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Numerical Analysis On MHD Stagnation-Point Flow of Casson Fluid and Heat Transfer Over a Stretching Sheet

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Abstract The MHD-stagnation point flow of Casson fluid over a stretching sheet is studied. The governing momentum and energy equations are transformed to similar nonlinear ODEs through similarity transformations, and then those equations are solved numerically using shooting method in Maple 19. If the magnetic parameter M increase, then the velocity boundary layer thickness becomes thinner. With the Casson parameter (β), the wall skin-friction coefficient's magnitude decreases. This study is really made an impact on demonstrating how mathematical and numerical analysis can assist us in solving problems and obtaining results. The numerical study of MHD stagnation point flow over a stretching sheet is a little difficult, but it has a lot of advantages. This model can be innovated by using multiple numerical method as a solution which many researchers were being not aware on this method since it's a golden opportunity for others to get involved into this method.

Keywords: Magnetohydrodynamic, Casson Fluid Flow, Stagnation-Point, Stretching Sheet

1. Introduction

In fluid flow, the impacts of an external magnetic field on a magnetohydrodynamic (MHD) stream over an extending sheet are exceptionally critical due to its applications in numerous engineering issues, such as glass fabricating, geophysics, paper production, and decontamination of rough oil. Both Andersson [1] and Mukhopadhyay et al. [2] reported the MHD flow and heat transfer over a stretching sheet with varying fluid viscosity. Andersson [3] examined the MHD flow of viscous fluid on a stretching sheet. In his study of the stagnation-point flow towards a stretching sheet with the stretching

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velocity of the plate being equal to the stagnation-point flow's straining velocity, Chiam [4] found no evidence of a boundary layer structure close to the sheet. Mahapatra and Gupta [5] examined the stagnation point flow problem towards a stretching sheet using various stretching and straining velocities and found two distinct types of boundary layers close to the sheet depending on the ratio of the stretching and straining constant.

The Casson fluid, which is one of the most important non-Newtonian rheological models, is a plastic fluid with shear subordinate properties and additional yield stress. When the shear stress exceeds the yield stress, Casson fluid flow occurs. The Casson model was developed for liquids containing bar-like solids and is frequently linked to model bloodstreams and other practical applications such as modern liquid chocolate and related foodstuff handling. The flow induced by stretching the boundary in polymer removal, drawing of copper wires, constant extending of plastic films and recreated strands, hot moving glass fibres, metal ejection, and metal turning are examples of situations where the stretching boundary phenomenon develops.

The non-Newtonian fluid is the fluid which does not follow Newton’s law of viscosity. The viscosity of the fluid change when external forces had been applied to it such as hit and shake. The condition of the fluid depends on the stress that had been applied to them. One of the astonishing examples of non-Newtonian fluid is Oobleck. Oobleck is a fluid made from a mixture of cornflour and water which temporarily become thick when an unusual force is implemented on it. If a force had hit the shear-thickening fluid slowly, the positions of the polymer chain have the time to change and rearrange themselves and the viscosity is not affected. In contrast, if a quick force is applied to it, the polymer chain will take a long time to rearrange itself and the viscosity will increase. For instance, there are a lot of products which have been produced such as ketchup, paint, toothpaste, and, melted butter.

Stagnation-point develops in the flow field at an object's surface when the fluid is stopped by the object. It is a location in a flow where the local velocity of the fluid is zero. According to the Bernoulli equation, static pressure is at its highest when velocity is zero, hence the highest value of static pressure is reached at stagnation points. Stagnation flow has various uses in engineering, including cooling nuclear reactors, polymer extrusion, cooling electronic devices using fans, drawing wire and plastic sheets, and many other hydrodynamic processes. The aim of this research are to transform the governing partial differential equation into ordinary differential equations by using the similarity transformation, to obtain the results using shooting technique of the velocity, temperature profiles, skin Friction and Nusselt number using the shooting technique and Runge-Kutta-Fehlberg and to investigate the effects of the magnetic field parameter, Casson fluid parameter and Prandtl number to the velocity and temperature profiles.

2. Mathematical formulation

The boundary layer of the continuous two-dimensional magnetohydrodynamic (MHD) flow of Casson fluid over the stretched sheet is analysed in this study.

