Enhanced Knowledge in Sciences and Technology Vol. 3 No. 1 (2023) 128-134
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**EKST**

Homepage: http://publisher.uthm.edu.my/periodicals/index.php/ekst
e-ISSN : 2773-6385

# Mathematical Simulation of Optimal Student Loan Repayment Using First-Order Linear Ordinary Differential Equation

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DOI: https://doi.org/10.30880/ekst.2023.03.01.015
Received 15 January 2023; Accepted 20 March 2023; Available online 3 August 2023

**Abstract:** The PTPTN student loan is the most accessible study loan for supporting Malaysian students to pursue their studies in tertiary education. However, the loan repayment stated in the PTPTN agreement is not made accordingly. This research aims to provide a mathematical simulation for the PTPTN student loan repayment using a first-order linear ordinary differential equation. For this purpose, variable and fixed interests are considered. First, the annual repayment amount is calculated, and the interest amount is determined. With these amounts, the repayment balance is obtained after solving the linear differential equation. Then, the repayment scheme for a chosen period is prepared. After this, a mathematical simulation for different repayments with the desired repayment periods is performed to settle the loan. In addition, the repayment amount decided by students is also simulated so that the possible repayment period in completing the loan will be suggested. From these simulation results, the relationship between the repayment amount and the repayment period is reciprocal, which indicates the increase in the repayment amount will shorten the repayment period and vice versa. In conclusion, the mathematical simulation through the first-order linear ordinary differential equation provides a flexible repayment plan for settling the PTPTN student loan repayment.

**Keywords**: Student Loan Repayment, Mathematical Simulation, First-Order Linear Ordinary Differential Equation, Flexible Repayment Plan

## 1. Introduction

Nowadays, many higher education institutions have been established in Malaysia and more study opportunities for students to pursue higher education levels, especially for tertiary education such as degree level. National Higher Education Fund Corporation (Perbadanan Tabung Pendidikan Tinggi Nasional, PTPTN) is under the Ministry of Higher Education and it was formed on 1 November 1997 [1]. PTPTN student loan was established under the National Higher Education Act 1997 (Act 566) with the main objectives to provide educational loans to eligible students for studying at higher learning

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institutions, provide saving schemes, and manage funds for higher education [2]. Therefore, the PTPTN student loan is one of the most accessible study loans for Malaysian students to support students who have limited financial resources and backgrounds to further their studies at the higher education level.

However, some students do not pay the amount accordingly, and even they do not pay the amount because they do not have any simulation on the loan repayment period and amount. For instance, 74% of PTPTN loan borrowers either have no regular income or earn less than RM2,000 a month, which is the reason they did not complete their PTPTN student loan repayments [3]. Failure to pay the PTPTN student loan will bring a lot of disadvantages, such as being blacklisted in the Central Credit Reference Information System (CCRIS), travel banned, and being rejected immediately from applications for credit cards, house loans or personal loans [4]. Not only that, failure to do the repayment will increase the debt by adding interest to the principal loan [5]. Moreover, the students cannot choose a fixed repayment amount they want to pay each month by using an online loan calculator such as the PTPTN loan calculator. Hence, they are not able to have a plan in shortening their repayment period.

The main focus of this paper is to solve the PTPTN student loan repayment problem with variable and fixed interests using a first-order linear ordinary differential equation. Here, variable interest refers to the amount of interest paid in a reduction value over a repayment period, while fixed interest is the amount of interest paid in a fixed value in a repayment period [5] [6]. Hence, this paper has the following objectives. First, to apply a first-order linear ordinary differential equation for the mathematical simulation of the PTPTN student loan repayment balance with variable and fixed interests. Second, to provide a mathematical simulation for the optimal decision on settlement of the PTPTN student loan repayment. Third, to provide the repayment scheme for illustrating the repayment balance of the PTPTN student loan repayment with variable and fixed interests. Throughout this study, the repayment balances with variable and fixed interests based on the first-order linear ordinary differential equation and its numerical solution will provide, as well as the repayment scheme.

