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AN ALGORITHM FOR SOLVING INTUITIONISTIC FUZZY LINEAR BOTTLENECK ASSIGNMENT PROBLEMS

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Abstract

The linear bottleneck assignment problem (LBAP), which is a variation of the classical assignment problem, seeks to minimize the longest completion time rather than the sum of the completion times when a number of jobs are to be assigned to the same number of workers. If the completion times are not certain, then it is said to be a fuzzy LBAP. Here we propose a new algorithm to solve fuzzy LBAP with completion times as intuitionistic fuzzy numbers.

Keywords: *Fuzzy linear bottleneck assignment problem, Intuitionistic fuzzy numbers, Generalized trapezoidal intuitionistic fuzzy numbers, Ranking of fuzzy numbers.*

1.0 Introduction

Linear bottleneck assignment problems were introduced by Fulkerson, Glicksberg and Gross (1953) and occur, e.g., in connection with assigning jobs to parallel machines so as to minimize the latest completion time. Like the classical assignment problem, LBAP arises in a wealth of practical settings. For example, consider a serial assembly line where each of the operators is to be assigned to one of the work stations to perform a specific task. Since the speed of the line is controlled by the slowest station (i.e., the bottleneck), it is important to identify the operator-station pairings with the minimum longest processing time to maximize the system's productivity. Another example of the LBAP pertains to the distribution of meals to patients in a hospital. The food is normally cooked in the main kitchen and delivered in bulk to serving stations in the building to be reheated. The meals are then placed in individual trays, loaded onto carts, and distributed to patients in different areas of the facility. In order to ensure that the food received by the patients is as warm as possible, it is desirable to allocate the carts to the wings so that the longest distance travelled (and hence the longest delivery time) is short as possible. Other applications of the concept of bottleneck assignment include partitioning an area into political districts so that the maximum deviation of any district population from the mean district population is as small as possible (Garfinkel & Nemhauser, 1970), transportation of perishable goods from warehouses to markets to minimize spoilage (Ravindran & Ramaswami, 1977), and location of fire stations within a city to reduce the longest response time to an emergency to the extent possible (Kuo, 2011).

In real life situations, the parameters of the LBAP are imprecise numbers instead of fixed real numbers because the distance (cost or time) vary due to different reasons. To overcome this, we make use of the theory of fuzzy sets introduced by Zadeh (1965) and the theory of intuitionistic fuzzy sets introduced by Atanassov (1986) and Atanassov (1989). If the completion time is a not a crisp value, then the corresponding problem becomes a fuzzy LBAP. Here, we are introducing a new algorithm for solving fuzzy a LBAP with parameters as generalized trapezoidal intuitionistic fuzzy numbers.

The rest of the paper is organized as follows. In Section 2, a comprehensive survey of the solution methods for the LBAP in the current literature is conducted. In section 3, we present the basic concepts of generalized trapezoidal intuitionistic fuzzy numbers, matchings and a method to rank the generalized trapezoidal intuitionistic fuzzy numbers. Following an extensive discussion of intuitionistic fuzzy LBAP in section 4, we suggest a new algorithm for solving an intuitionistic fuzzy LBAP with completion times as generalized trapezoidal intuitionistic fuzzy numbers in section 5 and provide a numerical example to illustrate its implementation in section 6. Finally, the conclusion is given in section 7.

2.0 Literature Review

The LBAP has been studied extensively over the past six decades, and a number of solution procedures have been proposed. The study of the LBAP dates back to 1953 when Fulkerson *et al.*, (1953) designed an algorithm to solve a production line assignment problem. Inspired by the seminal work, Gross (1959) developed a more efficient solution scheme for the LBAP. Ford and Fulkerson (1962) considered a maxi-min version of the LBAP in which agents were to be assigned to tasks to maximize the minimum efficiency. Subsequently, Page (1963) showed how to convert the LBAP into an equivalent CAP and solve it by using the Hungarian method.

In the early 1970s, Edmonds and Fulkerson (1970) suggested a threshold algorithm for a general class of bottleneck problems including the LBAP. Ravindran and Ramaswami (1977) noted that the maxi-min version of the LBAP can be treated as a maxi-min permutation problem and solved by using the procedure they developed. Meanwhile, Bhatia (1977) proposed an iterative process to find the minimum bottleneck by solving a series of special CAPs through the Hungarian method.