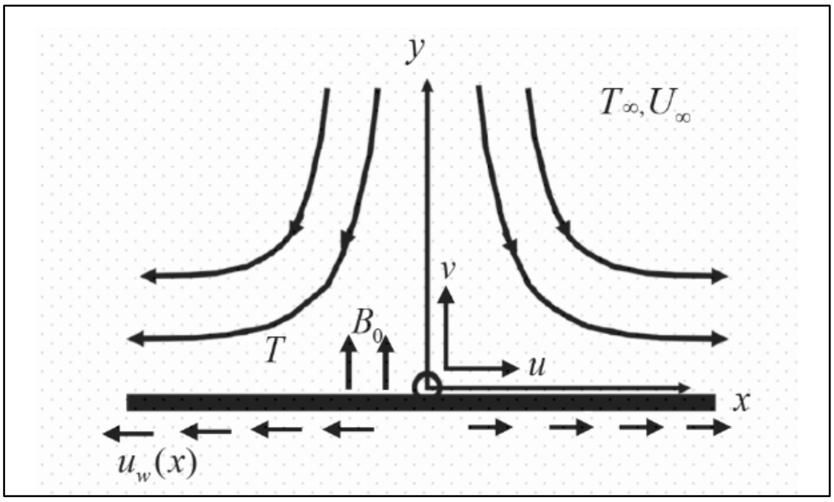

Figure 1: A sketch of the physical problem

It is assumed that Uw is the stretching velocity while u and v are the velocity components in x and y direction which the cartesian coordinates measured along the sheet and normal to it, respectively, B_0 is the application of the magnetic field. Under the previous conditions, the MHD boundary layer equations for steady stagnation-point flow can be written as [6]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad \text{Eq.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \upsilon\left(1 + \frac{1}{\beta}\right)\left(\frac{\partial^2 u}{\partial y^2}\right) - \frac{\sigma B_0^2}{\rho}u, \qquad \text{Eq.2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \kappa\frac{\partial^2 T}{\partial y^2} \qquad \text{Eq.3}$$

where u and v are the velocity components in x and y directions, respectively, $U_s = ax$ is the straining velocity of the stagnation-point flow with a (> 0) being the straining constant, υ is the kinematic fluid viscosity, ρ is the fluid density, $\beta = \mu_{\boldsymbol{B}}\sqrt{\frac{2\pi c}{py}}$ is the non-Newtonian or Casson parameter, σ is the electrical conductivity of the fluid, and B_0 is the strength of the magnetic field applied in the y direction, with the induced magnetic field being neglected. The boundary conditions are:

$$u = U_w, v = v = 0 \text{ at } y = 0$$

$$u \to U_s, T \to T_\infty \text{ as } y \to \infty \qquad \text{Eq.4}$$

From the expression above, $U_w = cx$ is the stretching velocity of c where c is the stretched content. The stream function ψ can be stated as:

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \qquad \text{and } \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \qquad \text{Eq.5}$$

For relations of Eq.5, Eq. 2 is satisfied automatically and Eq.3 takes the following form:

$$\frac{\delta\psi}{\partial y}\frac{\delta^2\psi}{\delta x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=U_s\frac{dU_s}{dx}+\left(1+\frac{1}{\beta}\right)\frac{\partial^3\psi}{\partial y^3}-\frac{\sigma H_0^2}{\rho}\left(\frac{\delta\psi}{\partial y}-U_s\right), \qquad v \qquad \text{Eq.6}$$

Also, boundary conditions in Eq.4 reduce to:

$$\frac{\delta\psi}{\partial y}=U_w, \quad \frac{\partial\psi}{\partial x}=0, \quad \text{at } y=0, \qquad \text{Eq.7}$$

$$\frac{\delta\psi}{\partial y}\to U_s \text{ as } y\to\infty,$$

The stream function's dimensionless variable is now implemented as

$$\psi=\sqrt{cv}xf\ (\eta) \qquad \text{Eq.8}$$

where the similarity variable η is denoted by the formula $\eta=y\sqrt{\frac{c}{v}}$. Finally, using relation Eq.8 and the similarity variable, Eq.6 takes the self-similar form shown below:

$$\left(1+\frac{1}{\beta}\right)f'''+ff''-f'^2-M(f'-B)+B^2=0 \qquad \text{Eq.9}$$

The prime number shows the derivative with respect to η, $M=\frac{\sigma B_0^2}{pc}$ is the magnetic parameter and $B = a/c$ is the velocity ratio parameter. The boundary conditions are as follows:

$$f(\eta)=0, \quad f'(\eta)=1 \quad \text{at } \eta=0,$$

$$f'(\eta)\to B \quad \text{as } n\to\infty. \qquad \text{Eq.10}$$

3 Results and Discussion

Using a shooting technique with RKF45, Maple19 was able to obtain the numerical solutions of the governing ordinary differential equations with the boundary conditions. The magnetic field parameter (M), the Casson fluid parameter (β) and the Prandtl number (Pr) will have an impact on how physical variables such as velocity, thermal fields, skin friction, and Nusselt number behave. The velocity and thermal fields graph for different values of governing parameters was carried out. Figure 2 – Figure 5 verify the roles of all the parameters. For the velocity and temperature fields, respectively, the results are shown in Figures 2 through 3 and Figures 4 through 5.

