Enhanced Knowledge in Sciences and Technology Vol. 3 No. 2 (2023) 016-024
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**EKST**

Homepage: http://publisher.uthm.edu.my/periodicals/index.php/ekst
e-ISSN : 2773-6385

# Numerical Analysis on MHD Stagnation-Point Flow Towards a Stretching Sheet using bvp4c Method with Slip Effect

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DOI: https://doi.org/10.30880/ekst.2023.03.02.003
Received 15 January 2023; Accepted 19 February 2023; Available online 30 November 2023

**Abstract:** The steady two-dimensional stagnation-point flow over a linearly stretching sheet in a viscous and incompressible fluid in the presence of a magnetic field is studied. The derivation of the governing equation from nonlinear partial differential equation to nonlinear ordinary differential equation are done by using similarity transformation. The problem is solved numerically by using shooting technique through bvp4c method in MATLAB R2015a. The results obtained are analysed, compared and are depicted in tables and graphs. It is found that the present results are in good agreement with the previous related results. For stretching sheet, as magnetic parameter $\boldsymbol{M}$ increases, the skin friction coefficient $\boldsymbol{f''(0)}$ and the Nusselt number $\boldsymbol{-\theta'(0)}$, decrease and increase respectively, while the heat transfer rate increases as the sheet stretch. Furthermore, both the skin friction coefficient and the heat transfer rate increase in the presence of slip.

**Keywords**: Magnetohydrodynamic, Stagnation-Point, Stretching Sheet, Slip

## 1. Introduction

Magnetohydrodynamic or MHD for short is a physical-mathematical framework for studying magnetic fields dynamic in electrically conducting fluids like plasmas and liquid metals. It was first introduced by Hannes Alfven in 1942. There is a difference between a stretching and shrinking surface. In comparison to shrinking sheets, research on stretching sheet has been more extensive over the years since the flow and heat transfer of a viscous and incompressible fluid over a stretching sheet has sparked significant interest among researchers due to its applications in many industrial processes such as, creating polymer fibers, metal continuous casting, manufacturing plastic covers, making paper plates, cooling large magnetic plates in baths, and suspending particles among others. Crane [1] was the first to study and provide an accurate solution in closed analytical form for the two-dimensional steady flow of an incompressible viscous fluid caused by a linearly stretched plate. Since then, various authors have conducted study on various elements of this issue.

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Mahapatra & Gupta [2] studied the steady two-dimensional stagnation-point flow of an incompressible viscous electrically conducting fluid over a flat deformable sheet when the sheet is stretched in its own plane with a velocity proportional to the distance from the stagnation-point. When the magnetic field is increased, the velocity at a place increase when the free stream velocity is larger than the stretching velocity, and the opposite is seen.

In a later study, Ishak et al. [3] made an analysis of the steady two-dimensional magneto-hydrodynamic flow of an incompressible viscous and electrically conducting fluid over a stretching vertical sheet in its own plane. In this study, physical parameters such as power-law velocity where it was assumed that the stretching velocity, the surface temperature and the transverse magnetic field are varied in a power-law with the distance from the origin. The equations are then numerically solved using the Keller-box technique of finite differences, and similarity solutions are achieved. The skin friction coefficient together with the local Nusselt number both fall as the magnetic parameter $M$ rises.

In addition, Ishak et al. [4] studied the steady two-dimensional MHD stagnation point flow towards a stretching sheet with variable surface temperature. The governing equations were transformed into ordinary differential equations using similarity transformation and then solved numerically using a finite-difference method, the Keller-box method. The result showed that the magnitude of the skin friction coefficient increases with the magnetic parameter $M$.

Most of the studies did not involve the presence of slips. Then, Wang [5] investigated the flow due to a stretching flat boundary due to partial slip and gave an exact similarity solution of Navier-Stokes equations of the problem.

Later, in the presence of a magnetic field, Aman et al. [6] studied the steady two-dimensional stagnation-point flow towards a linearly stretching and shrinking sheet in a viscous and incompressible fluid. The governing partial differential equations are transformed into nonlinear ordinary differential equations using a similarity transformation, before being solved numerically by a shooting technique. They found that the skin friction coefficient falls while the heat transfer rate at the surface rises when the effect of slip at the boundary is considered. There exist dual solutions for the shrinking sheet, but a unique solution is found in the stretching sheet.