One of the first inquiries into the LBAP in the 1980s was a shortest augmenting path method in combination with a labeling technique introduced by Derigs (1984). Mazzola and Neebe (1988) examined two versions of the bottleneck generalized assignment problem in which each machine is allowed to perform multiple tasks subject to the capacity constraint. Much of the research on the LBAP in the 1990s and 2000s was devoted to the analysis, comparison, and improvement of the computational efficiencies of various solution algorithms. One of the new schemes designed during this period of time was based on strong spanning trees due to Armstrong and Jin (1992). More recently, Pentico (2007) carried out a state-of-the-art survey of the CAP as well as many of its variations including the LBAP, while Kuo (2011) provided an in-depth treatment of the stochastic LBAP.

In sum, the LBAP has been researched for quite some time and there exists a body of literature on the solution algorithms. Of particular interest to us are the similar procedures developed by Page (1963) and Mazzola and Neebe (1988), respectively. Both studies tackle the LBAP by transforming it into equivalent CAPs, which represent a significant departure from the rest of the approaches. Recently, Kuo and Nicholls (2014) developed a turnpike approach for solving the LBAP. But no one has considered the parameters as fuzzy numbers. Thus, a more efficient algorithm is needed to solve the problem when the parameters are imprecise numbers.

3.0 Preliminary Concepts

3.1 Intuitionistic fuzzy sets and intuitionistic fuzzy numbers

In this section we will review the basic concepts of intuitionistic fuzzy sets and intuitionistic fuzzy numbers.

Definition 3.1.1 (Atanassov, 1986 & Atanassov, 1989): Let X be the universal set. An intuitionistic fuzzy set (IFS) A in X is given by

$$A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$$

where the functions $\mu_A(x), \nu_A(x)$ define respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 3.1.2: An IFS $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ of the real line $\mathbb{R}$ is called an intuitionistic fuzzy number (IFN) if

a) A is convex for the membership function $\mu_A(x)$, i.e., if
$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \mu_A(x_1) \wedge \mu_A(x_2)$ for all $x_1, x_2 \in \mathbb{R}$, $\lambda \in [0,1]$.

b) A is concave for the non-membership function $\nu_A(x)$, i.e., if
$\nu_A(\lambda x_1 + (1-\lambda)x_2) \leq \nu_A(x_1) \vee \nu_A(x_2)$ for all $x_1, x_2 \in \mathbb{R}$, $\lambda \in [0,1]$.

c) A is normal, that is, there is some $x_0 \in \mathbb{R}$ such that $\mu_A(x_0) = 1$ and $\nu_A(x_0) = 0$.

Definition 3.1.3 (**Generalized Trapezoidal Intuitionistic Fuzzy Number**): An intuitionistic fuzzy number A is said to be a generalized trapezoidal intuitionistic fuzzy number (GTIFN) with parameters

$$b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4,$$

and denoted by

$$A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4; \omega_A, u_A) \text{ or } A = ((a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4); \omega_A, u_A)$$

if its membership and non-membership functions are as follows:

$$\mu_A(x) = \begin{cases} 0 & \text{if } x < a_1 \\ \omega_A\left(\dfrac{x-a_1}{a_2-a_1}\right) & \text{if } a_1 \leq x \leq a_2 \\ \omega_A & \text{if } a_2 \leq x \leq a_3 \\ \omega_A\left(\dfrac{a_4-x}{a_4-a_3}\right) & \text{if } a_3 \leq x \leq a_4 \\ 0 & \text{if } x > a_4 \end{cases}$$

and

$$\begin{aligned} \nu_A(x) &= 1 && \text{if} \quad x < b_1 \\ &= \frac{(b_2 - x) + u_A(x - b_1)}{b_2 - b_1} && \text{if} \quad b_1 \le x \le b_2 \\ &= u_A && \text{if} \quad b_2 \le x \le b_3 \\ &= \frac{(x - b_3) + u_A(b_4 - x)}{b_4 - b_3} && \text{if} \quad b_3 \le x \le b_4 \\ &= 1 && \text{if} \quad x > b_4. \end{aligned}$$

where $0 < \omega_A \le 1, 0 \le u_A \le 1$ and $0 < \omega_A + u_A \le 1$.