## 2. Materials and Methods

Consider a general model for the student loan repayment [6], given by

$$\dot{x}(t) = px(t) - q, \qquad \textit{Eq. } 1$$

where $p$ is the interest rate and $q$ is the repayment of the loan, where $x(t) \in \Re$ is the balance of the loan at time $t$; while $\dot{x}(t)$ is the rate of change of the balance with respect to time and $px(t)$ is the variable interest to be paid at time $t$. Suppose the initial balance at the initial time $t_0$ is

$$x(t_0) = x_0 ,$$

the solution of the differential equation in *Eq*. 1, which is the balance of the loan given by $x(t)$ can be obtained numerically. This problem is known as the initial value problem of the student loan repayment [6] [7].

### 2.1 Analytical Solutions

Rewrite *Eq*. 1 in the following equation,

$$\dot{x}(t) - px(t) = q, \qquad \textit{Eq. } 2$$

and multiply $e^{-pt}$ to both sides of *Eq*. 2 to give

$$\dot{x}(t)e^{-pt} - px(t)e^{-pt} = qe^{-pt}. \qquad \textit{Eq. } 3$$

Notice that the terms at the left side of the equal sign in *Eq*. 3 are the outcome of the derivative, that is,

$$\frac{d}{dt}\left(x(t)e^{-pt}\right) = qe^{-pt}. \qquad Eq.\ 4$$

Hence, after integrating both sides of *Eq*. 4 and doing some algebraic manipulations, the repayment balance with variable interest is given by the following solution,

$$x(t) = \left(x_0 - \frac{q}{p}\right)e^{p(t-t_0)} + \left(\frac{q}{p}\right). \qquad Eq.\ 5$$

The method used to obtain the solution for *Eq*. 1 is known as the linear equation method [8] [9].

On the other hand, consider *Eq*. 1 in the following equation

$$\dot{x}(t) = px(t_0) + q, \qquad Eq.\ 6$$

with the fixed interest given by $px(t_0)$. Then, solving *Eq*. 6 by integrating both sides of the equation to give the repayment balance with fixed interest as follows,

$$x(t) = x(t_0) + (px(t_0) - q)(t - t_0)\ . \qquad Eq.\ 7$$

2.2 Simulation for Repayment

The simulation of the PTPTN student loan repayment is carried out based on the annual repayment amount and period. The equations used for calculating the annual repayment amount $q$ with variable and fixed interest [10] are given as follows.

(a) The annual repayment with variable interest

$$q = \frac{px_0}{(1-(1+p))^{-n})} \qquad Eq.\ 8$$

(b) The annual repayment with fixed interest

$$q = \frac{x_0(1+np)}{n} \qquad Eq.\ 9$$

The equations used for calculating the repayment period $n$ with variable and fixed interest [11] are provided below.

(a) The repayment period with variable interest

$$n = -\frac{\ln\left(1 - \frac{px_0}{q}\right)}{\ln(1+p)} \qquad Eq.\ 10$$

(b) The repayment period with fixed interest

$$n = \frac{x_0}{q - px_0} \qquad Eq.\ 11$$

## 3. Results and Discussion

Assume that a PTPTN student loan of RM50,000 has been approved with an annual interest of 1 percent and the repayment period is 15 years as shown in Table 1. The mathematical simulation on the loan repayment and the repayment balance will be performed using the GNU Octave.

**Table 1: PTPTN student loan information**

| Loan Amount (RM) | Interest Rate (%) | Repayment Period (Year) |
|---|---|---|
| 50,000.00 | 1.00 | 15 |

### 3.1 Loan Repayment with Variable Interest

The first-order differential equation with variable interest for the student loan repayment given in Table 1 is formulated by

$$x(t)=\left(50000-\frac{3606.19}{0.01}\right)e^{0.01t}+\left(\frac{3606.19}{0.01}\right). \qquad Eq.\ 12$$