Then, Agbaje et al. [7] studied a new numerical approach to MHD stagnation point flow and heat transfer towards a stretched sheet in a porous medium in the presence of a heat source/sink and suction/injection. The governing partial differential equations are solved by using Chebyshev spectral technique-based perturbation approach.

Recently, Mohamed et al. [8] studied the flow and heat transfer of a blood-based Casson ferrofluid at a stagnation point past a stretching sheet with Newtonian heating boundary conditions using magnetohydrodynamics (MHD). By using similarity transformation, the conversion of the nonlinear partial differential equations, into ordinary differential equations is then numerically solved using the RKF method. Due to the existence of magnetic effects, the blood-based Casson ferrofluid had up to 46% higher surface temperature than a blood-based fluid.

In this paper, we examine the behaviour of magnetohydrodynamic stagnation-point flow towards a stretching sheet with slip effect on the boundary. Based on previous research [6], the influence of magnetic, slip, and stretching factors on the skin friction coefficient and the rate of heat transfer at the surface are numerically calculated and analysed.

## 2. Mathematical formulation

Consider a two-dimensional stagnation-point flow towards a linearly stretching sheet of constant temperature $T_w$ immersed in an incompressible viscous fluid as shown in Figure 1.

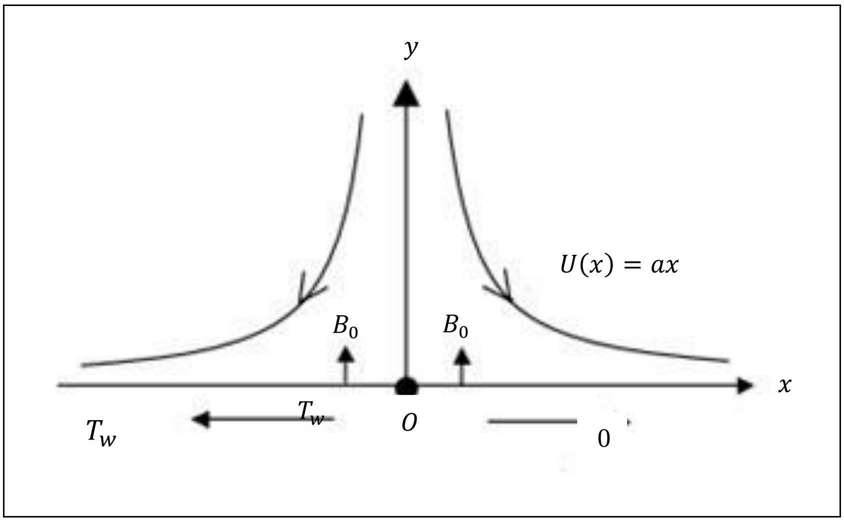


**Figure 1: A sketch of the physical problem**

Assume that the external flow velocity varies linearly along the $x$-axis, i.e., $U(x) = ax$ where $a$ is a positive constant. A uniform magnetic field of strength $B_0$ is assumed to be applied in the positive $y$-direction normal to the plate. The induced magnetic field is assumed to be small compared to the applied magnetic field and is neglected. Under these assumptions along with the boundary layer approximations, the system of equations, which model the boundary layer flow, are given by (see [4] and [9])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{Eq. 1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{\partial U}{\partial x} + \nu\frac{\partial^2 u}{\partial y} + \frac{\sigma B_0^2}{\rho}(U - u) \qquad \text{Eq. 2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \qquad \text{Eq. 3}$$

where $(u, v)$ are the fluid velocities in the $(x, y)$ directions, $T$ is the temperature in the boundary layer, $\nu$ is the kinematic viscosity, $\alpha$ is the thermal diffusity, $\rho$ is the fluid density and $\sigma$ is the electrical conductivity. The appropriate boundary conditions for the velocity components with slip condition at the surface and the temperature are given by (see [9])

$$u = cx + L(\partial u/\partial y), \quad v = 0, \quad T = T_w + S\frac{\partial T}{\partial y} \quad \text{at} \quad y = 0$$

$$u \to U(x), \qquad T \to T_\infty \quad \text{as} \quad y \to \infty \qquad \text{Eq. 4}$$

where $c$ is the stretching rate of the sheet with $c > 0$, $L$ denotes the slip length, $S$ is a proportionality constant and $T_\infty$ is the ambient temperature.

