_2, G_3 and G_4 are not in concordance, while A_1 is in concordance with A_2 . This choice was made to show that A_1 and A_2 can be in concordance without the existence of concordance between the G_i tables. It is sufficient to examine these tables to see that there is no element that can achieve concordance between them. The concordance element was not provided.

Case (III): G_1 and G_4 are not in direct concordance with G_2 and G_3

In this particular case, A_1 is still in concordance with A_2 , but we have made a choice at the level of the elements of T_G to hide the concordance that exists between the tables G_1, G_2, G_3 and G_4 . To this end, we have determined in advance the components of a concordance element from G_3 in such a way that, by selecting it and transferring it to table G_2 , we can achieve the concordance between G_1 and G_2 and at the same time between G_3 and G_4 . The highlighting of this case will be explained and detailed later. Consider a general table T_G of ten elements (from 1 to 10). In the same way as the previous examples, we will form, from the tables G_i , two tables A_1 and A_2 . All the tables are presented together in one **Table 9**. As can be seen, the concordance between the tables G_i does not appear, G_1 is not in concordance with G_2 and G_4 is not in concordance with G_3 .

Table 9 G_1 and G_4 are not in direct concordance with G_2 and G_3 . The gray color of the last line of each column represents the sum parameters and the arrow indicates the concordance between them

T_G		$G_1 (II)$		$G_2 (PI)$		$G_4 (PP)$		$G_3 (IP)$	
n	v	n	v	n	v	n	v	n	v
1	4	5	9	6	3	2	4	1	4
2	4	7	11	8	5	4	8	3	10
3	10	12	20	14	8	10	6	9	6
4	8					16	18	13	20
5	9	$A_1 (II + PP)$		$A_2 (IP + PI)$					
6	3	n	v	n	v	n	v	n	v
7	11	2	4	1	4	3	10	6	3
8	5	4	8	6	3	8	5	9	6
9	6	5	9	8	5	10	6	28	28
10	6	7	11	9	6	27	28		
		10	6	9	6				
55	66	28	38	27	28				

By examining **Table 9**, we see that A_1 and A_2 are in concordance while the G_i tables are not. However, there is an element that can achieve this non-apparent concordance. This is the element (1 4), which was initially planned by the designer of the general table in order to obtain the desired concordance between the G_i tables. The transfer that must be carried out to create the concordance between them, consists of moving the element (1 4) from G_3 to G_2 . (see **Table 10**).

Table 10 The tables G_i after the transfer of the element (1 4) from G_3 to G_2

$G_1 (II)$		$G'_2 (PI)$		$G_4 (PP)$		$G'_3 (IP)$	
n	v	n	v	n	v	n	v
5	9	6	3	2	4	-	-
7	11	8	5	4	8	3	10
		+1	+4	10	6	9	6
12	20	15	12	16	18	12	16

Particularly, this last case (III) has been provided in the structure of the Noble Quran, in which the concordance property is not apparent between the tables: G_1, G_2, G_3 , and G_4 . However, when the tables are

combined two by two (G_1 with G_4 and G_2 with G_3) we find the existence of a concordance property between A_1 and A_2 . The questions that arise are therefore the following:

- Why does the Quran use such a property and what is its purpose?
- Is there an element in the tables of the suras of the Quran that is capable of achieving concordance?
- If this element exists, what is its role and purpose?
- To answer these questions, it is first necessary to prove the existence of the element in the structure of the Noble Quran.

3.3 Necessary Quranic Data

In this part we will present the data necessary for this study. Some data has already been defined in the initial state, while the other data has been deduced from the final state.

a) In the initial state:

- 1- The total number of suras is 114. It is an even number which is a multiple of 19.
- 2- The suras number in G_1 is odd and equal to the suras number in G_2 . Similarly, the suras number in G_4 is even and equal to the suras number in G_3 . We have: $S_i(V_i) = S_p(V_i)$ and $S_i(V_p) = S_p(V_p)$
- 3- The 1st rule
- 4- Concordance relation: $N_i(S(V_i)) + N_p(S(V_p)) = V_p(S_i) + V_i(S_p)$
- 5- Concordance condition : $V_p(S_i) > N_p(S(V_p)) > N_i(S(V_i)) > V_i(S_p)$

b) In the final state:

These data are effective only when the key is activated.

- 1- The chaining condition of the tables.
- 2- The three rules.

4. The FOS Relation

4.1 Definition

Let G_u and G_m be two tables, their sum parameters are given by: $G_u s = (N_u V_u)$ and $G_m s = (N_m V_m)$. If the tables G_u and G_m are in concordance i.e. satisfy the condition $N_u = V_m$, we will then say that G_u and G_m are in the *FOS* relation. The term *FOS* is an abbreviation derived from the name of Sura Fussilat. This property will then be designated by $(G_u \text{ FOS } G_m)$, and we will write:

$$(G_u \text{ FOS } G_m) \Leftrightarrow N_u = V_m \quad (2)$$

This property will be called a secondary concordance relation.

4.2 Investigation of the Methodology Cases

As mentioned above, the author of the book has the possibility to create a direct concordance or an indirect concordance (not apparent) between the tables G_i . For this, we will study in a general way all the cases mentioned in the methodology, which can be determined by the following proposition:

A_1 is in FOS relation with $A_2 \Leftrightarrow G_1$ is in FOS relation with G_2 or G_4 is in FOS relation with G_3 , which can be written in the form:

$$A_1 \text{ FOS } A_2 \Leftrightarrow (G_1 \text{ FOS } G_2) \text{ or } (G_4 \text{ FOS } G_3) \quad (3)$$

According to the definition of the FOS relation, we can write:

$(A_1 \text{ FOS } A_2)$: A_1 is in FOS relation with A_2 if and only if we have:

$$A_1 \text{ FOS } A_2 \Leftrightarrow N_p(S(V_p)) + N_i(S(V_i)) = V_i(S_p) + V_p(S_i).$$

$(G_1 \text{ FOS } G_2)$: G_1 is in FOS relation with G_2 if and only if we have: $G_1 \text{ FOS } G_2 \Leftrightarrow N_i(S(V_i)) = V_i(S_p)$

$(G_4 \text{ FOS } G_3)$: G_4 is in FOS relation with G_3 if and only if we have: $G_4 \text{ FOS } G_3 \Leftrightarrow N_p(S(V_p)) = V_p(S_i)$

Depending on the values of the sum parameters, the main proposition (Eq.3) presents two possibilities:

If it is true (i.e., both implications are true), we will have a direct concordance between the tables (G_1 and G_2) or (G_4 and G_3) which is the first case of the methodology, and we write:

$$\begin{cases} N_i(S(V_i)) = V_i(S_p) \\ N_p(S(V_p)) = V_p(S_i) \end{cases} \Leftrightarrow N_p(S(V_p)) + N_i(S(V_i)) = V_i(S_p) + V_p(S_i)$$

If it is not true (i.e., both implications are not true), we write:

$$\begin{cases} N_p(S(V_p)) \neq V_p(S_i) \\ N_i(S(V_i)) \neq V_i(S_p) \\ N_i(S(V_i)) + N_p(S(V_p)) = V_i(S_p) + V_p(S_i) \end{cases}$$

This means that the tables (G_1 and G_2) or (G_4 and G_3) do not check the FOS relation. This leads us to consider two other possibilities. Either the concordance element exists in G_3 (case three of the methodology), or it does not exist (case two of the methodology) (see **section 3.2**).

