

An Artificial Neural Network with Bayes' Theorem (Hybrid) for Automated Bearing Faults Diagnosis

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DOI: <https://doi.org/10.30880/jsmt.2024.04.01.003>

Article Info

Received: 20 February 2024

Accepted: 4 June 2024

Available online: 30 September 2024

Keywords

Artificial neural network, Bayes' Theorem, automated bearing fault

Abstract

Bearing fault diagnosis plays a pivotal role in the realm of condition-based maintenance, with vibration spectra analysis standing out as a highly effective method for discerning issues in rotating machinery. Various signal processing tools, including wavelet analysis, empirical mode decomposition, and Hilbert-Huang transform, have been employed to scrutinize vibration spectra. However, these methodologies often necessitate human expertise to ensure optimal outcomes. In the pursuit of automated fault diagnosis, machine learning tools such as Artificial Neural Networks (ANN) and support vector machines (SVM) have emerged as viable alternatives. Over recent decades, considerable research efforts have been devoted to exploring the viability of employing Artificial Neural Networks (ANN) for automatic fault diagnosis, resulting in predominantly positive findings. Nevertheless, the accuracy of Artificial Neural Networks (ANN) is intricately linked to factors such as the neural network structure, encompassing considerations like the number of nodes, hidden layers, and the choice of activation function. Addressing this challenge, the present study introduces an innovative hybrid algorithm for automated bearing fault diagnosis, integrating Artificial Neural Networks (ANN) with the Bayes' Theorem (BT) theory. The hybrid algorithm exploits Bayes' Theorem (BT) theory to augment fault diagnosis results derived from Artificial Neural Networks (ANN), specifically by alleviating conflicting outcomes generated within the neural network. The study focuses on characterizing four conditions of bearings, namely a healthy state and three distinct fault types: rolling element, inner race, and outer race faults. Through the proposed hybrid algorithm, working in tandem with artificial neural networks, a demonstrated superiority is established, surpassing the results obtained by Artificial Neural Networks (ANN) in isolation. This pioneering approach not only underscores the potential for heightened accuracy but also underscores the enhanced reliability achievable in automated bearing fault diagnosis.

1. Introduction

In recent decades, significant advancements in industrial machinery have reshaped sectors such as power generation, oil and gas, aviation and manufacturing. Within these intricate machines, bearings play a critical role, influencing the overall integrity of rotating machinery. The failure of bearing can lead to catastrophic

malfunctions, underscoring the importance of precise health monitoring. Vibration spectra analysis emerges as the foremost diagnostic method for this purpose.

The evolution of vibration signal processing tools, including empirical mode decomposition, wavelet analysis and Hilbert-Huang transform, has marked a transition from non-adaptive to self-adaptive signal analysis (Hui et al., 2013) [1]. However, these methods heavily rely on the expertise of machine operators. Acknowledging the limitations of operator-dependent approaches, the literature increasingly emphasizes the role of fault diagnosis machine learning, promising a more consistent and automated diagnostic system.

While machine learning-based fault diagnosis offers enhanced consistency, the accuracy is contingent on the specific algorithm employed—whether Artificial Neural Network (ANN), support vector machine (SVM) or self-organizing maps (SOM). Noteworthy contributions by Yan et al. [2, 3, 4] and Kaisi et al. [5] have significantly advanced this domain. This study introduces an innovative perspective by applying Bayes' Theorem (BT) theory to augment the bearing fault diagnosis accuracy results based on Artificial Neural Networks (ANN). This integration introduces a novel dimension to diagnostic precision, promising a more reliable assessment of machine health. The use of Bayes' Theorem (BT) theory acts as a catalyst, elevating the diagnostic paradigm's precision and effectiveness, marking a promising stride in enhancing the reliability of bearing fault diagnosis.

2. Collection of Data

The dataset utilized in this study was acquired from the Case Western Reserve University Bearing Data Center's online repository. Specifically selected to encompass a comprehensive range of ball bearing conditions, the dataset includes instances of both healthy and faulty states, covering faults in rolling elements, inner raceways, and outer raceways. Figure 1 provides a visual representation of the experimental setup employed to simulate various bearing conditions. This setup incorporates essential components, including a 2 hp motor, a dynamometer and a torque transducer.