The following is the similarity transformation:

$$\eta = \left(\frac{U}{\nu x}\right)^{1/2} y, \qquad \psi = (\nu x U)^{1/2} f(\eta), \qquad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \qquad \text{Eq. 5}$$

where $\eta$ is the independent similarity variable, $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ is the dimensionless temperature and $\psi$ is the stream function defined as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, which identically satisfies Eq. 1. Using Eq. 5, we obtain

$$u = xaf'(\eta) \text{ and } v = -\nu^{1/2}a^{1/2}f(\eta) \qquad \text{Eq. 6}$$

where primes denote differentiation with respect to $\eta$.

Substituting Eq. 5 and 6 into Eq. 2 and Eq. 3, we obtain the following nonlinear ordinary differential equations:

$$f'''(\eta) + f(\eta)f''(\eta) + 1 - f'^2(\eta) + M\big(1 - f'(\eta)\big) = 0 \qquad \text{Eq. 7}$$

$$\theta''(\eta) + \mathrm{Pr}\, f(\eta)\theta'(\eta) = 0 \qquad \text{Eq. 8}$$

where $M = \sigma B_0^2/\rho a$ is the magnetic parameter and $\mathrm{Pr} = \nu/\alpha$ is the Prandtl number. The boundary conditions Eq. 4 now become

$$f(0) = 0, \qquad f'(0) = \varepsilon + \delta f''(0), \qquad \theta(0) = 1 + \gamma\theta'(0),$$

$$f'(\eta) \to 1, \qquad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty \qquad \text{Eq. 9}$$

where $\varepsilon = c/a$ is the stretching parameter, with $\varepsilon > 0$, $\delta = L(a/\nu)^{1/2}$ is the velocity slip parameter and $\gamma = S(a/\nu)^{1/2}$ is the thermal slip parameter.

The physical quantities of interest are the skin friction coefficient $C_f$ and the local Nusselt number $Nu_x$, which are defined as

$$C_f = \frac{T_w}{\rho U^2/2}, \quad Nu_x = \frac{xq_w}{k(T_w - T_\infty)} \qquad \text{Eq. 10}$$

where the surface shear stress $\tau_w$ and the surface heat flux $q_w$ are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \qquad \text{Eq. 11}$$

with $\mu$ and $k$ being the dynamic viscosity and the thermal conductivity, respectively. Using the similarity variables Eq. 5, we obtain

$$\frac{1}{2}C_f \mathrm{Re}_x^{1/2} = f''(0), \quad \frac{Nu_x}{\mathrm{Re}_x^{1/2}} = -\theta' \qquad \text{Eq. 12}$$

where $\mathrm{Re}_x = U_x/\nu$ is the local Reynolds number.

## 3. Results and Discussion

The transformed Eq. 7 and Eq. 8 subjected to the boundary conditions Eq. 9 are solved numerically using bvp4c method with shooting techniques for some values of the governing parameters, namely, the magnetic parameter $M$, the velocity slip parameter $\delta$, the thermal slip parameter $\gamma$, the stretching parameter $\varepsilon$ and the Prandtl number Pr. The numerical results are shown in the form of table for values of skin friction coefficient and graphs for velocity profiles and temperature profiles.

Comparison of the values of skin friction coefficient $f''(0)$ for the stretching parameter ($\varepsilon > 0$) with those obtained by Wang [10] for several values of $\varepsilon$ are listed in Table 1. It can be observed that the present results are in good agreement with the existing results which the values of $f''(0)$ decreases as the stretching parameter increases.

**Table 1: Comparison with previously published data for the values of $f''(0)$, when $M = 0$ and $\delta = 0$ (no slip) for stretching case ($\varepsilon > 0$)**

| $\varepsilon$ | $f''(0)$ | |
|---|---|---|
| | Present Results | Wang [10] |
| 0 | 1.232591 | 1.232588 |
| 0.1 | 1.146563 | 1.14656 |
| 0.2 | 1.051132 | 1.05113 |
| 0.5 | 0.713296 | 0.71329 |
| 1 | 0 | 0 |
| 2 | -1.887307 | -1.88731 |
| 5 | -10.264749 | -10.26475 |

Meanwhile, for the heat transfer rate at the surface, Table 2 depicts the value of $-\theta'(0)$ increases as the sheet stretching sheet increases.