4.2.1 The Element ($X_0 F$) Does Not Exist in G_3

If the element $K: (X_0 F)$ does not exist, the tables (G_1 and G_2) or (G_4 and G_3) are not in concordance, we deduce that the main proposition (Eq.3) is not true, (case two of the methodology).

4.2.2 The Element ($X_0 F$) Exists in G_3

We previously assumed the existence of a hidden concordance element $K: (X_0 F)$ belonging to the group G_3 . The transfer of this element to G_2 will result in the creation of a concordance between the tables (G_1 and G'_2) or (G_4 and G'_3). The tables G'_2 and G'_3 are the derivatives of G_2 and G_3 : G'_3 is the table G_3 without the element K , G'_2 is the table G_2 with the element K . The transfer of the element K creates the following conditions:

$$G_2s = \begin{pmatrix} N_p(S(V_i)) & V_i(S_p) \end{pmatrix} \text{ will be transformed into } G'_2s = \begin{pmatrix} N'_p(S(V_i)) & V'_i(S_p) \end{pmatrix}$$

$$G_3s = \begin{pmatrix} N_i(S(V_p)) & V_p(S_i) \end{pmatrix} \text{ will be transformed into } G'_3s = \begin{pmatrix} N'_i(S(V_p)) & V'_p(S_i) \end{pmatrix}$$

G'_3s and G'_2s are the sum parameters of G'_3 and G'_2 . The sum parameters of the tables G_i in the initial state and G'_i in the final state (see **Table 6**) can be written as follows:

$$\text{For table } G_3, \text{ we have: } G_3s = G'_3s + P \Rightarrow \begin{pmatrix} N_i(S(V_p)) & V_p(S_i) \end{pmatrix} = \begin{pmatrix} N'_i(S(V_p)) & V'_p(S_i) \end{pmatrix} + \begin{pmatrix} X_0 & F \end{pmatrix}$$

Which leads to the following equations:

$$N_i(S(V_p)) = N'_i(S(V_p)) + X_0 \tag{4}$$

$$V_p(S_i) = V'_p(S_i) + F \tag{5}$$

$$\text{For table } G_2, \text{ we have: } G'_2s = G_2s + P \Rightarrow \begin{pmatrix} N'_p(S(V_i)) & V'_i(S_p) \end{pmatrix} = \begin{pmatrix} N_p(S(V_i)) & V_i(S_p) \end{pmatrix} + \begin{pmatrix} X_0 & F \end{pmatrix}$$

Which leads to the following equations:

$$N'_p(S(V_i)) = N_p(S(V_i)) + X_0 \tag{6}$$

$$V'_i(S_p) = V_i(S_p) + F \tag{7}$$

The four equations obtained (Eqs. 4 – 7) will be called: “the transformation relations”.

According to the definition of the FOS relation, we can write:

$$(G_1 \text{ FOS } G'_2): G_1 \text{ is in FOS relation with } G'_2 \text{ if and only if we have: } G_1 \text{ FOS } G'_2 \Leftrightarrow N_i(S(V_i)) = V'_i(S_p)$$

$$(G_4 \text{ FOS } G'_3): G_4 \text{ is in FOS relation with } G'_3 \text{ if and only if we have: } G_4 \text{ FOS } G'_3 \Leftrightarrow N_p(S(V_p)) = V'_p(S_i)$$

The transfer of the concordance key leads to use of new tables and new sum parameters, which requires to determining a new proposition derived from the main proposition (Eq.3). We have:

$$A_1 \text{ FOS } A_2 \Leftrightarrow N_p(S(V_p)) + N_i(S(V_i)) = V_i(S_p) + V_p(S_i)$$

$$A_1 \text{ FOS } A_2 \Leftrightarrow N_p(S(V_p)) + N_i(S(V_i)) = V_i(S_p) + F + V_p(S_i) - F$$

$$\text{By using the transformation relations (Eq. 5 and 7): } V_p(S_i) - F = V'_p(S_i) \text{ and } V_i(S_p) + F = V'_i(S_p),$$

$$\text{And by posing } N_i(S(V_i)) = V'_i(S_p) \text{ and } N_p(S(V_p)) = V'_p(S_i)$$

We will have:

$$A_1 \text{ FOS } A_2 \Leftrightarrow G_1 \text{ FOS } G'_2 \text{ or } G_4 \text{ FOS } G'_3 \text{ is true if and only if } N_i(S(V_i)) = V'_i(S_p) \text{ and } N_p(S(V_p)) = V'_p(S_i).$$

We will call the following relations (Eqs. 8, 9): the secondary concordance relations:

$$N_i(S(V_i)) = V'_i(S_p) \tag{8}$$

$$N_p(S(V_p)) = V_p'(S_i) \quad (9)$$

4.3 Determination of the Component F

According to the transformation relation (Eq. 7) and the secondary concordance relation (Eq.8), we have:

$$\begin{cases} V_i'(S_p) = V_i(S_p) + F \\ N_i(S(V_i)) = V_i'(S_p) \end{cases} \Rightarrow N_i(S(V_i)) = V_i(S_p) + F$$

According to the transformation relation (Eq. 5) and the secondary concordance relation (Eq.9), we have:

$$\begin{cases} V_p'(S_i) = V_p(S_i) - F \\ N_p(S(V_p)) = V_p'(S_i) \end{cases} \Rightarrow N_p(S(V_p)) = V_p(S_i) - F$$

The value of F is therefore given by one of these two expressions:

$$F = V_p(S_i) - N_p(S(V_p)) \quad (10)$$

$$F = N_i(S(V_i)) - V_i(S_p) \quad (11)$$

4.4 Application of the FOS Property to the Noble Quran

Now we will apply the FOS property to the noble Quran to determine the second component F of the key $K : (X_0 F)$, while the first component X_0 will be found later by the rule 3.

The component F of the concordance key is calculated using the relations (Eq.10) or (Eq.11) and the values of the sum parameters recorded in **Table 5**, its value is found to be 54. After searching for the hidden concordance element K in table G_3 , where the number of its verses is 54, we found only one element, namely the Sura Fussilat (41 54). After transferring the concordance element K from G_3 to G_2 , we recalculated all the Quranic parameters. They are represented in the following **Tables 11, 12** and **13**.

The terms "even" and "odd" in columns 3 and 4 of **Tables 12** and **13** refer to the parity of the sum of the components of the suras ($N + V$), where the term odd or even is used instead of the values to indicate that it does not correspond to the parity of the sum and to avoid calculating it when adding. For **Table 12**, the column $N_p(S(V_p)) + V_p(S_p)$ must have an even value, if their sum is even, we show its value, otherwise we write the odd word. Similarly, the column $N_i(S(V_p)) + V_p(S_i)$ must have an odd value if their sum is odd, we show its value, otherwise we write the even word. The same procedure was applied to the column $N_i(S(V_i)) + V_i(S_i)$ and $N_p(S(V_i)) + V_i(S_p)$ and to **Table 13**.

Table 11 The Quranic structural parameters of: $G_1(II)$, $G_2'(PI)$, $G_3'(IP)$ and $G_4(PP)$ in the final state. The displacement of sura Fussilat (41 54) from G_3 to G_2 is represented by the grey color, the last line of each column represents the sum parameters and the arrow indicates the concordance between the tables G_i .