For the deliberate introduction of bearing faults, a fault diameter of 7 mils (178 microns) was precisely incorporated into the SKF bearing. Operating under conditions mimicking real-world scenarios, the motor maintained a speed of approximately 1772 rpm with a 1 hp load. To capture the nuances of bearing vibrations, data was meticulously collected at a high rate of sampling at 12,000 samples per second. This process involved strategically placing accelerometers on the bearing housing, ensuring the accurate representation of the bearing's vibrational characteristics.

By emphasizing the intentional selection of diverse bearing conditions and detailing the intricacies of the experimental setup, this study ensures a robust foundation for investigating ball bearing health and fault diagnosis, underlining the credibility and significance of the sourced dataset.

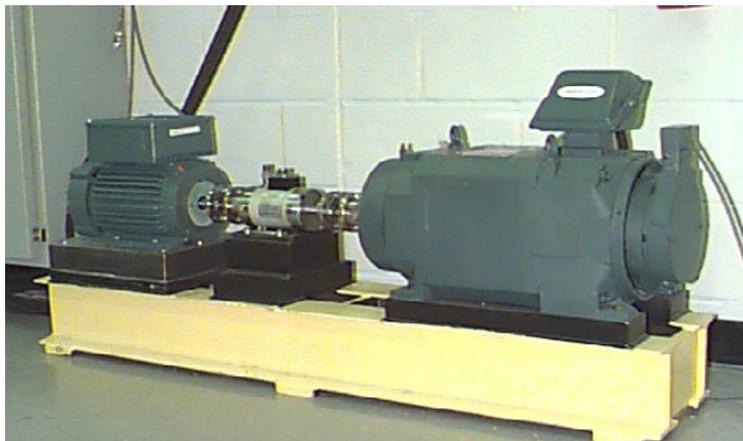


Fig. 1 *Experimental test rig*

To simulate the conditions prevalent in an industrial environment, where the signals of the bearing vibration may be susceptible to random noise, a deliberate introduction of white Gaussian noise was applied to the original vibration signal. As depicted in Figure 2, the unmodified vibration signal is contrasted with the adjusted signal, now featuring additional white Gaussian noise. Consequently, the signal-to-noise ratio (SNR) for the signal that already modified is established at 10 dB, reflecting the extent of noise introduced.

Following this noise augmentation, 1,000 sets of vibration time series were extrapolated from the time-domain signal of vibration. These sets were subsequently categorized into two (2) distinct inputs, with one set earmarked for training the machine learning model and another is designated for validation. The distribution of this comprehensive dataset of vibration is outlined in Table 1, elucidating the diversity of samples employed in

the study. The ensuing section delves into a detailed exploration of the parameters pivotal and statistical analysis methods to the feature extraction process for the machine learning diagnostic investigation. By intentionally elaborating on the noise simulation process and the comprehensive dataset utilization, this study establishes a robust foundation for the subsequent analytical procedures, reinforcing the credibility and relevance of the experimental design.

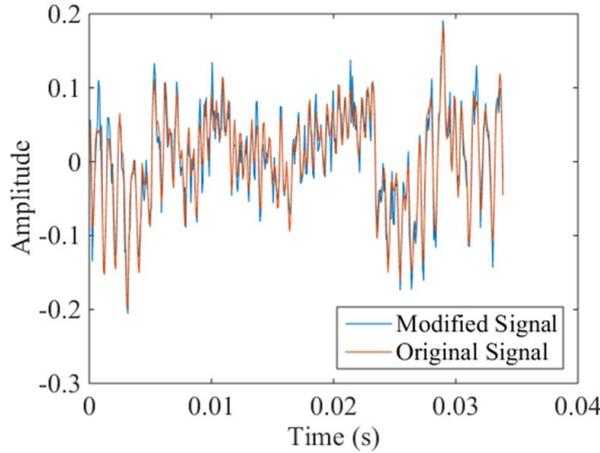


Fig. 2 The modified vibration signal vs the original signal with additive white Gaussian noise

Table 1 Data of vibration distributed in this study

		Bearing Condition			
		Healthy	Rolling Element Fault	Inner Raceway Fault	Outer Raceway Fault
Vibration Data	Training Data	200	200	200	200
	Testing Data	50	50	50	50