**Table 2: The values of $-\theta'(0)$, when $M = 0$ and $\delta = 0$ (no slip) for stretching case ($\varepsilon > 0$)**

| $c$ | $-\theta'(0)$ |
|---|---|
| | Present Results |
| 0 | 0.363361 |
| 0.1 | 0.373681 |
| 0.2 | 0.383339 |
| 0.5 | 0.409045 |
| 1 | 0.443806 |
| 2 | 0.494767 |
| 5 | 0.582700 |

The velocity and temperature profiles for different values of the magnetic parameter are depicted in Figure 2 and Figure 3, respectively when $\Pr = 1$, $\varepsilon = 1.5, \delta = 0.5$ and $\gamma = 0.1$. As seen in these figures, an increase in the magnetic parameter $M$ cause the velocity profile to become more curved and hence lead to an increase in the thickness the boundary layer. As a result, the skin friction coefficient $f''(0)$ decrease. Meanwhile, the increase in $M$ does not disturb the boundary layer, thus the temperature profile of the fluid did not changed. Overall, the skin friction coefficient $f''(0)$ and the heat transfer rate at the surface, $-\theta'(0)$, decreases and increases respectively as the magnetic parameter $M$ increases.

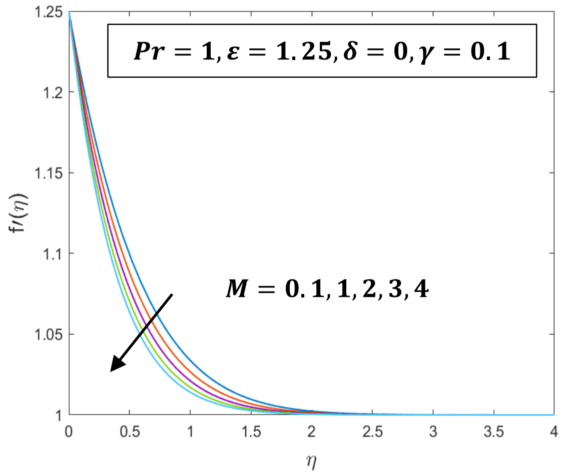


**Figure 2: Velocity profiles for several values of $M$**

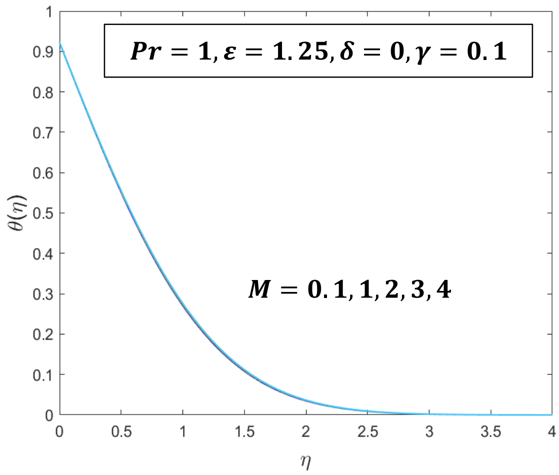


**Figure 3: Temperature profiles for several values of $M$**

Next, the velocity and temperature profiles for different values of the certain values of thermal slip $\gamma$ are depicted in Figure 4 and 5, respectively when $\mathrm{Pr} = 1, M = 0.5, \varepsilon = 1.5$ and $\delta = 0.5$ . As seen in these figures, Figure 2 shows changes in the value of thermal slip $\gamma$ do not affect the velocity profile since the momentum equation is not coupled tot the thermal slip parameter. However, rises of $\gamma$ leads to the decrease of the boundary layer thickness and a drop in the temperature profile as shown in Figure 3. Therefore, skin friction coefficient $f''(0)$ remains unchanged with different values of $\gamma$, while the heat transfer rate $-\theta'(0)$ at the surface decreases as $\gamma$ increases.