$G_1(II)$		$G_2'(PI)$		$G_4(PP)$		$G_3'(IP)$	
27 Suras		28 Suras		30 Suras		29 Suras	
n	v	n	v	n	v	n	v
1	7	6	165	2	286	3	200
9	129	8	75	4	176	5	120
11	123	10	109	14	52	7	206
13	43	12	111	16	128	19	98
15	99	20	135	18	110	21	112
17	111	26	227	22	78	23	118
25	77	36	83	24	64	31	34
27	93	41	54	28	88	37	182
29	69	42	53	30	60	-	-
33	73	42	53	32	30	47	38
35	45	44	59	34	54	49	18

39	75	46	35	38	88	51	60
43	89	48	29	56	96	53	62
45	37	50	45	58	22	55	78
57	29	52	49	64	18	59	24
63	11	54	55	66	12	61	14
81	29	60	13	68	52	65	12
87	19	62	11	70	44	67	30
91	15	76	31	72	28	69	52
93	11	82	19	74	56	71	28
97	5	84	25	78	40	73	20
101	11	86	17	80	42	75	40
103	3	92	21	88	26	77	50
105	5	96	19	90	20	79	46
107	7	100	11	94	8	83	36
111	5	104	9	98	8	85	22
113	5	108	3	102	8	89	30
		110	3	106	4	95	8
				112	4	99	8
				114	6	109	6
1551 1225		1595 1551		1752 1708		1657 1752	

Table 12 Initial state of tables A_3 and A_4 , before the displacement of sura Fussilat. The grey color indicates suras of (Ha-Mim), and the last line of each column represents the sum parameters

$A_3 (PP + IP)$ ($G_4 + G_3$): 60 Suras				$A_4 (II + PI)$ ($G_1 + G_2$): 54 Suras			
n	v	$N_p(S(V_p)) + V_p(S_p)$	$N_i(S(V_p)) + V_p(S_i)$	n	v	$N_i(S(V_i)) + V_i(S_i)$	$N_i(S(V_i)) + V_p(S_i)$
2	286	288	Even	1	7	8	Even
3	200	odd	203	6	165	odd	171
4	176	180	Even	8	75	odd	83
5	120	odd	125	9	129	138	Even
7	206	odd	213	10	109	odd	119
14	52	66	Even	11	123	134	Even
16	128	144	Even	12	111	odd	123
18	110	128	Even	13	43	56	Even
19	98	odd	117	15	99	114	Even
21	112	odd	133	17	111	128	Even
22	78	100	Even	20	135	odd	155
23	118	odd	141	25	77	102	Even
24	64	88	Even	26	227	odd	253
28	88	116	Even	27	93	120	Even
30	60	90	Even	29	69	98	Even
31	34	odd	65	33	73	106	Even
32	30	62	Even	35	45	80	Even

34	54	88	Even	36	83	odd	119
37	182	odd	219	39	75	114	Even
38	88	126	Even	40	85	odd	125
41	54	odd	95	42	53	odd	95
47	38	odd	85	43	89	132	Even
49	18	odd	67	44	59	odd	103
51	60	odd	111	45	37	82	Even
53	62	odd	115	46	35	odd	81
55	78	odd	133	48	29	odd	77
56	96	152	Even	50	45	odd	95
58	22	80	Even	52	49	odd	101
59	24	odd	83	54	55	odd	109
61	14	odd	75	57	29	86	Even
64	18	82	Even	60	13	odd	73
65	12	odd	77	62	11	odd	73
66	12	78	Even	63	11	74	Even
67	30	odd	97	76	31	odd	107
68	52	120	Even	81	29	110	Even
69	52	odd	121	82	19	odd	101
70	44	114	Even	84	25	odd	109
71	28	odd	99	86	17	odd	103
72	28	100	Even	87	19	106	Even
73	20	odd	93	91	15	106	Even
74	56	130	Even	92	21	odd	113
75	40	odd	115	93	11	104	Even
77	50	odd	127	96	19	odd	115
78	40	118	Even	97	5	102	Even
79	46	odd	125	100	11	odd	111
80	42	122	Even	101	11	112	Even
83	36	odd	119	103	3	106	Even
85	22	odd	107	104	9	odd	113
88	26	114	Even	105	5	110	Even
89	30	odd	119	107	7	114	Even
90	20	110	Even	108	3	odd	111
94	8	102	Even	110	3	odd	113
95	8	odd	103	111	5	116	Even
98	8	106	Even	113	5	118	Even
99	8	odd	107				
102	8	110	Even				
106	4	110	Even				
109	6	odd	115				
112	4	116	Even				
114	6	120	Even				
3450	3514	3460	3504	3105	2722	2776	3051

Table 13 Final state of tables A'_3 and A'_4 , after the displacement of sura Fussilat. The grey color indicates the suras of (Ha-Mim), the last line of each column represents the sum parameters, the arrows indicate the concordance between the tables

$A'_3 (PP + IP)$ ($G_4 + G'_3$): 59 Suras				$A'_4 (II + PI)$ ($G_1 + G'_2$): 55 Suras			
n	v	$N_p(S(V_p))$ + $V_p(S_p)$	$N'_i(S(V_p))$ + $V'_p(S_i)$	n	v	$N_i(S(V_i))$ + $V_i(S_i)$	$N'_i(S(V_p))$ + $V'_p(S_i)$
2	286	288	Even	1	7	8	Even
3	200	-	203	6	165	odd	171
4	176	180	Even	8	75	odd	83
5	120	-	125	9	129	138	Even
7	206	odd	213	10	109	odd	119
14	52	66	Even	11	123	134	Even
16	128	144	Even	12	111	odd	123
18	110	128	Even	13	43	56	Even
19	98	odd	117	15	99	114	Even
21	112	odd	133	17	111	128	Even
22	78	100	Even	20	135	odd	155
23	118	odd	141	25	77	102	Even
24	64	88	Even	26	227	odd	253
28	88	116	Even	27	93	120	Even
30	60	90	Even	29	69	98	Even
31	34	odd	65	33	73	106	Even
32	30	62	Even	35	45	80	Even
34	54	88	Even	36	83	odd	119
37	182	odd	219	39	75	114	Even
38	88	126	Even	40	85	odd	125
				41	54	odd	95
47	38	odd	85	42	53	odd	95
49	18	odd	67	43	89	132	Even
51	60	odd	111	44	59	odd	103
53	62	odd	115	45	37	82	Even
55	78	odd	133	46	35	odd	81
56	96	152	Even	48	29	odd	77
58	22	80	Even	50	45	odd	95
59	24	odd	83	52	49	odd	101
61	14	odd	75	54	55	odd	109
64	18	82	Even	57	29	86	Even
65	12	odd	77	60	13	odd	73
66	12	78	Even	62	11	odd	73
67	30	odd	97	63	11	74	Even
68	52	120	Even	76	31	odd	107
69	52	odd	121	81	29	110	Even
70	44	114	Even	82	19	odd	101

71	28	odd	99	84	25	odd	109
72	28	100	Even	86	17	odd	103
73	20	odd	93	87	19	106	Even
74	56	130	Even	91	15	106	Even
75	40	odd	115	92	21	odd	113
77	50	odd	127	93	11	104	Even
78	40	118	Even	96	19	odd	115
79	46	odd	125	97	5	102	Even
80	42	122	Even	100	11	odd	111
83	36	odd	119	101	11	112	Even
85	22	odd	107	103	3	106	Even
88	26	114	Even	104	9	odd	113
89	30	odd	119	105	5	110	Even
90	20	110	Even	107	7	114	Even
94	8	102	Even	108	3	odd	111
95	8	odd	103	110	3	odd	113
98	8	106	Even	111	5	116	Even
99	8	odd	107	113	5	118	Even
102	8	110	Even				
106	4	110	Even				
109	6	odd	115				
112	4	116	Even				
114	6	120	Even				
3409	3460	3460	3409	3146	2776	2776	3146

The values of sum parameters of different tables in the initial state are summarized in **Table 14**. As can be seen, the parameters marked in gray color between A_1 and A_2 show that the tables are in concordance. Likewise, the values of sum parameters of different tables in the final state, after the transfer of sura Fussilat, are summarized in represented in **Table 15**. As we can see, $G_4(PP)$ and $G_3(IP)$ verify the *FOS* property at the same time as $G_1(II)$ and $G_2'(PI)$. The number of elements of the G_i tables (G_1, G_2, G_3 and G_4) in the initial state and in the final state are presented in the **Table 16**.