The behaviour of the Casson fluid parameter, on the velocity, is explained in Figure 2. The various values of β are set as follows, β = 0.7, β =1.4, β = 2.0 and β = 3.0 when magnetic parameter, M = 0.1. The figure shows that the boundary layer velocity and thickness decrease as the Casson fluid parameter increases. As the Casson fluid parameter increases, the yield stress decreases, which in turn constrains the fluid velocity. Additionally, Increasing the value of non-Newtonian parameters can create resistance to fluid movement.

Figure 3 shows the characteristics of the magnetic field parameters, M on the velocity, $f'(\eta)$. The magnetic field parameter M is set as M = 0.0, M = 0.4, M = 0.8 and M =1.2 when Casson fluid parameter, β = 0.1. It is mandatory to observe that both the velocity magnitude and the boundary layer thickness decrease when using larger magnetic field parameters. This phenomenon occurs when the magnetic field of a conducting liquid at the interface produces an opposing force on the liquid called the Lorentz force. This force can slow down the movement of liquids.

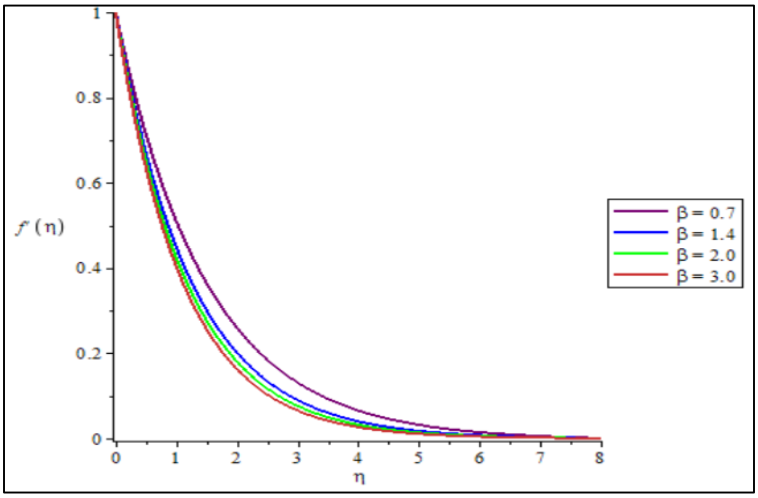

Figure 2: Velocity profiles $f(\eta)$ for several values of β

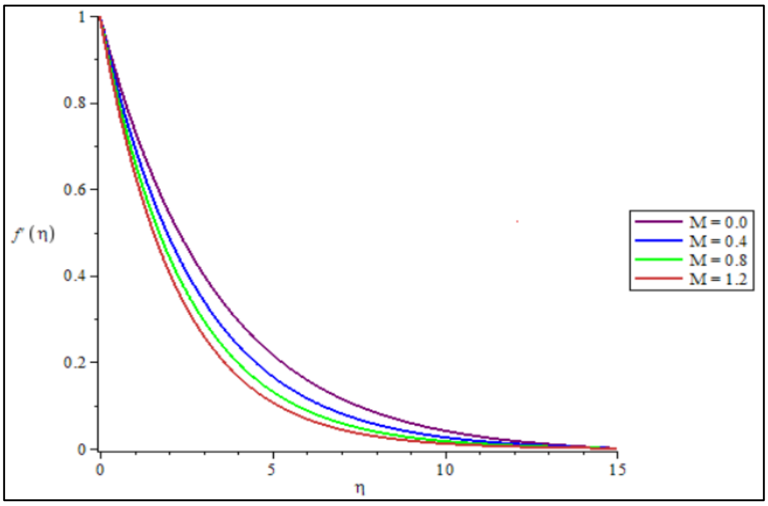

Figure 3: Velocity profiles $f(\eta)$ for several values of M

Figure 4 is set up with M = 0.0, M = 0.4, M = 0.8, and M = 1.2 when the Casson fluid parameter, β = 2.0 and Pr = 1.2. The figure shows that a larger magnetic field parameter, M corresponds to a higher thermal field, and boundary layer thickness. Moreover, due to the large magnetic field parameter, the Lorentz force opposes and amplifies the fluid motion, converting valuable energy into heat. This process causes an increase in the thermal field,$\theta(\eta)$.