If $b_1 = a_1, b_2 = a_2,\ b_3 = a_3,\ b_4 = a_4$, then the corresponding intuitionistic fuzzy number is of the form

$$A = ((a_1, a_2, a_3, a_4); \omega_A, u_A).$$

3.2 Arithmetic operations on generalized trapezoidal intuitionistic fuzzy numbers

Let

$$A = ((a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4); \omega_A, u_A)$$

And

$$B = ((c_1, c_2, c_3, c_4), (d_1, d_2, d_3, d_4); \omega_B, u_B)$$

be two GTIFNs and λ be a real number.

Then

(i) $A + B = ((a_1 + c_1, a_2 + c_2, a_3 + c_3, a_4 + c_4),(b_1 + d_1, b_2 + d_2, b_3 + d_3, b_4 + d_4);\ \omega, u)$
where $\omega = \min\{\omega_A, \omega_B\}$ and $u = \max\{u_A, u_B\}$.

(ii) $A - B = ((a_1 - c_4, a_2 - c_3, a_3 - c_2, a_4 - c_1), (b_1 - d_4, b_2 - d_3, b_3 - d_2, b_4 - d_1);\ \omega, u)$
where $\omega = \min\{\omega_A, \omega_B\}$ and $u = \max\{u_A, u_B\}$.

(iii) $\lambda A = ((\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4), (\lambda b_1, \lambda b_2, \lambda b_3, \lambda b_4); \omega_A, u_A)$ if $\lambda > 0$
$= ((\lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1), (\lambda b_4, \lambda b_3, \lambda b_2, b a_1); \omega_A, u_A)$ if $\lambda < 0$.

3.3 Matchings

Let G be a graph with vertex set $V = V(G)$ and edge set $E = E(G)$ and G has no loops.

Definition 3.3.1: A subset M of E, i.e., a subset M of edges of G, is called a matching in G if no two of the edges in M are adjacent, in other words, if for any two edges e and f in M the two end vertices of e are both different from the two end vertices of f.

If the vertex v of the graph G is the end vertex of some edge in the matching M then v is said to be M-saturated and we say that Msaturates v. Otherwise v is M-unsaturated.

Definition 3.3.2: If M is a matching in G such that every vertex of G is M-saturated, then M is called a perfect matching.

Definition 3.3.3: A matching M in G is called maximum if G has no matching M' with a greater number of edges than M has.

3.4. Ranking of generalized trapezoidal intuitionistic fuzzy numbers

The ranking order relation between two GTIFNs is a difficult problem. However, GTIFNs must be ranked before the action is taken by the decision maker. In this paper we use the following method for ranking generalized trapezoidal intuitionistic fuzzy numbers (Gani & Mohamed, 2015).

If $A = ((a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4); \omega_A, u_A)$, then $\Re(A) = \frac{\omega_A S(\mu_A) + u_A S(\nu_A)}{\omega_A + u_A}$, where

$$S(\mu_A) = \left(\frac{2a_1 + 7a_2 + 7a_3 + 2a_4}{18}\right)\left(\frac{7\omega_A}{18}\right) \text{ and } S(\nu_A) = \left(\frac{2b_1 + 7b_2 + 7b_3 + 2b_4}{18}\right)\left(\frac{11 + 7u_A}{18}\right).$$

If $A = ((a_1, a_2, a_3, a_4); \omega_A, u_A)$, then

$$S(\mu_A) = \left(\frac{2a_1 + 7a_2 + 7a_3 + 2a_4}{18}\right)\left(\frac{7\omega_A}{18}\right) \text{ and } S(\nu_A) = \left(\frac{2a_1 + 7a_2 + 7a_3 + 2a_4}{18}\right)\left(\frac{11 + 7u_A}{18}\right).$$

