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Numerical Analysis on a Boundary Layer Flow of a Dusty Fluid Over a Permeable Shrinking Surface

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Abstract: The numerical analysis of the boundary layer flow of a dusty fluid over a permeable shrinking surface is studied. The purpose of this research is to study the effects of the governing parameters on the profiles of fluid and dust phases when there is fluid suction at the surface. Using the similarity transformation, the problem's governing equations are transformed into a system of ordinary differential equations. Then the Runge-Kutta-Fehlberg method in the Maple software is used to numerically solve the problem by using the shooting technique. The obtained results are observed, compared, and represented in tables and graphs. The current findings are found to be in strong agreement with the results of previously published. The fluid phase and dust phase velocities increased and the particle velocities decreased at the surface due to the drag coefficient L. The dimensionless fluid-phase velocity profiles and the dimensionless dust-phase velocity profiles both increase as a result of the increase in the suction velocity through the shrinking sheet. The findings of this research add to the growing body of literature on the dusty fluid boundary layer problem. The dusty fluid is important in a variety of practical applications, including the transport of suspended powdered materials through pipes, rocket propulsion and combustion, blood flow in arteries, wastewater treatment, and as corrosive particles in engine oil flow.

Keywords: Boundary Layer, Dusty Fluid, Fluid Suction, Shrinking Surface, Skin Friction

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1. Introduction

A dusty fluid is a liquid or gas combination containing tiny dust particles. They are very useful in a variety of applications, including carrying suspended powdered products through pipes, rocket propulsion and combustion, and blood flow in arteries. Furthermore, dusty fluid is utilized to treat wastewater and acts as corrosive particles in engine oil flow. To simplify the apparent challenges inherent in dusty fluid dynamics, various assumptions have been made in the majority of the existing literature [1].

Although most people understand that the term fluid refers to a wide range of materials, including water and air, a volume of fluid cannot keep its form for long period unless it is restricted by surrounding surfaces. It is undeniable that if the walls of a barrel of water were abruptly removed, the cylinder of the fluid inside would immediately collapse and spread out into a thin layer over a wide area. If the water reaches a new set of boundaries the motion will stop. In addition, a fluid is a substance that deforms constantly when exposed to shear stress, regardless of how small the shear force is. A fluid can also be described as any substance that, while at rest, cannot endure a shear stress, since fluid motion continues when a shear stress is applied [2].

The flow of the boundary layer generated by a shrinking sheet has recently attracted a lot of attention. The shrinking sheet's boundary velocity is directed towards a fixed point, unlike the stretching sheet. The flow towards the shrinking sheet is likely to occur under two physical conditions, and the shrinking sheet's velocity can be limited in the boundary layer. One is that the boundary is subjected to enough suction. Then, the other one is that a stagnation flow is considered. Many researchers have researched the shrinking sheet boundary layer flow problem because of its relevance in industries that require packaging procedures, such as shrink wrapping [3].

Hamid et al. [4] studied the unstable stretching/shrinking flow of a fluid-particle suspension in the presence of continuous suction and dust particle slip on the surface using numerical methods. The governing partial differential equations for the fluid and dust particle two-phase flows are reduced to the applicable ordinary differential equations using a similarity transformation. The numerical results are obtained using MATLAB software's bvp4c function. The results showed that in a decelerating shrinking flow, the dusty fluid had higher wall skin friction than the clean fluid. In addition, in a shrinking flow, the influence of the fluid-particle interaction parameter on the fluid-phase may be seen more clearly. Other non-dimensional physical characteristics studied and shown in the figures include unsteadiness, mass suction, viscosity ratio, particle slip, and particle loading. Moreover, in this problem, the second solution is identified, and higher unsteadiness and suction values are added to the solution. As a result, a stability analysis is performed, and the second solution is proven to be unstable.

In this study, the problem of boundary layer flow of a dusty fluid is analyzed over a permeable shrinking surface researched by [4] by using a different method which is the shooting technique with Runge-Kutta-Fehlberg (RKF45) in Maple software. The sheet's surface is assumed to be permeable, allowing suction to be imposed.

2. Mathematical formulation

Consider the steady boundary layer flow of an incompressible viscous dusty fluid over a shrinking surface as shown in Figure 1, in which the x-axis is measured alongside the surface and y-axis is normal to it.