Table 14 Sum parameters of the tables ($G_1, G_2, G_3, G_4, A_1, A_2, A_3, A_4$ and T_G) and their values in the initial state.

Sum parameters	N	V
$G_4s(PP)$	1752	1708
$G_3s(IP)$	1698	1806
$G_1s(II)$	1551	1225
$G_2s(PI)$	1554	1497
$A_1s(II + PP)$	3303	2933
$A_2s(PI + IP)$	3252	3303
$A_3s(PP + IP)$	3450	3514
$A_4s(II + PI)$	3105	2722
T_G	6555	6236

Table 15 Sum parameters of the tables ($G_1, G'_2, G'_3, G_4, A_1, A_2, A'_3, A'_4$ and T_G) and their values in the final state

Sum parameters	N	V
G_4s (PP)	1752	1708
G'_3s (IP)	1657	1752
G_1s (II)	1551	1225
G'_2s (PI)	1595	1551
A_1s (II + PP)	3303	2933
A_2s (PI + IP)	3252	3303
A'_3s (PP + IP)	3409	3460
A'_4s (II + PI)	3146	2776
T_G	6555	6236

Table 16 The number of elements of the G_i tables (G_1, G_2, G_3 and G_4) in the initial state and in the final state

Tables	Number of elements	
	Initial state	Final state
G_1	$S_i(V_i) = 27$	$S_i(V_i) = 27$
G_2 or G'_2	$S_p(V_i) = 27$	$S'_p(V_i) = 28$
G_3 or G'_3	$S_i(V_p) = 30$	$S'_i(V_p) = 29$
G_4	$S_p(V_p) = 30$	$S_p(V_p) = 30$

5. Rules Revealed by the Transfer of Sura Fussilat

After performing the first steps of the methodology, namely determining the concordance key, the secondary concordance relations and the transformation relations, the author of the book creates three other rules in order to complete the system of equations. Some of these rules do not appear in the structure of the book, especially in the initial state, with the exception of the first rule. The rest of the rules can be observed after the discovery and implementation of the key, i.e. in the final state. These relations are simple mathematical equations which link the parameters together, either in the initial state, or in the final state (transformed), or between the two states. In this study, these relations will be called "the Rules". We will then provide the necessary details regarding their choice.

5.1 Rule 1 (In the Initial State)

Announcement

The ratio between the sum of the suras numbers with odd verses and the sum of the suras numbers with even verses is equal to the ratio between the number of suras with odd verses $S(V_i)$ and the number of suras with even verses $S(V_p)$.

$$\frac{N_i(S(V_i)) + N_p(S(V_i))}{N_i(S(V_p)) + N_p(S(V_p))} = \frac{S(V_i)}{S(V_p)} = \frac{9}{10} \tag{12}$$

Consequences

If n is the total number of suras and it is even, then according to the rule 1:

$$\begin{aligned} \text{We have: } & \begin{cases} S(V_i) + S(V_p) = n \\ S(V_i)/S(V_p) = 9/10 \end{cases} \Rightarrow \begin{cases} S(V_p) = \frac{10n}{19} \\ S(V_i) = \frac{9n}{19} \end{cases} \\ \text{and: } & \begin{cases} N(S(V_i)) + N(S(V_p)) = \frac{n(n+1)}{2} \\ N(S(V_i))/N(S(V_p)) = \frac{9}{10} \end{cases} \Rightarrow \begin{cases} S(V_p) = \left(\frac{2}{n+1}\right) N(S(V_p)) \\ S(V_i) = \left(\frac{2}{n+1}\right) N(S(V_i)) \end{cases} \Rightarrow \begin{cases} N(S(V_p)) = \frac{5n(n+1)}{19} \\ N(S(V_i)) = \frac{9n(n+1)}{38} \end{cases} \Rightarrow \frac{S(V_i)}{N(S(V_i))} = \frac{S(V_p)}{N(S(V_p))} \end{aligned}$$

5.2 Rule 2 (Relation Between the Initial and the Final States)

Announcement

The difference between the square of the number of suras with even verses $S(V_p)$ and the square of the number of suras with odd verses $S(V_i)$ in the initial state is equal to the difference between the sum of the even verses V_p' and the sum of the odd verses V_i' in the final state (transformed).

$$S(V_p)^2 - S(V_i)^2 = V_p' - V_i' \quad (13)$$

We can write the second rule in another form, using secondary concordance relations (Eq.8) and (Eq.9) as follow:

$$\begin{cases} V_i'(S_p) = N_i(S(V_i)) \\ V_p'(S_i) = N_p(S(V_p)) \end{cases} \Rightarrow \begin{cases} V_p' = V_p(S_p) + V_p'(S_i) = V_p(S_p) + N_p(S(V_p)) \\ V_i' = V_i(S_i) + V_i'(S_p) = V_i(S_i) + N_i(S(V_i)) \end{cases}$$

Rule 2 will become in the initial state:

$$S(V_p)^2 - S(V_i)^2 = N_p(S(V_p)) - N_i(S(V_i)) + V_p(S_p) - V_i(S_i) \quad (14)$$

5.3 Rule 3 (In the Final State: Transformed)

Announcement

In the transformed state, the difference between the sum of the even verses $V_p(S_p)$ and the sum of the odd verses $V_i'(S_p)$ of the even suras is equal to the difference between the sums of their numbers.

$$V_p(S_p) - V_i'(S_p) = N_p(S(V_p)) - N_i'(S(V_i)) \quad (15)$$

The rule 3 can be used to find the mathematical expression of second component X_0 of concordance element $K: (X_0, F)$, by writing the relation (Eq. 15) in the initial state as follows:

$$X_0 = V_i(S_p) - V_p(S_p) + N_p(S(V_p)) - N_p(S(V_i)) + F \quad (16)$$

After the compensation of the values in (Eq.16), we get $X_0 = 41$, which represents the sura number of Fussilat.

5.4 Rule 4 (In the Final State: Transformed)

Announcement

In the transformed state, the difference between the sums of even numbers $N_p(S(V_p))$ and odd numbers $N_i'(S(V_p))$ of suras having even verses is equal to the sum of the components of sura Fussilat (Eq. 17).

$$X_0 + F = N_p(S(V_p)) - N_i'(S(V_p)) \quad (17)$$

5.5 Methods for Discovering the Four Quranic Rules

In this part, we will give the necessary explanations concerning the discovery of these rules. To determine the relations that exist between the different Quranic sum parameters between $G_{2S}(PI)$ and $G_{3S}(IP)$ with $G_{1S}(II)$ and $G_{4S}(PP)$, which are represented in **Table 6**, we tried to find the links that could exist between their values. For this we will use the two secondary concordance relations (Eq. 8) and (Eq. 9), the two relations of component F (Eq.10) and (Eq.11) as well as the sums of the even numbers N_p and the odd numbers N_i of suras.