3. Statistical Analysis

Having established the groundwork with the comprehensive dataset, the subsequent phase involved utilizing the vibration data (1,000 sets) vibration data for statistical analysis and other purposes. Each statistical analysis method, crucial for subsequent investigations, is succinctly detailed in the following paragraphs. One such method, the root-mean-square (RMS) value, recognized for its effectiveness in identifying imbalances in rotating machinery (Yang et al., 2003) [6], plays a pivotal role in the analysis. The ensuing equation (1) presents the mathematical function governing RMS. This meticulous approach to dataset utilization and analytical methods ensures a robust and credible foundation for the ensuing diagnostic study.

$$x_{RMS} = \sqrt{\frac{1}{N} \sum_{i=1}^n (x_i)^2} \tag{1}$$

The standard deviation (σ) of a vibration time series characterizes the energy content within the signal of the vibration, serving as a discerning metric (Sarabi et al, 2013) [7]. Equation (2) delineates the mathematical expression for standard deviation:

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{n=1}^N (x(n) - \bar{x})^2} \tag{2}$$

Skewness serves as a metric for gauging the extent of asymmetry in the distribution of a dataset around its mean. This dimensionless parameter proves to be effective in fault diagnosis for rotating machinery (Lei et al., 2009) [8]. Equation (3) outlines the mathematical function representing skewness:

$$\text{Skewness} = \frac{\frac{1}{N} \sum_{n=1}^N (x(n) - \bar{x})^3}{\left(\frac{1}{N} \sum_{n=1}^N (x(n) - \bar{x})^2 \right)^{\frac{3}{2}}} \quad (3)$$

Kurtosis, serving as a statistical parameter, elucidates the arrangement of data points relative to the mean, providing insight into the peakedness of a statistical frequency curve (Kankar et al, 2011) [9]. This dimensionless metric plays a pivotal role in characterizing the shape of the distribution, offering a quantitative measure of the tails and sharpness of the curve.

Furthermore, kurtosis serves as a valuable tool in discerning the degree of deviation from a normal distribution, with higher kurtosis indicating heavier tails and a more peaked curve. The interpretation of kurtosis in statistical analysis aids in understanding the distribution's behavior and potential outliers, contributing to a comprehensive analysis of the dataset. Equation (4) articulates the mathematical expression governing kurtosis:

$$\text{Kurtosis} = \frac{\frac{1}{N} \sum_{n=1}^N (x(n) - \bar{x})^4}{\left(\frac{1}{N} \sum_{n=1}^N (x(n) - \bar{x})^2 \right)^2} \quad (4)$$

The crest factor, a crucial metric defined as the peak value ratio to the root-mean-square (RMS) value within an input signal, assumes a pivotal role in discerning variations in signal patterns induced by impulsive vibration sources. Its applicability is notably pronounced in the identification of defects within ball bearings, particularly those situated on the outer raceway, as outlined by Yang et al. in their study from 2003 [6]. The mathematical expression governing the crest factor is explicitly elucidated through Equation (5), providing a concise representation of this significant parameter in the context of vibration signal analysis.

This fundamental measure, encapsulated by the crest factor, holds significance in detecting impulsive vibrations indicative of specific faults, contributing to the diagnostic efficacy of the overall study. The incorporation of this metric within the analytical framework underscores the meticulous consideration given to diverse features for a comprehensive machinery fault diagnosis:

$$\text{Crest Factor} = \frac{\max|x(n)|}{\sqrt{\frac{1}{N} \sum_{n=1}^N x(n)^2}} \quad (5)$$

This parameter proves to be a significant contributor to understanding signal characteristics, particularly in the nuanced realm of impulsive vibrations associated with defects in the outer raceway of ball bearings.

Figure 3 meticulously portrays the data distribution for key parameters such as skewness, kurtosis, and crest factor across a spectrum of experimental bearing conditions. Within this dataset, consisting of 250 samples for each bearing condition, a deliberate selection process was undertaken, with 80% of these samples chosen at random for the purpose of training the machine learning model. The 20% of samples that remains were judiciously reserved for the critical task of validating the trained machine learning model. The visualization of this distribution, encapsulating the three crucial feature parameters—crest factor, skewness, and kurtosis—is vividly depicted in the Figure 4.