4.0 Intuitionistic Fuzzy Linear Bottleneck Assignment Problems

Let n jobs and n machines be given. The cost coefficient $\tilde{c}_{ij}$ is the time needed for machine j to complete job i, where $\tilde{c}_{ij}'s$ are generalized trapezoidal intuitionistic fuzzy numbers. If the machines work in parallel and we want to assign the jobs to the machines such that the latest completion time is as early as possible, we get a linear bottleneck assignment problem (LBAP) of the form

$$\min_{\varphi \in S_n} \max_{1 \le i \le n} \tilde{c}_{i\varphi(i)}.$$

If we describe permutations by the corresponding permutation matrices $X = (x_{ij})$, a LBAP can be modeled as

$$\min \max_{1 \le i,j \le n} \tilde{c}_{ij} x_{ij}$$

such that

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1,2,\dots,n$$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1,2,\dots,n$$

$$x_{ij} \in \{0,1\}.$$

A slight generalization of LBAPs is min-cost maximum matching problems with a bottleneck objective: Let $G = (U, V; E)$ be a bipartite graph with edge set E. Every edge $[i, j]$ has length c_{ij}, where $c_{ij} = \Re(\tilde{c}_{ij})$The bottleneck min-cost maximum matching problem can be formulated as follows:

Find a maximum matching in G such that the maximum length of an edge in this matching is as small as possible:

$$\min\{\max_{[i,j]\in M} c_{ij} : M \text{ is a maximum macthing}\}.$$

Associated with each intuitionistic fuzzy LBAP, there is an$n \times n$ matrix

$$\tilde{C} = [\tilde{c}_{ij}] = \begin{bmatrix} \tilde{c}_{11} & \tilde{c}_{12} & \dots & \tilde{c}_{1n} \\ \tilde{c}_{21} & \tilde{c}_{22} & \dots & \tilde{c}_{2n} \\ \dots & \dots & \dots & \dots \\ \tilde{c}_{n1} & \tilde{c}_{n2} & \dots & \tilde{c}_{nn} \end{bmatrix}$$

of intuitionistic fuzzy numbers, we call it the cost matrix.

5.0 The Proposed Algorithm

The proposed algorithm alternates between two phases. In the first phase a cost element c^* −the threshold value − is chosen from the matrix $C = [c_{ij}] = [\Re(\tilde{c}_{ij})]$and a threshold matrix $\overline{C}$ is defined by

$$\bar{c}_{ij} = \begin{cases} c_{ij}, & \text{if } c_{ij} > c^* \\ 0, & \text{otherwise.} \end{cases}$$

In the second phase it is checked whether for the cost matrix $\bar{C}$ there exists an assignment with total cost 0. To check this we construct a bipartite graph $G = (U, V; E)$ with $|U| = |V| = n$ and edges $[i, j] \in E$ if and only if $\bar{c}_{ij} = 0$. In other words, we have to check whether a bipartite graph with threshold matrix $\bar{C}$ contains a perfect matching or not. The fuzzy element corresponding to the smallest value c^*, for which the corresponding bipartite graph contains a perfect matching gives the optimum value of the intuitionistic fuzzy LBAP.

Algorithm

Let $\tilde{C} = [\tilde{c}_{ij}]$ be the $n \times n$ cost matrix and $z = \min\max_{1\le i,j\le n} \tilde{c}_{ij} x_{ij}$

Step 1: First we form the matrix $C = [c_{ij}] = [\Re(\tilde{c}_{ij})]$ of ranks of the fuzzy costs by using the given ranking method.

Step 2: Find $c_0{}^* = \min_{ij}\{c_{ij}\}$ and $c_1{}^* = \max_{ij}\{c_{ij}\}$.

Step 3: If $c_0{}^* = c_1{}^*$, then $z = c_0{}^*$ and any permutation of $\{1,2,\ldots,n\}$ is optimal. Otherwise go to **Step 4**.