Relation between the sum parameters: $N_p(S(V_i))$ and $N_i(S(V_p))$ with $N_i(S(V_i))$ and $N_p(S(V_p))$

Let n be the total number of even and odd integers from 1 to n with n even. Since n is an even parameter that appears in all equations, we will calculate the sum of even and odd numbers based on this parameter. We can write the sum N of the n natural numbers as follows:

$$N = 1 + 2 + 3 + \dots + (n - 1) + n = n + n - 1 + \dots + 2 + 1 \Rightarrow N = \frac{n(n + 1)}{2}$$

The sum of all the even and odd numbers of suras is: $N_p + N_i = n(n + 1)/2$, where:

The sum of the odd numbers is: $N_i = n^2/4$, while the sum of the even numbers is: $N_p = n(n + 2)/4$.

We write:

$$N_i = N_i(S(V_i)) + N_i(S(V_p)) = n^2/4 \quad \text{and} \quad N_p = N_p(S(V_p)) + N_p(S(V_i)) = n(n + 2)/4$$

The difference between the sums of even and odd numbers is: $N_p - N_i = n/2$

$$\Rightarrow N_p(S(V_p)) + N_p(S(V_i)) - N_i(S(V_i)) - N_i(S(V_p)) = n/2$$

In this last equality, we transpose $n/2$ and we add $(F - F)$:

$$N_p(S(V_p)) - N_i(S(V_p)) + N_p(S(V_i)) - N_i(S(V_i)) - n/2 + (F - F) = 0$$

$$N_p(S(V_p)) - N_i(S(V_p)) - F + N_p(S(V_i)) - N_i(S(V_i)) - (n/2 - F) = 0$$

By comparing with the values of the Quranic sum parameters (see **Table 5**), we find:

$$1752 - 1698 - 54 = 0 \quad \text{and} \quad 1554 - (57 - 54) - 1551 = 0$$

we conclude that this equation can have these solutions:

$$N_p(S(V_p)) - N_i(S(V_p)) - F = 0$$

$$N_p(S(V_i)) - N_i(S(V_i)) - (n/2 - F) = 0$$

Therefore, we obtain:

$$N_i(S(V_p)) = N_p(S(V_p)) - F \tag{18}$$

$$N_p(S(V_i)) = N_i(S(V_i)) + (n/2 - F) \tag{19}$$

Relation between the sum parameters: $V_i(S_p)$ and $V_p(S_i)$ with $N_i(S(V_i))$ and $N_p(S(V_p))$

According to the two relations (Eq. 10) and (Eq. 11), we have:

$$V_i(S_p) = N_i(S(V_i)) - F$$

$$V_p(S_i) = N_p(S(V_p)) + F$$

By replacing the expressions of the following sum parameters $N_i(S(V_p))$, $N_p(S(V_i))$, $V_i(S_p)$ and $V_p(S_i)$ by the equations (Eq. 18), (Eq. 19), (Eq. 10) and (Eq. 11), respectively in the **Table 6**, we will obtain the **Table 17**.

Table 17 The sum parameters of the tables G_2 and G_3 as function of $N_i(S(V_i))$, $N_p(S(V_p))$, n and F

Tables	Sum Parameters	N (1 st column)	V (2 nd column)
G_1	$G_1S (II)$	$N_i((S(V_i)))$	$V_i(S_i)$
G_2	$G_2S (PI)$	$N_i((S(V_i))) + (n/2 - F)$	$N_i((S(V_i))) - F$
G_4	$G_4S (PP)$	$N_p(S(V_p))$	$V_p(S_p)$
G_3	$G_3S (IP)$	$N_p(S(V_p)) - F$	$N_p(S(V_p)) + F$

Looking at **Table 17**, we notice that only the parameters $V_i(S_i)$ and $V_p(S_p)$ have not been determined. We then thought that these parameters would be determined using other rules that will be defined.

How to discover the rule 1

we have :
$$\begin{cases} N(S(V_i)) + N(S(V_p)) = n(n + 1)/2 \\ S(V_i) + S(V_p) = n \end{cases}$$
 , n designates the number of suras.

After compensation, we will have:

$$N(S(V_i)) + N(S(V_p)) = S(V_i) \left(\frac{(S(V_i)) + S(V_p) + 1}{2} \right) + S(V_p) \left(\frac{(S(V_i)) + S(V_p) + 1}{2} \right)$$

By replacing the sum parameters by their values (see **Table 5**), and taking into account the correspondence and symmetry used by the structure of the Noble Quran, we can extract the two following expressions which can be a solution of the above equality:

$$N(S(V_i)) = S(V_i) \left(\frac{(S(V_i)) + S(V_p) + 1}{2} \right) \quad \text{and} \quad N(S(V_p)) = S(V_p) \left(\frac{(S(V_i)) + S(V_p) + 1}{2} \right)$$

From here we conclude the following ratio, which represents the rule 1:

$$\frac{N(S(V_i))}{N(S(V_p))} = \frac{S(V_i)}{S(V_p)} = \frac{9}{10}$$

How to discover the rule 2

We have $V = V_p + V_i$, and we can write:

$$V_p = V_p(S_p) + V_p(S_i) \quad \text{and} \quad V_i = V_i(S_i) + V_i(S_p)$$

By making the difference between the number of even and odd verses and using the relations of the component F (Eq.10) and (Eq.11), we will have:

$$V'_p - V'_i = V_p(S_p) + N_p(S(V_p)) - V_i(S_i) - N_i(S(V_i))$$

We replace the sum parameters with their values:

$$1708 + 1752 - 1225 - 1551 = 684$$

We note that:

$$684 = 114 \times \frac{114}{19} = n \times \frac{n}{19}$$

n is the total number of suras. We write:

$$V'_p - V'_i = \frac{n^2}{19}$$

According to the consequences of rule 1: $S(V_p) = \frac{10n}{19}$ and $S(V_i) = \frac{9n}{19}$, We write:

$$\begin{cases} S(V_p) - S(V_i) = \frac{n}{19} \\ S(V_p) + S(V_i) = n \end{cases}$$

After replacement :

$$V'_p - V'_i = (S(V_p) + S(V_i)) \times (S(V_p) - S(V_i))$$

We deduce the rule 2:

$$S(V_p)^2 - S(V_i)^2 = V'_p - V'_i$$

How to discover the rule 3

As for rule 3, We thought that it serves to determine at the same time the parameters X_0 and $V_p(S_p)$.

Table 18 The sum parameters of the tables G'_2 and G'_3 as function of $N_i(S(V_i))$, $N_p(S(V_p))$, F and X_0 , after the transfer of the concordance key $K: (X_0 F)$ from G_3 to G_2 .