This deliberate allocation of samples for training and validation underscores the meticulous approach adopted in this study, ensuring a robust and comprehensive exploration of the feature parameters essential for effective machine learning diagnostic analysis.

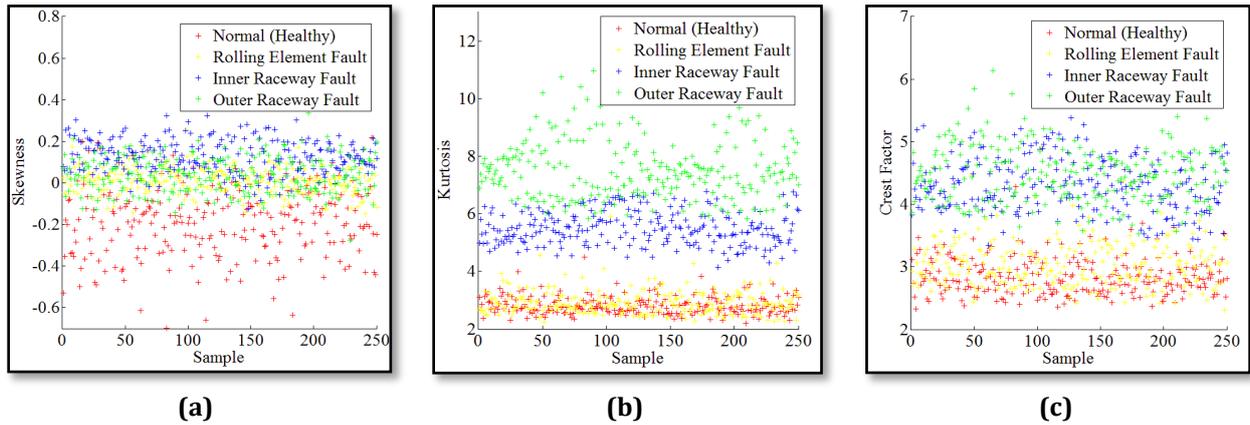


Fig. 3 Skewness; (a) Kurtosis; (b) Crest factor; (c) (Distribution of data) for experimental bearing conditions

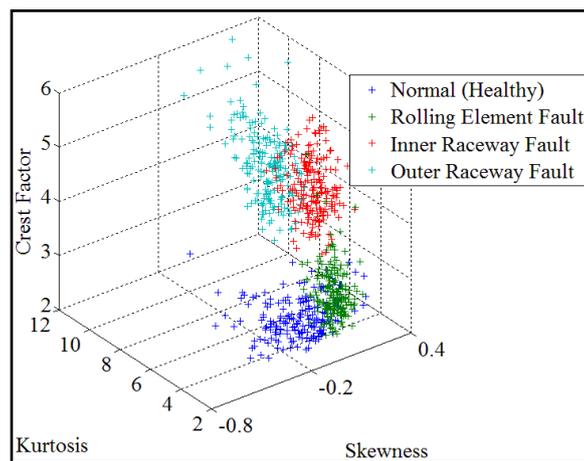


Fig. 4 All training data distribution

4. Introduction to Artificial Neural Network (ANN)

Over the course of the past century, the widespread adoption of Artificial Neural Networks (ANN) has been a conspicuous trend, finding applications across diverse domains, with machinery fault diagnosis emerging as a prominent and impactful use case. Functioning on the principles of supervised machine learning, Artificial Neural Networks (ANN) establish a sophisticated parallel information processing framework characterized by a network of interconnected artificial neurons, as vividly depicted in Figure 5 (Liu et al, 2002) [10].

The incorporation of Artificial Neural Networks (ANN) typically unfolds through two pivotal phases: the training phase and the testing phase (Saravanan et al., 2010) [11]. The training phase is dedicated to discerning the specific tasks that the network can adeptly tackle in subsequent real-world applications. In contrast, the testing phase is geared towards processing the representative features of the inputs to evaluate the network's performance. A comprehensive review conducted by Lee et al. in 2014 [12] thoroughly assessed various algorithms, including Artificial Neural Networks (ANN), Support Vector Machines (SVM), Hidden Markov Model, Bayesian Belief Networks (BBN) and Feature Map, with a specific focus on their characteristics in the domain of machinery fault diagnostics. Conclusively, they unequivocally identified Artificial Neural Networks (ANN) as the most suitable and effective tool for addressing the intricate challenges posed by machinery fault diagnosis.