Consequently, the repayment scheme is shown in Figure 1 and the repayment results (in RM) are shown in Table 2. In summary, a monthly repayment of RM300.25 should be made, and a total repayment of 54,092.84 with an interest of RM4,092.84 is recorded. Hence, at the end of the repayment period of 15 years, the principal of RM50,000 is settled.

```
         --- Repayment Scheme ---
year     balance          principal        interest         repayment
   1     50000.000        3106.189         500.000          3606.189
   2     46893.811        3137.251         468.938          3606.189
   3     43756.560        3168.623         437.566          3606.189
   4     40587.937        3200.310         405.879          3606.189
   5     37387.627        3232.313         373.876          3606.189
   6     34155.314        3264.636         341.553          3606.189
   7     30890.678        3297.282         308.907          3606.189
   8     27593.396        3330.255         275.934          3606.189
   9     24263.141        3363.558         242.631          3606.189
  10     20899.584        3397.193         208.996          3606.189
  11     17502.390        3431.165         175.024          3606.189
  12     14071.225        3465.477         140.712          3606.189
  13     10605.749        3500.132         106.057          3606.189
  14     7105.617         3535.133         71.056           3606.189
  15     3570.484         3570.484         35.705           3606.189
         --- End of Statement ---
```

**Figure 1: Repayment scheme with variable interest**

**Table 2: Repayment result with variable interest**

| Total Principal | Total Interest | Total Repayment | Monthly Repayment |
|---|---|---|---|
| 50,000.00 | 4,092.84 | 54,092.84 | 300.52 |

### 3.2 Loan Repayment with Fixed Interest

The first-order differential equation with fixed interest for the student loan repayment using the information given in Table 1 is

$$x(t)=50000-3333.33t. \qquad Eq.\ 13$$

As a result, Figure 2 shows the repayment scheme, and Table 3 shows the repayment results (in RM). It is summarized that the total repayment for 15 years is RM57,500 with an interest of RM7,500. The principal of RM50,000 is settled at the end of 15 years with repayment amount of RM319.44 in each month.

```
         --- Repayment Scheme ---
year     balance          principal        interest         repayment
   1     50000.000        3333.333         500.000          3833.333
   2     46666.667        3333.333         500.000          3833.333
   3     43333.333        3333.333         500.000          3833.333
   4     40000.000        3333.333         500.000          3833.333
   5     36666.667        3333.333         500.000          3833.333
   6     33333.333        3333.333         500.000          3833.333
   7     30000.000        3333.333         500.000          3833.333
   8     26666.667        3333.333         500.000          3833.333
   9     23333.333        3333.333         500.000          3833.333
  10     20000.000        3333.333         500.000          3833.333
  11     16666.667        3333.333         500.000          3833.333
  12     13333.333        3333.333         500.000          3833.333
  13     10000.000        3333.333         500.000          3833.333
  14     6666.667         3333.333         500.000          3833.333
  15     3333.333         3333.333         500.000          3833.333
         --- End of Statement ---
```

**Figure 2: Repayment scheme with fixed interest**

**Table 3: Repayment result with fixed interest**

| Total Principal | Total Interest | Total Repayment | Monthly Repayment |
|---|---|---|---|
| 50,000.00 | 7,500.00 | 57,500.00 | 319.44 |

3.3 Simulation Results on Repayment Amount

The simulation results (in RM) on repayment amounts for variable and fixed interests are shown in Tables 4 and 5, respectively. The repayment period is set from 5 to 15 years, and changes in the annual repayment are recorded. From the observation, the annual repayment is increased when the repayment period is decreased. The relationship between the repayment period and the annual repayment is reciprocal.

**Table 4: Simulation results on repayment amount with variable interest**

| Repayment Period | Total Repayment | Total Interest | Annual Repayment | Monthly Repayment |
|---|---|---|---|---|
| 5 years | 51,509.95 | 1,509.95 | 10,301.99 | 858.50 |
| 6 years | 51,764.51 | 1,764.51 | 8,627.42 | 718.95 |
| 7 years | 52,019.90 | 2,019.90 | 7,431.41 | 619.29 |
| 8 years | 52,276.12 | 2,276.12 | 6,534.52 | 544.54 |
| 9 years | 52,533.16 | 2,533.16 | 5,837.02 | 486.42 |
| 10 years | 52,791.04 | 2,791.04 | 5,279.10 | 439.93 |
| 11 years | 53,049.74 | 3,049.74 | 4,822.70 | 401.89 |
| 12 years | 53,309.27 | 3,309.28 | 4,442.44 | 370.20 |
| 13 years | 53,569.63 | 3,569.63 | 4,120.74 | 343.39 |
| 14 years | 53,830.82 | 3,830.82 | 3,845.06 | 320.42 |
| 15 years | 54,092.84 | 4,092.84 | 3,606.19 | 300.52 |