Tables	Sum Parameters	N (1 st column)	V (2 nd column)
G_1	$G_1s(II)$	$N_i((S(V_i)))$	$V_i(S_i)$
G'_2	$G'_2s(PI)$	$N_i((S(V_i)))+(n/2 - F + X_0)$	$N_i((S(V_i)))$
G_4	$G_4s(PP)$	$N_p(S(V_p))$	$N_p(S(V_p)) - (n/2 - F + X_0)$
G'_3	$G'_3s(IP)$	$N_p(S(V_p)) - (F + X_0)$	$N_p(S(V_p))$

We note in the transformed table (**Table 18**) the following:

$$N_p(S(V_p)) - N'_p(S(V_i)) = N_p(S(V_p)) - N_i((S(V_i))) - (n/2 - F + X_0)$$

We set $t = (n/2 - F + X_0)$

$$N_p(S(V_p)) - N'_p(S(V_i)) = N_p(S(V_p)) - N_i((S(V_i))) - t$$

We transpose $N_i((S(V_i)))$

$$N_p(S(V_p)) - N'_p(S(V_i)) + N_i((S(V_i))) = N_p(S(V_p)) - t$$

According to the two secondary concordance relations (Eq. 8) and (Eq. 9), we have:

$$N_p(S(V_p)) - N'_p(S(V_i)) + V'_i(S_p) = V'_p(S_i) - t$$

We add to both member: $-V_p(S_p)$.

$$N_p(S(V_p)) - N'_p(S(V_i)) + V'_i(S_p) - V_p(S_p) = V'_p(S_i) - t - V_p(S_p)$$

Replacing the parameters of the first part of equation by their values we find:

$$1752 - 1595 + 1551 - 1708 = 0$$

The equality is verified by the values of the sum parameters. The second part of equation is therefore zero.

$$V_p'(S_i) - t - V_p(S_p) = 0$$

$V_p(S_p) = V_p'(S_i) - t = N_p(S(V_p)) - (n/2 - F + X_0)$ which is also verified by the values of the sum parameters. This allowed us to state the third rule by checking that the first part of the equality is zero, which gives us:

$$N_p(S(V_p)) - N_p'(S(V_i)) = V_p(S_p) - V_i'(S_p)$$

How to discover the rule 4

As for rule 4, it appears clearly in **Table 18**, and we observe its effect in the last line G_3' of the 1st column.

$$N_i'(S(V_p)) = N_p(S(V_p)) - (X_0 + F)$$

We deduce the rule 4:

$$X_0 + F = N_p(S(V_p)) - N_i'(S(V_p))$$

Where: $N_p(S(V_p)) = 1752$, $N_i'(S(V_p)) = 1657$, $X_0 = 41$ and $F = 54$

$$\Rightarrow X_0 + F = N_p(S(V_p)) - N_i'(S(V_p)) = 95 = 5 \times 19$$

Table 18 highlights the effect of the key. Once the key is moved, we notice a surprising interaction between the parameters of the structure, a fabulous structure emerges. This allows us to determine four rules except the first one which already existed in the initial state.

6. Discussion of Results

By examining this study more closely, we see that the Fussilat key $K: (X_0 F)$ is the crucial element in highlighting mathematical equations. Without its primordial existence, this study could not be undertaken. Thus, the definition and choice of the components of the key made it possible to find all the necessary relations which lead to the establishment of the system of equations. Until now, this study has only allowed us to determine the value of F . The value of X_0 , it is deduced from a table G_3 . Because the only element whose second component is equal to 54 is (41 54). The value of X_0 can also be given by the expression (Eq.16), deduced from the rule 3.

Extraordinary, it is with great astonishment that we discover a book, decried by most western scholars, revealing a divinely organized structure, thanks to one sura, which presents a unique solution to this study. This incredible result is sura Fussilat. Its name, according to some Muslim scholars, generally means "expose in detail, tailor, ...", but, according to this study, its title also means "separate". That is to say, she was separated from her Ha-Mim family. This sura, incredibly, is the only sura of the Ha-Mim family signaled by so many indices. It is the only sura which has an even number of verses, unlike other suras of Ha-Mim which contain an odd number of verses, this is an expressive index that makes us wonder why it has this difference with its sisters?

It was therefore separated from the G_3 (or A_3) and transferred to G_2 (or A_4). Its position and its particularity in the Quran are remarkable, we find it with a group of suras which follow one another in time (chronological order from 59 to 66,) and consecutive in the order of classification (order of collection from 39 to 46), (see **Table 19, 20**) During the establishment of **Tables**, it was found that all the suras which follow each other chronologically and at the same time in the order of collection are located in table A_4 except sura Fussilat (41,54) which is in table A_3 !

Isn't it exceptional that the author of the book points out this difference to us and that he invites us to put Fussilat in its place, by separating it from table A_3 and inserting it in table A_4 where it joins its family?

Table 19 A part of table A₃ and A₄ showing the order of classification of the Ha-Mim family to which sura Fussilat should be belong

Part of the table A ₄			Part of the table A ₃		
36	Ya-Sin	83			
39	Az-Zumar	75	38	Sad	88
40	Ghafir	85	41	Fussilat	54
42	Ash-Shuraa	53	47	Muhammad	38
43	Az-Zukhruf	89			
44	Ad-Dukhan	59			
45	Al-Jathiyah	37			
46	Al-Ahqaf	35			
48	Al-Fath	29			

Table 20 All suras of the Ha-Mim family, which follow each other chronologically and consecutively in classification, after adding the sura Fussilat.

Chronological Order	Family	Number (n)	Suras Name	Number of verses (v)
59	/	39	Az-Zumar	75
60	Ha-Mim	40	Ghafir	85
61	Ha-Mim	41	Fussilat	54
62	Ha-Mim	42	Ash-Shuraa	53
63	Ha-Mim	43	Az-Zukhruf	89
64	Ha-Mim	44	Ad-Dukhan	59
65	Ha-Mim	45	Al-Jathiyah	37
66	Ha-Mim	46	Al-Ahqaf	35

It is therefore thanks to the wisdom of the author of the Holy book, who provided us with the concordance property (Eq.1) which was the essential argument for the discovery of the key and the four rules. But without the existence of this voluntarily planned key (concordance element), we will not know this unexpected result which is the authenticity of the Noble Quran. Fussilat is a significant, distinctive sura. Its name and its characteristics are in perfect agreement with the role attributed to it in the organization and harmony of the structure of the Noble Quran. Which means that his choice was made with full knowledge of the facts! The role assigned to it as well as the indices which characterize it, even if they are all supposed to be the result of coincidences, clearly show that the choice of Fussilat, with all the weight of its significant and decisive connotation, rules out any inclination to believe that this solution is the result of coincidences. It is clear that the entire structure of the Quran is based on the choice of sura Fussilat, considered as an unapparent key revealing a precise mathematical organization. God chose this method to show us that he is the only guarantor of the preservation of the Quran and to confirm to us in particular the absolute certainty in verse 9 of suras (15: 99) and in verse 19 of suras (75: 40). Therefore, there is only one answer, the Quran comes from God, the Almighty, the Wise.

AL-Hijr (15: 99), verse 9: "It is certainly We who have revealed the reminder, and it is certainly We who will preserve it."

Al-Qiyamah (75: 40), verse 19: "Then it is surely upon Us to make it clear to you."

We concluded that:

- 1) The choice of this sura with its scientific evidence, its confirmation by signs and the resulting organization allows us to say that it was wanted, planned and decided from the beginning by the author of this Noble book which is God the greatest.

- 2) This Author also tells us that it is he who will take care of the preservation, the collection of the Quran and that it is he who will expose it and bring it to light.
- 3) Who is this Author who can predict all this and who possesses these qualities beyond the reach of ordinary mortals, and who guarantees us in a certain and absolute way that these predictions will undoubtedly come true?
- 4) No one has the science to predict the future and program code that will stand the test of time and achieve all the predictions announced!
- 5) Fussilat is a sura with multiple roles :its first effect is to reveal the homogeneity between the tables, which was initially hidden, and this thanks to its integration into the Ha-Mim family.
- 6) Then unsuspected relations between the sum parameters are then revealed thanks to the consistency established between these tables.
- 7) Finally, very important rules although pre-existing are highlighted and clearly formulated.