Step 4: Let $C^* = \{c_{ij}: c_0{}^* < c_{ij} < c_1{}^*\}$. If $C^* = \emptyset$, then go to **Step 7**. If $C^* \neq \emptyset$, then find the median c^* of C^*, which is given by

$$c^* = \min\left\{c \in C^*: \left|\{c_{ij} \in C^*: c_{ij} \leq c\}\right| \geq \frac{1}{2}|C^*|\right\}.$$

Step 5: Find the threshold matrix $\bar{C}[c^*] = (\bar{c}_{ij})$ corresponding to c^* and construct a bipartite graph $G = (U, V; E)$ with $|U| = |V| = n$ and edges $[i,j] \in E$ if and only if $\bar{c}_{ij} = 0$.

Step 6: Find a maximum cardinality matching in $G[c^*]$. If the cardinality of the maximum matching is n, then $G[c^*]$ allows a perfect matching and set $c_1{}^* = c^*$, otherwise set $c_0{}^* = c^*$ and go to **Step 4**.

Step 7: If $C^* = \emptyset$, then either $G[c_0{}^*]$ or $G[c_1{}^*]$ allows a perfect matching.

Step 8: Find $c_{ij}^* = \min\{c^*: G[c^*]$ allows a perfect matching$\}$. It will give the optimal value of z

6.0 Numerical Example

Consider an intuitionistic fuzzy LBAP with rows representing 4 jobs x_1, x_2, x_3, x_4 and columns representing the 4 machines y_1, y_2, y_3, y_4. The cost matrix $[\tilde{c}_{ij}]$ is given whose elements are generalized trapezoidal intuitionistic fuzzy numbers. The objective is to assign the jobs to the machines such that the latest completion time is as early as possible, if the machines work in parallel.

Jobs	Machines			
	1	2	3	4
A	((3,5,6,8), (2,4,7,10);0.6,0.1)	((5, 8, 11, 13), (4,6,12,14);0.7,0.2)	((8, 10, 11, 15), (7,9,13,17);0.5,0.3)	((5, 8, 10, 12), (4,7,11,13);0.5,0.3)
B	((7, 9, 10, 12), (6,8,11,13);0.7,0.1)	((3, 5, 6,8), (1,4,7,10);0.4,0.3)	((6, 8, 10, 12), (5,7,11,13);0.7,0.1)	((5, 8, 10, 12), (4,6,11,13);0.8,0.1)
C	((2, 4, 5,7), (1,3,6,8);0.6,0.1)	((5, 7, 10, 12), (4,6,11,14);0.7,0.1)	((8, 11, 13, 15), (7,9,14,16);0.6,0.2)	((4, 6, 7, 10), (2,5,8,11);0.8,0.1)
D	((6, 8, 10, 12), (5,7,11,13);0.8,0.1)	((2, 5, 6, 8), (1,3,7,9);0.7,0.1)	((5, 7, 10, 14), (4,6,12,15);0.6,0.2)	((2, 4, 5, 7), (1,3,6,8);0.7,0.1)

Solution: We first form the matrix $C = [c_{ij}] = [\Re(\tilde{c}_{ij})]$ of ranks by using the given ranking method. It is given by

$$C = [c_{ij}] = \begin{bmatrix} 1.621 & 3.366 & 4.366 & 3.506 \\ 3.035 & 2.204 & 2.875 & 3.072 \\ 1.318 & 2.724 & 4.061 & 2.298 \\ 3.139 & 1.690 & 3.096 & 1.438 \end{bmatrix}$$

Here $c_0^* = 1.318$ and $c_1^* = 4.366$. Therefore,

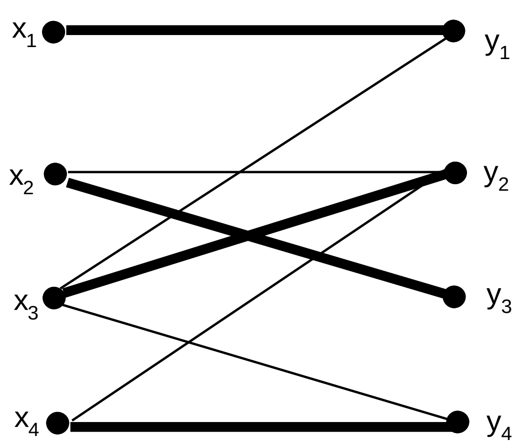
$$C^* = \{c_{ij}: 1.318 < c_{ij} < 4.366\}$$
$$= \{1.438, 1.621, 1.690, 2.204, 2.298, 2.724, 2.875, 3.035, 3.072, 3.096, 3.139, 3.366, 3.506, 4.061\}$$

and median c^* of C^* is 2.875.