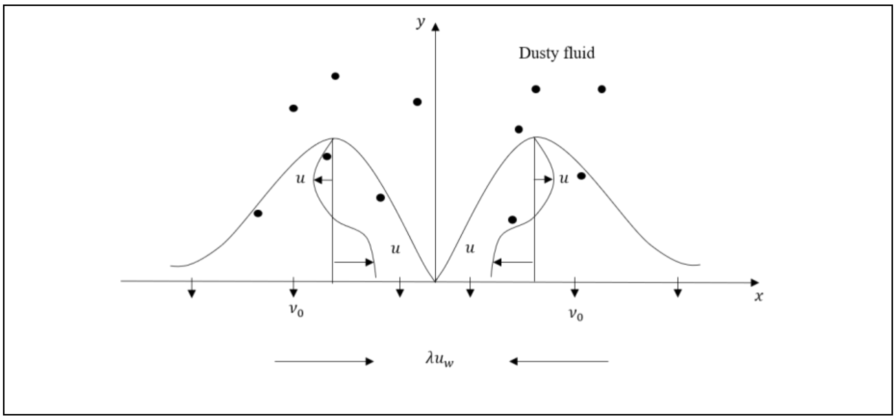

Figure 1: Physical model and coordinate system of shrinking sheet [4]

It is assumed that the sheet is contracted along the x axis with the velocity $u_w(x)$, with the origin being fixed inside the fluid, which is the normal shrinking ($\lambda < 0$) parameter. The constant mass flux velocity is v_0 where $v_0 > 0$ indicates suction and $v_0 > 0$ indicates injection. The fluid and dust particle clouds are meant to be static at the beginning. The dust particles are considered to be spherical and equal in size, with a constant number density throughout the flow.

In the typical area of notation, the basic boundary layer equations for two-dimensional flow [5] are as follows:

For fluid phase

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad \text{Eq. 1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} + \frac{KN}{\rho}\left(u_p - u\right), \qquad \text{Eq. 2}$$

For the dust phase:

$$\frac{\partial}{\partial x}\left(\rho_p\, u_p\right) + \frac{\partial}{\partial y}\left(\rho_p v_p\right) \;=\; 0, \qquad \text{Eq. 3}$$

$$u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial y} = \frac{K}{m}\left(u - u_p\right), \qquad \text{Eq. 4}$$

$$u_p\frac{\partial v_p}{\partial x} + v_p\frac{\partial v_p}{\partial y} = \frac{K}{m}\left(v - v_p\right), \qquad \text{Eq. 5}$$

The appropriate boundary conditions are as follows [5]:

$$v = v_0, \quad u = \lambda\, u_{w(x)} \text{ at y} = 0,$$

$$u \to 0,\ u_p \to 0,\ v_p \to v,\ \ \rho_p \to \rho\omega \text{ as } y \to \infty. \qquad \text{Eq. 6}$$

(u, v) and (u_p, v_p) are the velocity components of the fluid and dust particle phases along x and y directions, respectively. ω is the density ratio while kinematic viscosity is represented by υ. Furthermore, N represents the particle phase's number density, whereas K represents Stokes' resistance (drag coefficient). Then, m is the dust particle's mass and $\rho_p = \rho\rho_r$ is the particle phase's density with ρ_r being the relative density.

The similarity transformation is expressed below:

$$u = cx\, f'(\eta), \quad v = -\sqrt{vc}\, f(\eta), \quad u_p = cx\, F(\eta), \quad v_p = \sqrt{vc}\, G(\eta),$$

$$\rho_r = H(\eta), \quad \eta = \left(\frac{c}{v}\right)^{1/2} y, \qquad \text{Eq. 7}$$

where prime denotes differentiation with respect to η. By substituting Eq. 7 into Eq. 2 to Eq. 5, the following ordinary differential equations are obtained:

$$f'''(\eta) + ff''(\eta) - f'(\eta)^2 + LH\big(F(\eta) - f'(\eta)\big) = 0, \qquad \text{Eq. 8}$$

$$H'(\eta)G(\eta) + H(\eta)G'(\eta) + F(\eta)H(\eta) = 0, \qquad \text{Eq. 9}$$

$$F(\eta)^2 + G(\eta)F'(\eta) + \delta\big(F(\eta) - f'(\eta)\big) = 0, \qquad \text{Eq. 10}$$

$$G(\eta)G'(\eta) + \delta\big(f(\eta) + G(\eta)\big), \qquad \text{Eq. 11}$$

and the boundary condition becomes:

$$f(0) = s, f'(0) = \lambda,$$

$$f'(\eta) \to 0, F(\eta) \to 0, G(\eta) \to -f(\eta), H(\eta) \to \omega \text{ as } \eta \to \infty. \qquad \text{Eq. 12}$$