7. Setting up the System of Equations and Its Resolution

According to step 3 of the methodology, the system of equations of six unknowns, in the final state, will be determined below.

7.1 Statement of Problem Data

To do this, we will use the data in the initial state and the final (transformed) state as shown in **Table 5**, as follows:

- 1) The data of the structure of the Noble Quran (initial and final states)
- 2) The four tables of suras G_i ,
- 3) The principal and secondary concordance relations (Eqs. 1, 8, 9),
- 4) The transformation relations (Eqs. 4, 5, 6, 7),
- 5) The four rules (Eqs. 12, 13, 15, 17),
- 6) The Fussilat key $K: (X_0 F)$.

7.2 Establishment of a System of Equations in Final State

We have:
$$\begin{cases} N_i = N_i(S(V_i)) + N_i(S(V_p)) = n^2/4 \\ N_p = N_p(S(V_p)) + N_p(S(V_i)) = n(n+2)/4 \end{cases}$$

By replacing the secondary concordance relations (Eq. 8) and (Eq. 9), we obtain :

$$\begin{cases} V'_i(S_p) + N'_i(S(V_p)) = n^2/4 \\ V'_p(S_i) + N'_p(S(V_i)) = n(n+2)/4 \end{cases}$$

By using the transformation relations (Eq. 4) and (Eq. 6), we obtain:

$$\begin{cases} N'_i(S(V_p)) = N_i(S(V_p)) - X_0 \\ N'_p(S(V_i)) = N_p(S(V_i)) + X_0 \end{cases}$$

we finally get:

$$V'_i(S_p) + N'_i(S(V_p)) = -X_0 + \frac{n^2}{4} \quad (20)$$

$$V'_p(S_i) + N'_p(S(V_i)) = X_0 + \frac{n(n+2)}{4} \quad (21)$$

Application of the rule 1 (Eq. 12) gives:

$$\Rightarrow N_i(S(V_p)) + N_p(S(V_p)) = \frac{10}{9}(N_i(S(V_i)) + N_p(S(V_i)))$$

And knowing that the consequences of the rule 1 are:
$$\begin{cases} N_i(S(V_p)) + N_p(S(V_p)) = \frac{5n(n+1)}{19} \\ N_i(S(V_i)) + N_p(S(V_i)) = \frac{9n(n+1)}{38} \end{cases}$$

$$\Rightarrow N_i(S(V_p)) + N_p(S(V_p)) = 5n(n + 1)/19$$

By using the transformation relation (Eq.4) and secondary concordance relation (Eq.9), we obtain:

$$N'_i(S(V_p)) + V'_p(S_i) = \frac{(5n(n + 1) - 19 X_0)}{19} \tag{22}$$

Application of the rule 2 (Eq. 13) gives:

$$V'_p - V'_i = \frac{n^2}{19}$$

Where: $V'_p = V_p(S_p) + V'_p(S_i)$ and $V'_i = V_i(S_i) + V'_i(S_p)$

Finally, we obtain:

$$(V_p(S_p) + V'_p(S_i)) - (V_i(S_i) + V'_i(S_p)) = \frac{n^2}{19} \tag{23}$$

Application of the rule 3 gives:

By replacing the secondary concordance relations (Eq. 9) in the rule 3, we obtain :

$$V_p(S_p) - V'_i(S_p) = V'_p(S_i) - N'_p(S(V_i)) \tag{15}$$

Application of the rule 4 gives:

By replacing the secondary concordance relations (Eq. 9) in the rule 4, we obtain :

$$V'_p(S_i) - N'_i(S(V_p)) = X_0 + F \tag{17}$$

7.3 Resolution of the System of Equations

We obtain, in the final state, a system composed of 6 equations with six unknowns of sum parameters. Solving the equations (Eqs. 15, 17, 20, 21, 22, 23) leads to obtaining 6 solutions that depend on three Quranic quantities: the number of suras (n) and the components of the concordance element (X_0 F). The six sum parameters in the final (transformed) state, $N'_i(S(V_p))$; $N'_p(S(V_i))$; $V'_i(S_p)$, $V'_p(S_i)$, $V_i(S_i)$ and $V_p(S_p)$, are determined as follows :

Using Eqs (17, 22):

$$N'_i(S(V_p)) = (5n(n + 1) - 19(2X_0 + F))/38 \tag{24}$$

Using Eqs (17, 24):

$$V'_p(S_i) = (19F + 5n(n + 1))/38 \tag{25}$$

Using Eqs (20, 24):

$$V'_i(S_p) = (9n^2 - 10n + 38F)/76 \tag{26}$$

Using Eqs (21, 25):

$$N'_p(S(V_i)) = (9n^2 + 28n + 76X_0 - 38F)/76 \tag{27}$$

Using Eqs (15, 25, 26, 27):

$$V_p(S_p) = (5n^2 - 14n + 57F - 38X_0)/38 \tag{28}$$

Using Eqs (23, 25, 26, 28):

$$V_i(S_i) = (7n^2 - 8n + 114F - 76X_0)/76 \tag{29}$$

Using the six sum parameters in the final state, the four transformation relations (Eq.4, 5, 6, 7) and the secondary concordance relation (Eq.8, 9), we can obtain eight equations for the sum parameters in the initial state, which are as follows: $N_i(S(V_i))$; $N_p(S(V_p))$; $N_i(S(V_p))$; $N_p(S(V_i))$; $V_i(S_i)$; $V_p(S_p)$; $V_i(S_p)$ and $V_p(S_i)$.

Using Eqs (4, 24):

$$N_i(S(V_p)) = (5n(n + 1) - 19F)/38 \tag{30}$$

Using Eq (25):

$$N_p(S(V_p)) = (19F + 5n(n + 1))/38 \quad (31)$$

Using Eqs (5, 25):

$$V_p(S_i) = (57F + 5n(n + 1))/38 \quad (32)$$

Using Eq (26):

$$N_i(S(V_i)) = (9n^2 - 10n + 38F)/76 \quad (33)$$

Using Eqs (7, 26):

$$V_i(S_p) = (9n^2 - 10n - 38F)/76 \quad (34)$$

Using Eqs (6, 27):

$$N_p(S(V_i)) = (9n^2 + 28n - 38F)/76 \quad (35)$$

Using Eq (28):

$$V_p(S_p) = (5n^2 - 14n + 57F - 38X_0)/38 \quad (28)$$

Using Eq (29):

$$V_i(S_i) = (7n^2 - 8n + 114F - 76X_0)/76 \quad (29)$$

Finally, we get all the equations, the eight sum parameters in the initial state and the eight sum parameters in the final state, as a function of the Quranic quantities n , X_0 and F .

The sum parameters were calculated using the above mathematical expressions in order to validate them, using as data, the suras number $n = 114$ and the components of sura Fussilat ($X_0 F$) = (41 54).

For example:

$$N_i(S(V_i)) = (9 \times 114 \times 114 - 10 \times 114 + 38 \times 54)/76 = 1551$$

$$V_i(S_i) = (7 \times 114 \times 114 - 8 \times 114 + 114 \times 54 - 76 \times 41)/76 = 1225$$

8. Determination of the Number of Suras n and Verses V of the Holy Quran

In this study, the main objective is to show that the number of suras (n) and the number of verses (V) have not changed, that is, if we have 114 suras, we must find 6236 verses or vice versa, which is more than enough. But considering the uniqueness of the Noble Quran, which has no equivalent, we asked ourselves the following question: "Can we determine both the number of suras and the number of verses?". In response to this question and taking into account all the criteria that are mentioned in **section 3.3** concerning the data of the structure of the Noble Quran, we thought that the author of the book would certainly have integrated the number of suras in the form of equations as was the case for the total number of verses.