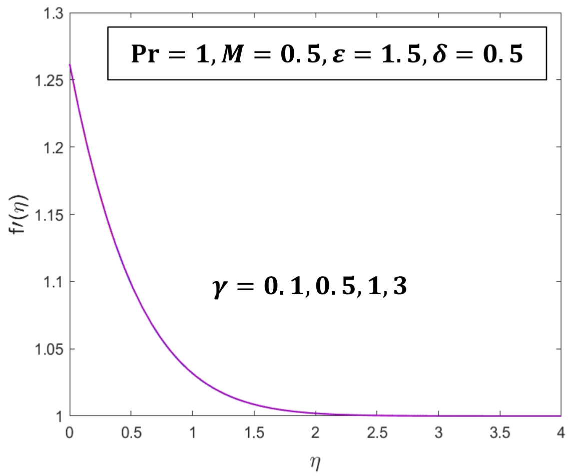


**Figure 4: Velocity profiles for several values of $\gamma$**

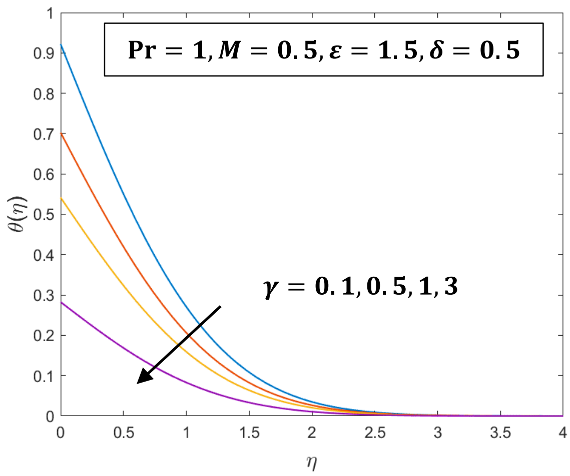


**Figure 5: Temperature profiles for several values of $\gamma$**

Additionally, Figures 6 and 7 show the velocity profile and temperature profile for certain values of velocity slip $\delta$, respectively when $\mathrm{Pr} = 1, M = 0.5, \varepsilon = 1.5$ and $\gamma = 0.5$. As seen in Figure 6, the boundary layer thickness and the velocity profile decrease as the $\delta$. On the other hand, as shown in Figure 7, an increase in $\delta$ leads to decrease in the temperature profile though there is no much differences. The $\delta$ does not effect on the boundary layer thickness. Thus, it can be concluded that as $\delta$ increases, the skin friction coefficient $f''(0)$ and the Nusselt number $-\theta'(0)$ increases.

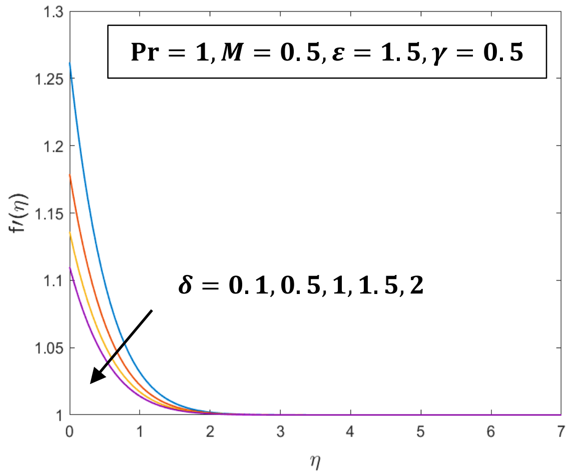


**Figure 6: Velocity profiles for several values of $\delta$**

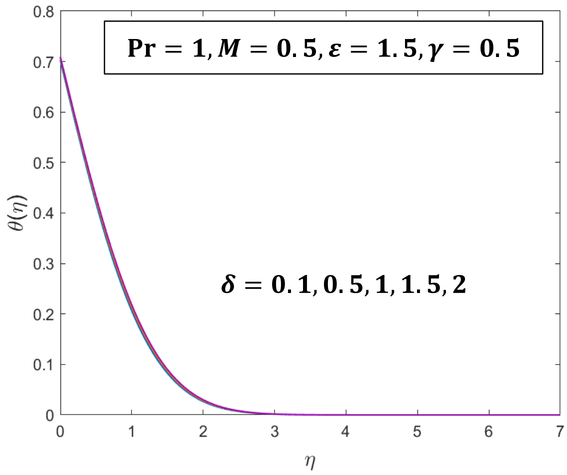


**Figure 7: Temperature profiles for several values of $\delta$**

Lastly, Figures 8 and 9 show the velocity profile and temperature profile for specific values of the Prandtl number Pr, respectively when $M = 0.5, \varepsilon = 1.5, \delta = 0.5$ and $\gamma = 0.5$. Figure 8 indicates the velocity profile remains unchanged with different values of Prandtl number Pr as the equations governing the velocity and temperature profiles are not connected. In contrast, the temperature profile decreases as Pr increases since thermal boundary layer thickness decreases as the Pr increases which lead to a decline in the values of the wall temperature in the boundary layer. This implies the Nusselt number also decreases. The Prandtl number influences the fluid's thermal conductivity; as the Prandtl number decreases, the fluid's thermal conductivity becomes higher. It is worth noting that as the Prandtl number value increases, the thermal diffusivity value decreases, leading to a reduction in energy transfer and a decrease in the thermal boundary layer.