3. Results and Discussion

To solve the system of ordinary differential equations of this problem, we used the RKF45 method in the Maple software, with shooting techniques for some values of the governing parameters, such as fluid-phase skin friction coefficient $f''(0)$, particle-phase velocity profiles, $F(0)$ and dust-phase velocity profiles, $G(0)$. For the values of the skin friction coefficient and the velocity functions for the dust phase, the numerical results are presented in tables and graphs.

Table 1 lists the comparisons between [4] results for various values of δ and ω and the values of fluid phase skin friction coefficient $f''(0)$. The current results can be seen to be in good agreement with earlier research results in which the values of $f''(0)$ increases when values of ω increase.

Table 1: Comparison values of fluid-phase skin friction coefficient, $f''(0)$ when $L = \delta$, $s = 0$ and $\lambda = 1$ with various values of δ and ω

δ	ω	[4] $-f''(0)$	Present Results $-f''(0)$
0.5	0.2	1.03	1.03
	0.5	1.08	1.08
	1	1.16	1.16
0.2	0.5	1.04	1.04

In the meantime, Table 2 shows the values of particle-phase velocity profiles, $F(0)$ is constantly the same and dust-phase velocity profiles, $G(0)$ decrease when the value of ω increases.

Table 2: Comparison values of fluid-phase skin friction coefficient, $f''(0)$ when $L = \delta$, $s = 0$ and $\lambda = 1$ with various values of δ and ω

δ	ω	[4]		Present Results	
		$F(0)$	$-G(0)$	$F(0)$	$-G(0)$
0.5	0.2	0.339	0.579	0.341	0.581
	0.5	0.339	0.555	0.341	0.557
	1	0.339	0.521	0.341	0.522
0.2	0.5	0.169	0.789	0.166	0.788

Furthermore, various values of the fluid suction parameter, s in the shrinking sheet ($\lambda = -1$) when $L = 0.1$, $\delta = 0.5$ and $\omega = 0.2$ affect the velocity profiles for fluid-phase and dusty-phase. Figures 2 and 3 have shown how the $f'(\eta)$ and $F(\eta)$ both increase as a result of the increase in the suction velocity through the shrinking sheet. The reason is that as more fluid is sucked into the sheet's surface, more fluid flow will accelerate toward the surface, and therefore increasing the fluid velocity. The dusty-phase velocity increases because of the particles being dragged by the fluid flow.

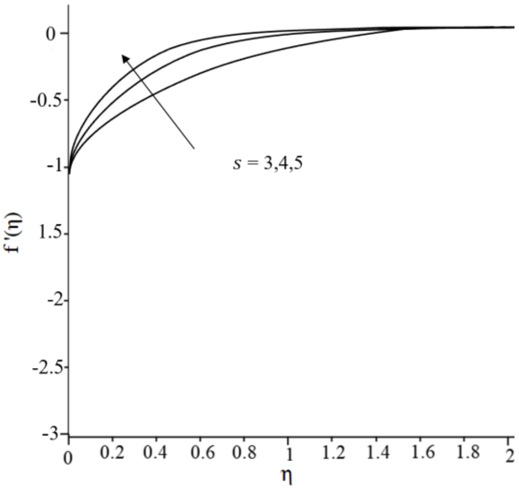

Figure 2: Fluid phase velocity profiles for several values of s in the shrinking sheet