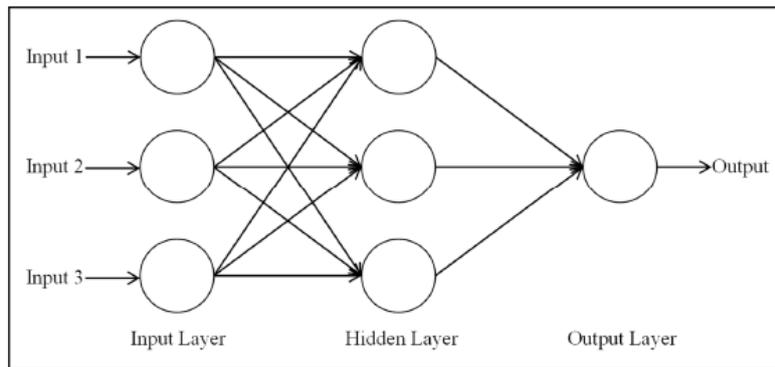


Fig. 5 Artificial neural networks (ANN) schematic structure

4.1 Introduction to Bayes' Theorem Theory

Named after the 18th-century British mathematician Thomas Bayes, Bayes' Theorem (BT) stands as a foundational concept in the realms of probability and statistics, providing a robust framework for determining the event probability based on the occurrence of another. At its core, this theorem serves as a mathematical tool for calculating conditional probability, unraveling the likelihood of an event by considering its relationship to one or more other events.

Bayes' Theorem (BT) serves as the cornerstone of the Bayesian statistics field, often referred to as Bayes Law or Bayes Rules. In the financial sector, it finds practical application in assessing the risk associated with customers seeking loans from banks. Beyond the confines of finance, Bayes' Theorem (BT) proves invaluable in critically evaluating the medical test results accuracy, facilitating the determination of the probability of an individual harboring a specific illness, such as the prevalence of cancer.

The adaptability of Bayes' Theorem (BT) extends across various domains, leaving its imprint in fields such as medicine (Dienes, 2008) [13], human language (Frank and Goodman, 2012) [14], machinery condition monitoring (Bishop et al., 2006) [15], psychology (Jones and Love, 2011) [16], ecological data analysis (Parent and Rivot, 2012) [17], and beyond. In the contemporary landscape, Bayes' Theorem (BT) continues to inspire confidence in the nuanced assessment of relevant events, contributing significantly to diverse areas of inquiry and application. The expression of Bayes' Theorem (BT) is encapsulated in the following formula:

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)} \tag{6}$$

5. Automated Bearing Fault Diagnosis Model Structure

The envisaged methodology unfolds as a sophisticated two-layer classification system, seamlessly integrating the prowess of Artificial Neural Networks (ANN) and the robust principles of Bayes' Theorem (BT) theory. In the inaugural layer, a meticulously crafted Artificial Neural Networks (ANN) model takes shape, with the assimilation of comprehensive training data that encompasses all pertinent features, including crest factor, kurtosis, and skewness, expertly fed into the Artificial Neural Networks (ANN) algorithm. Subsequently, the rigorously prepared testing data assumes a pivotal role in subjecting the trained Artificial Neural Networks (ANN) algorithm to a meticulous evaluation. Notably, this stage unfolds with a discerning eye on potential conflicting results that may emerge during the assessment of the testing data, a scenario thoughtfully illustrated in Table 2.

This meticulous two-layer classification system, combining the computational power of Artificial Neural Networks (ANN) and the probabilistic insights of Bayes' Theorem (BT), underlines a nuanced approach to automated fault diagnosis, promising robustness and accuracy in the intricate task of bearing fault assessment.