The corresponding threshold matrix $\overline{C}[c^*]$ is given by

$$\overline{C}[2.875] = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

The corresponding bipartite graph is



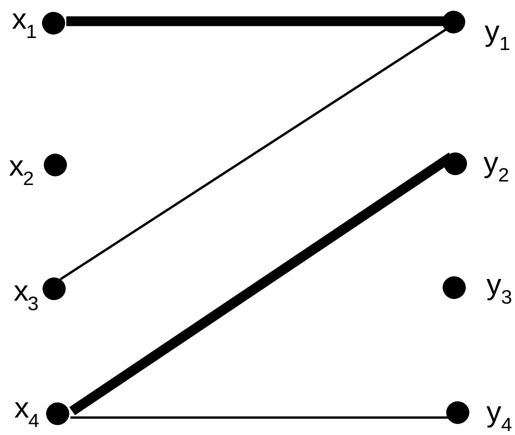
The maximum cardinality matching, shown by the thick lines, has 4 edges. Thus the graph $G[c^*] = G[2.875]$ allows a perfect matching. So set $c_0^* = 1.318$ and $c_1^* = 2.875$. Then

$$C^* = \{c_{ij}: 1.318 < c_{ij} < 2.875\} = \{1.438, 1.621, 1.690, 2.204, 2.298, 2.724\}$$

and median c^* of C^* is 1.690. The corresponding threshold matrix $\overline{C}[c^*]$ is given by

$$\overline{C}[1.690] = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

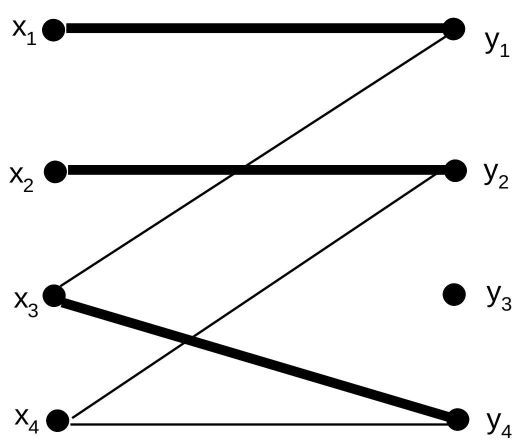
The corresponding bipartite graph is



The maximum cardinality matching, shown by the thick lines, has 2 edges. Thus the graph $G[c^*] = G[1.690]$ does not allow a perfect matching. So set $c_0^* = 1.690$ and $c_1^* = 2.875$. Then $C^* = \{c_{ij}: 1.690 < c_{ij} < 2.875\} = \{2.204, 2.298, 2.724\}$ and median c^* of C^* is 2.298. The corresponding threshold matrix $\overline{C}[c^*]$ is given by

$$\overline{C}[2.298] = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

The corresponding bipartite graph is

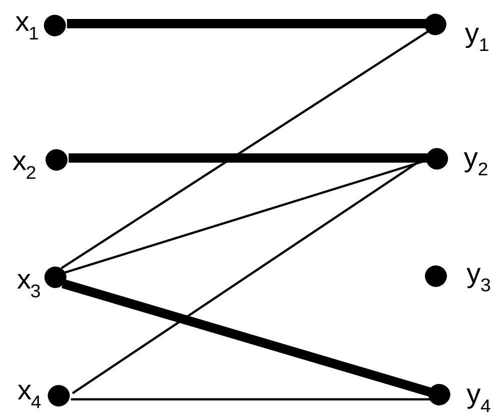


The maximum cardinality matching, shown by the thick lines, has 3 edges. Thus the graph $G[c^*] = G[2.298]$ does not allow a perfect matching. So set $c_0^* = 2.298$ and $c_1^* = 2.875$. Then $C^* = \{c_{ij}: 2.298 < c_{ij} < 2.875\} = \{2.724\}$and median c^* of C^* is 2.724.