Besides, Figure 5 shows the variation of Prandtl Number, Pr on the thermal fields $\theta(\eta)$ if the magnetic field parameter, M = 1.0 and the Casson fluid parameter, $\beta = 2.0$. It has been deeply researched that an increase in the Prandtl number Pr decreases the thermal field, $\theta(\eta)$ and the boundary layer thickness. The Prandtl number is the ratio of kinematic viscosity to thermal conductivity. Therefore, the higher the Pr value, the lower the thermal diffusivity.

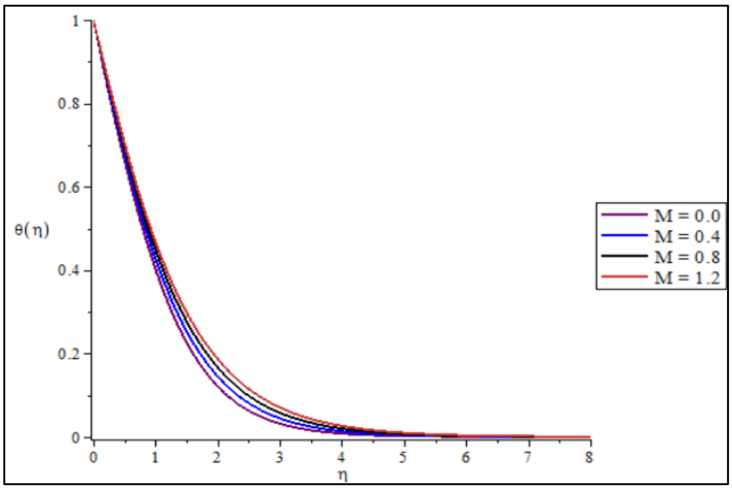

Figure 4: Temperature profiles $\theta(\eta)$ for several values of *M*

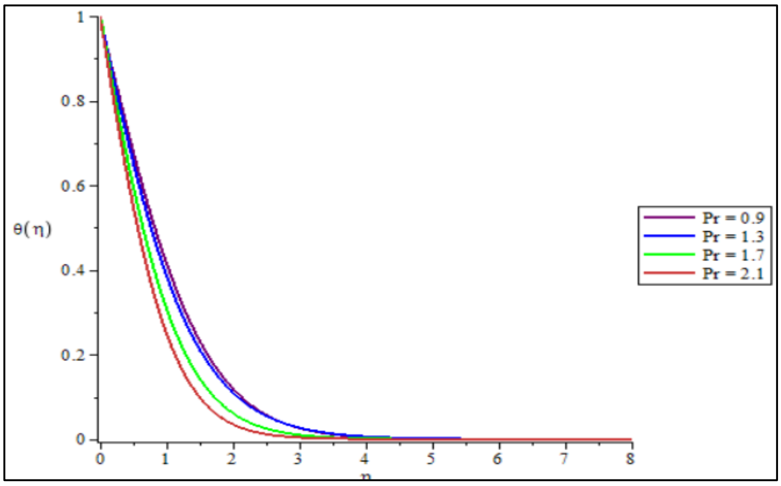

Figure 5: Temperature profiles $\theta(\eta)$ for several values of Pr

4 Conclusion

This research shows magnetohydrodynamic (MHD) implications in Casson fluid stretching sheet flow to analyse the characteristics of temperature and velocity fields. The research has been resolved quantitatively in Maple19 utilizing a shooting method with RK45. Using Maple19, a similarity transformation was used to convert the governing partial differential equations into ordinary differential equations with the boundary conditions. Finding numerical solutions for velocity, thermal fields, skin friction, and Nusselt number as well as looking at the effects of magnetic field parameters, Casson fluid parameters, and Prandtl number are the main objective of this research.

From the results obtained, it can be concluded that:

- When the yield stress reduces with an increase in the Casson fluid parameter (β), the fluid's velocity and boundary layer thickness will decrease.
- The velocity strength and boundary layer thickness will be reduced if a larger magnetic field parameter (M) is used.
- The larger the magnetic field parameter (M) corresponds to the thermal field and the higher the boundary layer thickness.
- Increasing Prandtl number (Pr) can lead to a decrease in the thermal field.

- The incremental values of the Casson liquid parameter (β) and the magnetic field parameter (M) lead to increased skin friction.
- The behaviour of the Nusselt number will be increased if the larger values of the Prandtl number (Pr) are used but the opposite situation occurs for the higher magnetic field parameter (M).

Future research is advised to look at different flow conditions, such as an unsteady flow, viscous flow, or non-viscous flow, to see additional information about the behaviour. In order to see changes in physical behaviour in velocity, thermal fields, skin friction coefficient, and Nusselt number, this study should be improved by employing more than one fluid. For the purpose of comparing the behaviour, this study can be improved by taking into account various stretching sheets conditions, such as a stretching cylinder or a shrinking sheet.

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