Table 2 The results generated by Artificial Neural Networks (ANN) (examples)

		Sample					
		A	B	C	D	E	F
Bearing Condition	Healthy	1	-	-	-	-	-
	Rolling Element Fault	-	1	-	-	-	1
	Inner Raceway Fault	-	-	1	-	-	1

				1		
Final Decision	Healthy	Rolling Element Fault	Inner Raceway Fault	Outer Raceway Fault	Conflict	Conflict

Transitioning to the second layer of classification, the methodology unfolds with the establishment of three distinct Artificial Neural Networks (ANN) models, each meticulously constructed on the foundation of training data associated with a specific feature—namely, skewness, kurtosis, or crest factor. The testing data, marked by conflicting outcomes from the initial layer, undergoes a meticulous reevaluation within this classification framework of second layer. This subsequent model of classification seamlessly integrates all three Artificial Neural Networks (ANN) models, aligning them through the application of the Bayes’ Theorem (BT) theory.

The noteworthy feature of this dual-layered approach is the perceptible superiority exhibited by the Artificial Neural Networks (ANN) models with a single-feature focus, showcasing enhanced classification curves. This distinctive capability translates into an improved discernment of samples positioned in close proximity to the boundary of the first layer classification. Collectively, these models play a pivotal role in shaping the final decision pertaining to the condition of a bearing. An Artificial Neural Network (ANN) processes input data through multiple layers of interconnected nodes (neurons). The ANN’s output is typically a set of probabilities for each class, and the class with the highest probability is chosen as the final decision. The comprehensive workflow of the diagnosis model for automated bearing fault finds visual representation in the insightful flowchart presented in Figure 6. This visual aid serves as a guiding map, elucidating the intricacies of the model’s decision-making process.

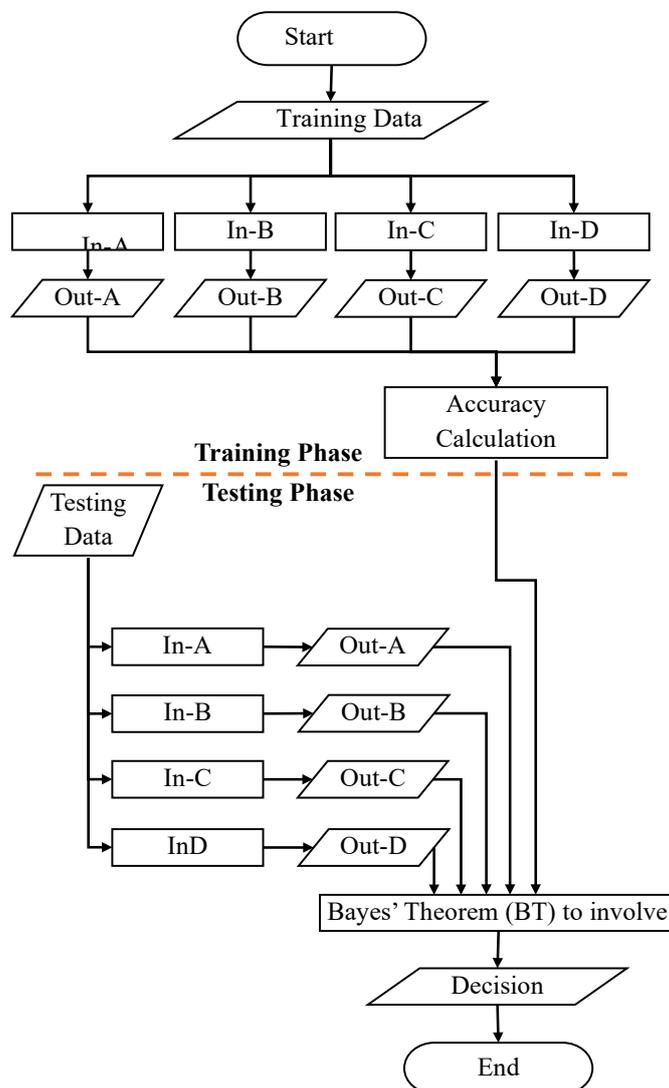


Fig. 6 Flowchart - diagnosis of automated bearing fault

6. Results and Discussion - Artificial Neural Network (ANN)

In the initial phase of bearing condition classification, the Artificial Neural Network (ANN) exhibited commendable proficiency, successfully classifying a majority of the testing data into four discernible bearing conditions: healthy, outer raceway fault, inner raceway fault, and rolling element fault. The architecture of the employed Artificial Neural Networks (ANN) in this study adheres to a feed-forward backpropagation neural network model, characterized by two layers and ten neurons positioned in the first layer. Noteworthy is the adoption of the Levenberg-Marquardt training algorithm (trainlm), acclaimed for its expeditious training capabilities, contributing to the efficiency of the model. A visual representation of the Artificial Neural Networks (ANN)'s structure is thoughtfully depicted in Figure 7, providing a tangible insight into the intricate neural network configuration employed in the study.