8.1 Determination of the Number of Suras n

If n is the number of suras, we have: $S(V_i) = S_i(V_i) + S_p(V_i)$ and $S(V_p) = S_i(V_p) + S_p(V_p)$

$$\text{We have also: } \begin{cases} S(V_i) + S(V_p) = n \\ S(V_i)/S(V_p) = 9/10 \end{cases} \Rightarrow \begin{cases} S(V_p) = 10n/19 \\ S(V_i) = 9n/19 \end{cases}$$

We can write the number of suras in table $G_1(II)$ or $G_2(PI)$ as follow: $S_i(V_i) = S_p(V_i) = S(V_i)/2 = 9n/38$

We can write the number of suras in table $G_3(IP)$ or $G_4(PP)$ as follow: $S_i(V_p) = S_p(V_p) = S(V_p)/2 = 5n/19$

We deduce from these equalities that:

$$\begin{aligned} S_i(V_p) &= 10 S_i(V_i)/9 \\ n &= 2 (S_i(V_i) + S_i(V_p)) \end{aligned}$$

Finally, we obtain the expression of suras number as follow:

$$n = 38 \frac{S_i(V_i)}{9} \quad (36)$$

Regarding the number of suras n , when the key is activated, its values follow each other and the tables G_i become in order: $S_i(V_i) = 27$, $S'_p(V_i) = 28$, $S'_i(V_p) = 29$ and $S_p(V_p) = 30$. Where:

$$n = S_i(V_i) + S'_p(V_i) + S'_i(V_p) + S_p(V_p)$$

We can therefore write: $S'_p(V_i) = S_i(V_i) + 1$, $S'_i(V_p) = S'_p(V_i) + 1$, $S_p(V_p) = S'_i(V_p) + 1$.
 Finally, we obtain the chaining condition of the tables as follow:

$$n = 4S_i(V_i) + 6 \tag{37}$$

We therefore deduce that the number of suras n must simultaneously verify the two equalities (Eq.36 and Eq.38)

$$\begin{cases} n = 38 S_i(V_i)/9 \\ n = 4 S_i(V_i) + 6 \end{cases}$$

Solving the two equations gives us:

$$\begin{cases} S_i(V_i) = 27 \\ n = 114 \end{cases}$$

We have just demonstrated that there is only one possible solution which allows both a successive sequence of the values of the numbers of suras of the tables G_i in the final state: $S_i(V_i) = 27$, $S'_p(V_i) = 28$, $S'_i(V_p) = 29$ and $S_p(V_p) = 30$ and meets all the conditions of the initial state as well as: $S_i(V_i) = 27$, $S_p(V_i) = 27$, $S_i(V_p) = 30$ and $S_p(V_p) = 30$.

8.2 Determine the Number of Verses V

We can find the total number of verses V by adding all the sum parameters of the verses (Eqs. 28, 29, 32, 34) as follows:

$$\begin{aligned} V = V_i + V_p &= [V_i(S_i) + V_i(S_p)] + [V_p(S_p) + V_p(S_i)] \\ V &= \frac{9n(n-1) + 38(2F - X_0)}{19} \end{aligned} \tag{38}$$

Calculation:

$$V = \frac{9 \times 114 \times (114 - 1)}{19} + 2(2 \times 54 - 41) = 6236$$

The number of suras (114) and the number of verses (6236) as well as the values obtained are all identical to the original. These results also confirm that all the relations, rules and equations adopted in this study are correct and integrated. The main objective of this study is not only to prove an aspect of the inimitability of the Noble Quran, but rather to emphasize that its homogeneous structure is based on astonishing scientific foundations mathematically precise, and that the number of its parameters is extremely high and that they are completely interconnected. In fact, this study represents only a small part in this immeasurable magnitude.

9. Analysis of Results

It was previously mentioned that the Quranic parameters adopted and highlighted in this study represent only a very minimal part from the structural elements of the Holy Quran. While the other Quranic parameters, related to other parts such as the classification of the suras, the number of verses associated with each sura, the number of words or the number of letters as well as many others present in the Holy book are not considered in this approach. However, this study remains the first step which is very important, because it definitively puts an end to the doubts about the authenticity of the divine message. This study, thanks to Allah (SWT) who planned the key of Fussilat, rigorously proves to us that the number of suras and the number of verses did not know any modification (Eqs. 36, 37 et 38). In the following discussion, we will verify, on the basis of the results found, if certain parameters of the study can be modified without effect on the structure?

- **Modification of the number of suras (n):**

All the parameters are directly related to the number of suras n (see the formulas), so any change in n automatically leads to change in all the other parameters. We therefore deduce that any addition or deletion of a single sura leads to a modification of the structure.

- **Modification of the number of verses (V):**

The number of verses is given by the formula of (Eq. 38): $V = (9n(n-1) + 38(2F - X_0))/19$. If n remains constant and V changes, then $2F - X_0$ changes, which means that the key must be modified to change the number of verses. We therefore deduce that the addition of a single verse or its deletion leads to a modification of the key, but knowing that the key is present, we conclude that the number of verses has not been modified. It is thanks to God (SWT).

- **Inversion of number of verses and suras:**

This part of the analysis will be the subject of another study relating to the classification of the suras of the Noble Quran which will be the solution and the logical complement.

10. Conclusion

In summary, the numerical structure of the Holy Quran has been successfully studied using a rigorous scientific method based on simple and precise mathematical concepts. The results obtained show that the calculated total number of suras (114) and the total number of verses (6236), have not undergone any change and correspond exactly to that of the noble Quran, which proves its authenticity and reveals its miraculous character. The method of exposing even and odd numbers in the tables and performing sum calculations was allowed to discover the principal concordance relation between the Quranic parameters and the first rule. In light of this discovery, it was believed that there was a hidden continuity of the concordance property between the sub-tables. The application of the proposed methodology led to the discovery of a concordance key which is Sura Fussilat. The integration of this sura into the Ha-Mim family has allowed the discovery of secondary concordance relations and transformation relations as well as other fundamental Quranic rules. These rules, which play an essential role in the formulation of the system of mathematical equations, are skillfully chosen to closely correlate the different Quranic parameters. Indeed, the 1st rule allows the even and odd suras to be related to their sum numbers in a well-defined ratio, equal to 9/10. The 2nd and 3rd rules relate the Quranic parameters to each other, in the initial state and in the final state. The 4th rule relates the sum of the components of the Fussilat (X_0, F) and the Quranic parameters. Finally, the system of equations obtained is the real guarantee that the structure of the Quran is protected against any risk of deformation or manipulation.

What is astonishing in this study is the question of why the Author of this book has veiled the key and the resulting mathematical arrangement which is in no way the result of coincidences..., it is clearly to show us that the Noble Quran cannot be a human work, but rather the work of a Wise, perfectly knower, whose abilities are superhuman, he is God the Almighty, the Guardian of the Reminder, as he promised in his book. Another study on the classification of the suras of the book is being finalized in order to complete this study and to verify the correct order of the suras.

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Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

Benzitouni Saadane confirms sole responsibility for the following: study conception and design, data collection, analysis and interpretation of results, and manuscript preparation. Benzitouni Sara confirms sole responsibility for the following: supervision, analysis and interpretation of results, and manuscript preparation. All authors reviewed the results and approved the final version of the manuscript.

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