The corresponding threshold matrix $\overline{C}[c^*]$ is given by

$$\overline{C}[2.724] = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

The corresponding bipartite graph is



The maximum cardinality matching, shown by the thick lines, has 3 edges. Thus the graph $G[c^*] = G[2.724]$ does not allow a perfect matching. So set $c_0^* = 2.724$ and $c_1^* = 2.875$. Then $C^* = \left\{c_{ij}: 2.724 < c_{ij} < 2.875\right\} = \emptyset$.

So either $G[c_0^*] = G[2.724]$ or $G[c_1^*] = G[2.875]$ allows a perfect matching. Since we have already checked the feasibilities of the current values, we have obtained that that $G[2.875]$ allows a perfect matching and $G[2.724]$ does not allow a perfect matching. Now

$$c_{ij}^* = \min\{c^*: G[c^*] \text{allows a perfect matching}\} = 2.875$$

Hence the fuzzy element corresponding to $c_{ij}^* = 2.875$, that is,
$\tilde{c}_{23} = ((6,8,10,12), (5,7,11,13); 0.7, 0.1)$ is the optimal value of z.

7. 0 Conclusion

In this paper, a new algorithm has been developed for solving intuitionistic fuzzy linear bottleneck assignment problems with completion times as generalized trapezoidal intuitionistic fuzzy numbers, by using the given ranking method. There are several papers in the literature for solving LBAP, but no one has used completion times as generalized intuitionistic fuzzy numbers. The algorithms is easy to understand and can be used for all types of linear bottleneck assignment problems with completion times as crisp, fuzzy and intuitionistic fuzzy numbers.

References

Armstrong, R. D., & Jin, Z. (1992). Solving linear bottleneck assignment problems via strong spanning trees. *Operations Research Letters*, *12*(3), 179-180.

Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *20*(1), 87-96.

Atanassov, K. T. (1989). More on intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *33*(1), 37-45.

Bhatia, H. L. (1977). Time minimizing assignment problem. *Systems and Cybernetics in Management*, *6*(3), 75-83.

Derigs, U. (1984). Alternate strategies for solving bottleneck assignment problems—analysis and computational results. *Computing*, *33*(2), 95-106.

Edmonds, J., & Fulkerson, D. R. (1970). Bottleneck extrema. *Journal of Combinatorial Theory*, *8*(3), 299-306.

Ford, L. R., & Fulkerson, D. R. (1962). *Flows in networks* (Vol. 1962). Princeton University Press: Princeton.

Fulkerson, DR, Glicksberg, I, Gross, O (1953). *A production line assignment problem*. Research Memorandum RM-1102, Santa Monica, CA: Rand Corporation.

Gani, A. N., & Mohamed, V. N. (2015). A method of ranking generalized trapezoidal intuitionistic fuzzy numbers. *International Journal of Applied Engineering Research*, *10*(10), 25465-25473.

Garfinkel, R. S., & Nemhauser, G. L. (1970). Optimal political districting by implicit enumeration techniques. *Management Science*, *16*(8), B-495.

Gross, O. (1959). *The bottleneck assignment problem* (No. P-1630). RAND CORP SANTA MONICA CALIF.

Kuo, C. C. (2011). Optimal assignment of resources to strengthen the weakest link in an uncertain environment. *Annals of Operations Research*, *186*(1), 159-173.

Kuo, C. C., & Nicholls, G. (2014). A turnpike approach to solving the linear bottleneck assignment problem. *The International Journal of Advanced Manufacturing Technology*, *71*(5-8), 1059-1068.

Mazzola, J. B., & Neebe, A. W. (1988). Bottleneck generalized assignment problems. *Engineering Costs and Production Economics*, *14*(1), 61-65.

Page, E. S. (1963). A note on assignment problems. *The Computer Journal*, *6*(3), 241-243.

Pentico, D. W. (2007). Assignment problems: A golden anniversary survey. *European Journal of Operational Research*, *176*(2), 774-793.

Ravindran, A., & Ramaswami, V. (1977). On the bottleneck assignment problem. *Journal of Optimization Theory and Applications*, *21*(4), 451-458.

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*(3), 338-353.