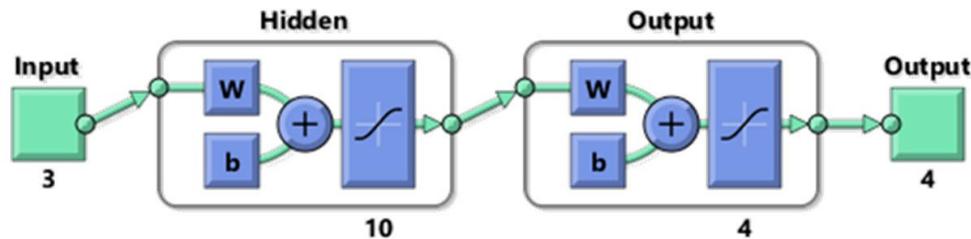


Fig. 7 The Artificial Neural Networks (ANN) structure

The dynamic evolution of the training performance of the Artificial Neural Network (ANN) is meticulously captured in Figure 8, where the validation curve mirrors the test curve with striking resemblance, a telling sign of the absence of significant concerns such as overfitting during the intricate training stage. Noteworthy is the validation performance's journey, reaching a noteworthy minimum at the culmination of 12 iterations. The regression plot, an insightful visual component in this study, unveils a robust R-value of approximately 0.9 across all the pivotal stages—training, validation, and testing. This resounding R-value underscores a formidable relationship between the Artificial Neural Network (ANN)'s outputs and the intended targets, affirming the authors' assertion regarding the model's acceptable performance. The comprehensive visualization and statistical metrics collectively lead to a nuanced understanding of the Artificial Neural Network (ANN) model's proficiency and reliability in the intricate realm of bearing fault diagnosis.

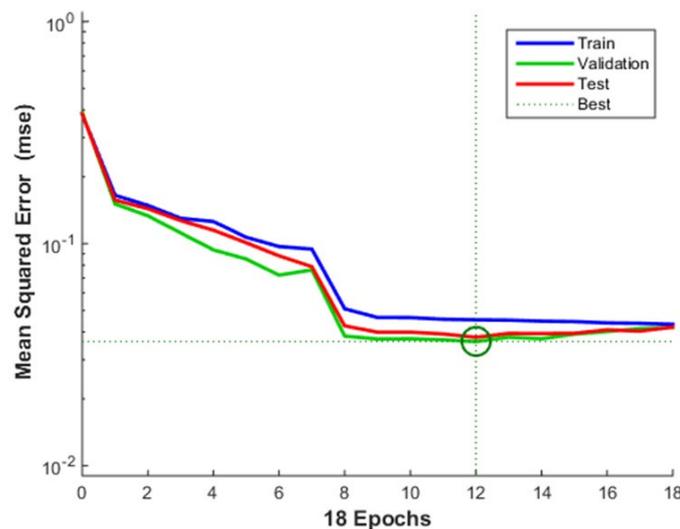


Fig. 8 The Artificial Neural Network (ANN) training performance

6.1 Results and Discussion - Bayes' Theorem (BT)

In this critical phase, the outcomes of having conflicts arising from the Artificial Neural Network (ANN) model underwent a meticulous process of analysis and amalgamation utilizing Bayes' Theorem (BT) theory. The primary objective of this process was to eliminate conflicting decisions and yield a definitive result for bearing fault diagnosis. Each set of input data associated with conflicting results was individually routed to the distinct Artificial Neural Network (ANN) models—crest factor, kurtosis, and skewness—for classification. The outcomes from

these individual Artificial Neural Networks (ANN) models were subsequently harmonized through the application of Bayes' Theorem (BT) theory, culminating in the formulation of the ultimate decision. The visual representation of comparative decision-making outcomes between the Artificial Neural Network (ANN) model and the hybrid Artificial Neural Network-Bayes' Theorem (ANN-BT) model is vividly depicted in Figure 9, underscoring the hybrid model's proficiency in eradicating conflicting decisions emanating from the Artificial Neural Networks (ANN) model and facilitating a conclusive decision based on the available dataset.