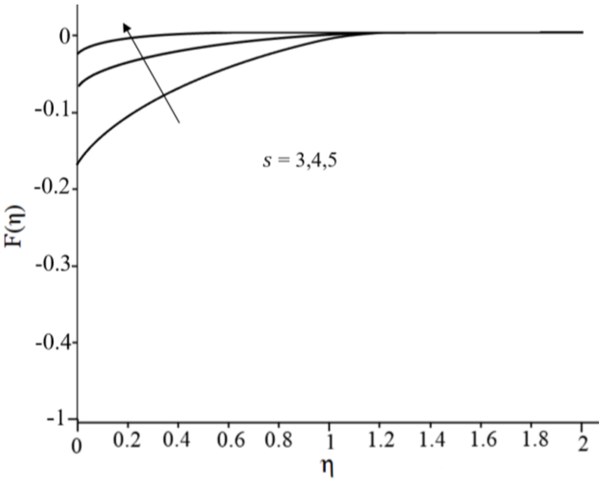

Figure 3: Dust-phase velocity profiles for several values of s in the shrinking sheet

Other than that, with various values of the suction parameter, s, the effect of parameter L on the velocities of the fluid-phase and dust-phase in the shrinking sheet flow are shown. Figures 4 and 5 show that as L increases, the velocity magnitude eventually increases while the thickness of the boundary layer decreases. This can be explained as an increase in Stoke's drag force aims to increase fluid flow resistance and reduce the thickness of the boundary layer. In addition, when the shrinking sheet flow caused the interaction between the fluid and the particles to increase, then the fluid and dust velocities are also increased.