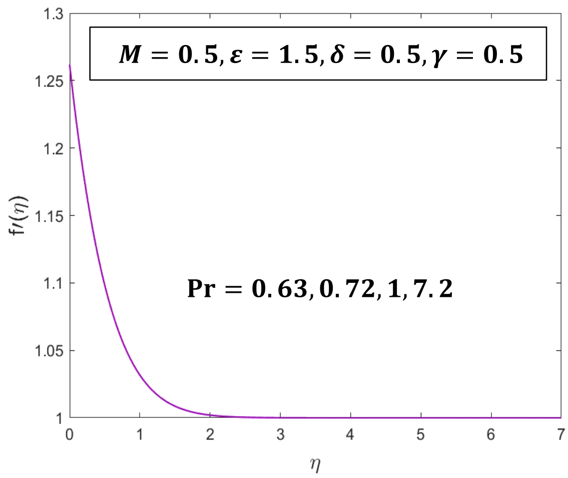


**Figure 8: Velocity profiles for several values of Pr**

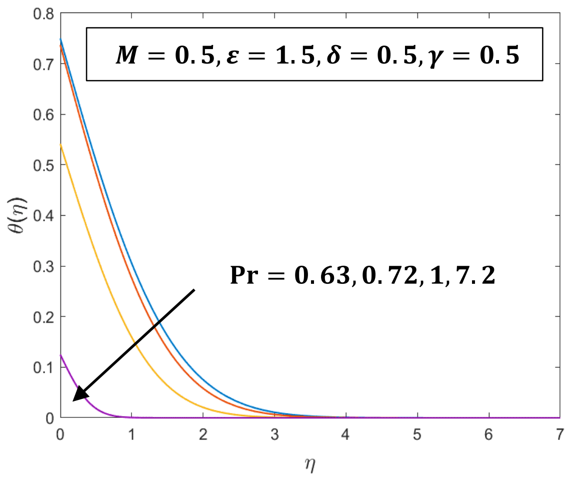


**Figure 9: Temperature profiles for several values of Pr**

## 4. Conclusion

The problem of MHD stagnation point flow towards stretching sheet with slip has been solved numerically. The conversion of the governing equations from the nonlinear partial differential equations to nonlinear ordinary differential equations of the problem has been done by similarity transformation.

Next, the numerical solution for MHD stagnation-point flow towards a stretching sheet has been obtained numerically using the shooting technique and the bvp4c method in MATLAB R2015a. From the results obtained, it can be concluded that:

- The comparison has been made and the present results are in good agreement with the previous results.
- The value of Nusselt number $-\theta'(0)$ increases as the sheet stretched.
- The skin friction coefficient $f''(0)$ and the heat transfer rate at the surface, $-\theta'(0)$, decrease and increase respectively as the magnetic parameter M increases.
- The heat transfer rate $-\theta'(0)$ at the surface decreases as the thermal slip parameter $\gamma$ increases.
- As velocity parameter $\delta$ increases, the skin friction coefficient $f''(0)$ and the Nusselt number $-\theta'(0)$ increases.
- The heat transfer rate at the surface becomes lower as Prandtl number Pr increases.

The limitation of current study can be seen on method comparison as different researchers may use different numerical methods to solve MHD flow problems, each with its own advantages and limitations. For example, the bvp4c method used in this current research is a boundary value problem solver that can handle problems with nonlinear boundary conditions. However, it may not be as effective as other methods, such as finite element methods or finite volume methods, in handling more complex geometries or flow configurations.

For further study, it is recommended to study on the solution for the local Nusselt number for stretching sheet and compare with the present result. The research can be extended to know whether the solution will get dual solutions or unique solution. In addition, we can also obtain the governing equation and find the numerical solution using alternative methods, rather than using similarity transformation and MATLAB R2015a.

## Acknowledgement

The authors would like to thank the Faculty of Applied Sciences and Technology, Universiti Tun Hussein Onn Malaysia for its support.

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