Remarkably, the accuracy metrics for the Artificial Neural Networks (ANN) model and the hybrid Artificial Neural Network-Bayes' Theorem (ANN-BT) model were determined to be 72% and 87%, respectively. While the increment in accuracy may be considered moderate, it proved to be a pivotal enhancement in the context of eliminating conflicting results through the application of the hybrid Artificial Neural Network-Bayes' Theorem (ANN-BT) model for diagnosis of automated bearing fault. This notable accuracy improvement can be predominantly credited to the successful mitigation of conflict decisions that were prevalent in the standalone Artificial Neural Networks (ANN) model. Notably, the hybrid model demonstrated a substantial 15% surge in accuracy compared to the standalone Artificial Neural Networks (ANN) model, underscoring its efficacy in refining the precision and reliability of the bearing fault diagnosis process. Accuracy is a widely used metric for assessing the performance of classification models. It is defined as the proportion of correctly identified instances out of the total number of instances. The equation for accuracy is:

$$(\text{Accuracy}) = \frac{TP + TN}{TP + TN + FP + FN} \quad (7)$$

If we denote:

- TP as true positives (correctly classified faults)
- TN as true negatives (correctly classified healthy states)
- FP as false positives (incorrectly classified faults)
- FN as false negatives (missed faults)

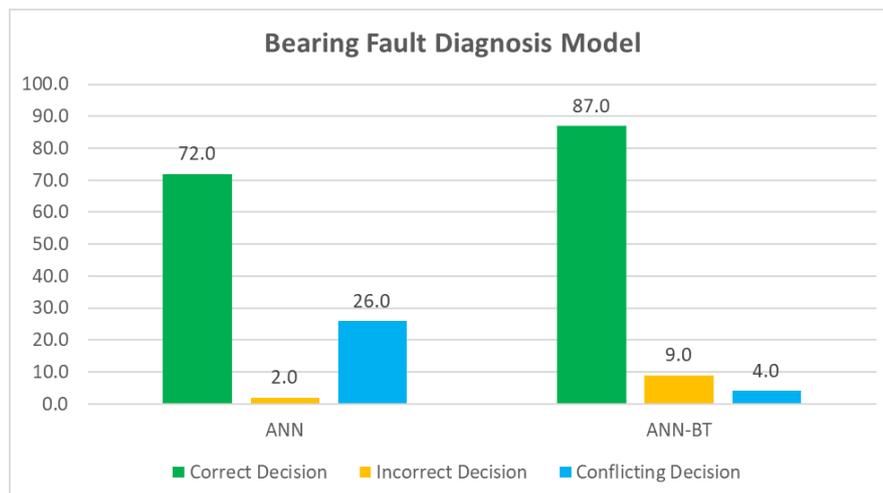


Fig. 9 The accuracy of decisions by Artificial Neural Network (ANN) and Artificial Neural Network-Bayes' Theorem (ANN-BT)

7. Conclusion

The research presents a groundbreaking hybrid Artificial Neural Network-Bayes' Theorem (ANN-BT) model designed for the automated diagnosis of bearing faults. Employing the four bearing conditions simulated by the Case Western Reserve University Bearing Data Center as input parameters for the machine learning models, the findings underscore the profound impact of Bayes' Theorem (BT) on augmenting the accuracy of the Artificial Neural Network (ANN) model. This enhancement is manifested through the systematic elimination of all conflicting results, showcasing the efficacy of the Artificial Neural Network-Bayes' Theorem (ANN-BT) hybrid model. In summary, the integration of Artificial Neural Network (ANN) with Bayes' Theorem (BT) theory emerges as a better and more accurate approach for diagnosis of bearing fault in comparison to relying solely on the standalone Artificial Neural Network (ANN) model.

Acknowledgement

A special thanks of gratitude to my supervisor, Associate Professor Ir. Dr. Lim Meng Hee, who gave me the opportunity to work on this wonderful research topic "A hybrid Artificial Neural Network with Bayes-Theorem for Automated Bearing Fault Diagnosis", which also encouraged me to explore the applications of artificial intelligence in field of machinery fault diagnosis.

Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The authors confirm the equal contribution in each part of this work. All authors reviewed and approved the final version of this work.

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