**Table 5: Simulation results on repayment amount with fixed interest**

| Repayment Period | Total Repayment | Total Interest | Annual Repayment | Monthly Repayment |
|---|---|---|---|---|
| 5 years | 52,500.00 | 2,500.00 | 10,500.00 | 875.00 |
| 6 years | 53,000.00 | 3,000.00 | 8,833.33 | 736.11 |
| 7 years | 53,500.00 | 3,500.00 | 7,642.86 | 636.90 |
| 8 years | 54,000.00 | 4,000.00 | 6,750.00 | 562.50 |
| 9 years | 54,500.00 | 4,500.00 | 6,055.56 | 504.63 |
| 10 years | 55,000.00 | 5,000.00 | 5,500.00 | 458.33 |
| 11 years | 55,500.00 | 5,500.00 | 5,045.46 | 420.45 |
| 12 years | 56,000.00 | 6,000.00 | 4,666.67 | 388.89 |
| 13 years | 56,500.00 | 6,500.00 | 4,346.15 | 362.18 |
| 14 years | 57,000.00 | 7,000.00 | 4,071.43 | 339.29 |
| 15 years | 57,500.00 | 7,500.00 | 3,833.33 | 319.44 |

From these simulation results, the relationship of annual repayment with the respective repayment period is reciprocal as presented in Figure 3. The repayment period of 15 years and the monthly repayment of RM319.44 are stated in the PTPTN agreement. However, through these simulation results, students could choose the repayment period for paying the related repayment amount. Although in this study, the loan repayments with variable and fixed interest are studied, indeed, the loan repayment with fixed interest is provided to students.

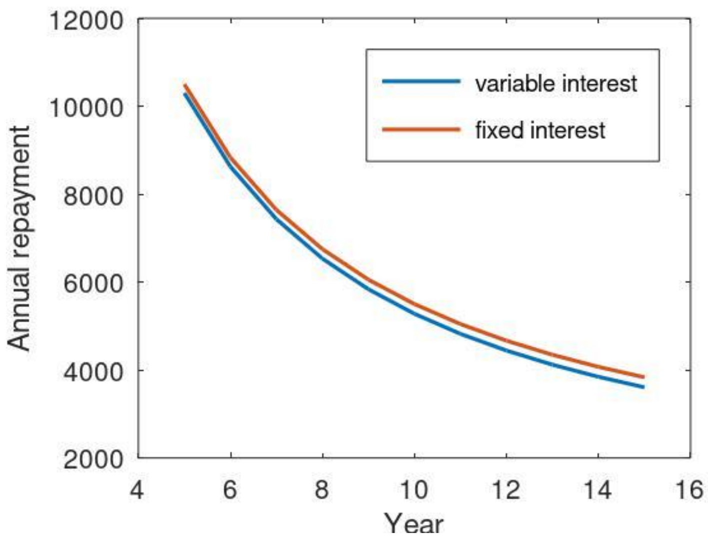


**Figure 3: Relation of annual repayment vs period**

## 4. Conclusion

In this paper, a first-order linear ordinary differential equation was studied to formulate loan repayment as an initial value problem. In addition, the next repayment balance could be identified since the current principal would be updated when the repayment amount was made each year. Therefore, the repayment scheme for 15 years was prepared for illustration, where variable and fixed interests were considered in the differential equations. Moreover, the optimal decision for settling the loan repayment with different repayment amounts and periods was also suggested. For future studies, a nonlinear differential equation is recommended to formulate the complex loan repayment problem. An efficient numerical solver is required to solve the nonlinear model of loan repayment problems.

**Acknowledgement**

The authors would like to thank the Faculty of Applied Sciences and Technology, Universiti Tun Hussein Onn Malaysia for its support.

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