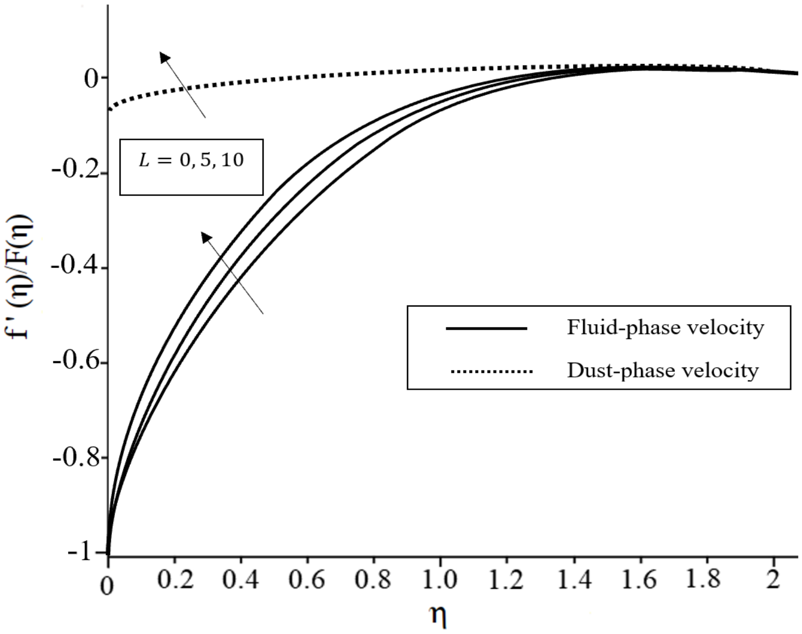

Figure 4: Effects of L on $f'(\eta)$ and $F'(\eta)$ when $s = 3$

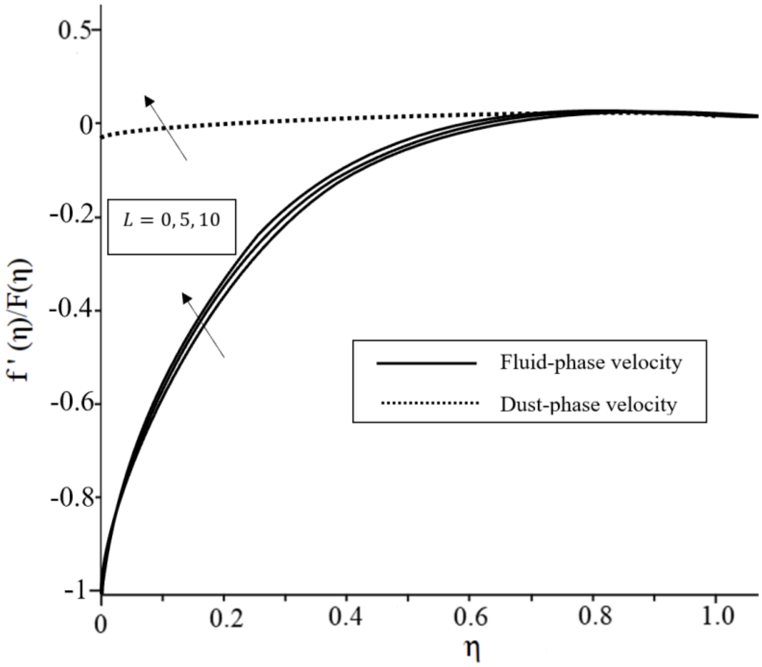

Figure 5: Effects of L on $f'(\eta)$ and $F'(\eta)$ when $s = 5$

Figure 6 shows the effect of fluid-particle interaction parameter δ on the dust phase velocity profiles for different values of s when $L = 0.1, \lambda = -1, \omega = 0.2$. It can be concluded that when δ increases, there is also an increase in the thickness of the boundary layer. In the dust phase velocity, on the other hand, the boundary layer thickness is reduced. Furthermore, when additional fluid is imposed, the thickness of the boundary layer decreases. These results are the result of observing a decrease in the mass concentration of the particle m when the parameter δ is increased.

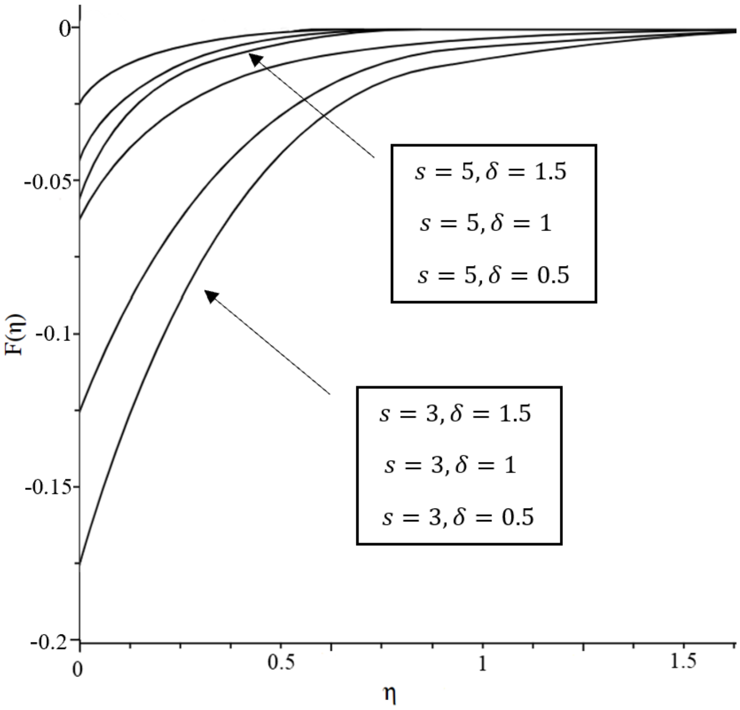

Figure 6: Effects of δ on the dust phase velocity profiles for different values of s

4. Conclusion

The effects of various parameters such as L, s, δ and ω on the numerical analysis of the boundary layer flow of a dusty fluid effect on a shrinking sheet have been studied and obtained. The following are some of the important findings in this issue.

The problem of the boundary layer flow of a dusty fluid over a permeable shrinking sheet has been analyzed using RKF45 method in this study. It has been determined how to transform the governing equation for convective boundary condition problems from a partial differential equation to an ordinary differential equation using a similarity transformation. According to the results, there is a good agreement between the current studies and the previous study [4]. To conclude the results, it has been discovered that:

- The drag coefficient L parameter increased both the fluid-phase and dust-phase velocities, and it is more noticeable at lower suction values.
- Reduced particle phase velocity and reduced particle velocity at the surface are the results of the fluid-particle interaction parameter δ
- The velocity of dust phase at the surface and the fluid skin friction both increased as a result of the particle loading parameter ω.

It is recommended that further research be performed on the second solution in the dusty fluid problem and that a stability analysis be carried out to determine the solution that is stable and physically realizable. The same study should be researched using several approaches, such as the Keller-box method and the Homotopy Analysis Method (HAM), to discover whether there are any parallels or changes in the research results.

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