

# Laplace Transform and Finite Difference Method for Solving Problems of Mechanical Systems

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DOI: <https://doi.org/10.30880/ekst.2024.04.01.017>

## Article Info

Received: 27 December 2023

Accepted: 3 June 2024

Available online: 27 July 2024

## Keywords

Laplace transform, Finite Difference Method, Mechanical System, Ordinary Differential Equation

## Abstract

This research explores the application and efficiency of the Laplace transform and the finite difference method on several mechanical equations. The mechanical equations utilised in this research can be divided into three types, namely, homogenous, non-homogenous, and non-linear. The numerical results obtained using both Laplace transform and the finite difference method are compared in terms of absolute error, and graphically illustrated using MATLAB 2023. The results show that the absolute errors between both methods are minimal.

## 1. Introduction

Laplace transform and the finite difference method (FDM) are part of mathematical tools used in mathematical modelling that translate real-life phenomena into mathematical equations, which is important because it allows us to analyse the phenomena's behaviour and predict its outcome. As for the mechanical system, it refers to a group of components that work together to carry out a certain task. It can be mathematically described using ordinary differential equations (ODEs) and partial differential equations (PDEs) to dictate the system's behaviour, describing the dynamics of the system.

As a mathematical tool, the Laplace transform is suitable for determining the behaviour of dynamic systems, including mechanical systems. [1] applied the Laplace transform and the nonstandard finite difference method to solve fractional order electrical circuits and compared with Runge-Kutta (RK) method. The results showed that both proposed methods are more efficient than RK method. In another study [2], they demonstrate that Laplace transform is applicable in economic studies by constructing several economic models and solving them. The results showed that the Laplace transform is effective in solving such models. Besides, [3] investigated the difference of the Laplace transform and the numerical method for systems of ODEs in a naphtha reforming process and the results showed that the Laplace transform produced a better result.

On the other hand, the FDM is used for approximating derivatives of differential equations by dividing the domain into discrete points. In a study done by [4], the author used the FDM and Laplace transform to solve boundary value problems (BVPs) and the results showed that the error of FDM is minimally deviated from Laplace transform. Research in [5] investigated human breast cyst's bio-heat transfer using FDM and they discovered that the numerical results of FDM does not deviate too much with existing analytical results. Moreover, according to research carried out in [6], they used FDM in an Sc-Drying Mass Transport Model and found that FDM exhibited good accuracy and computational efficiency.

Mechanical systems are widely used in real-life applications to simulate more complex systems. However, using only one method may occasionally cause the results obtained to be inaccurate and inapplicable to complex systems. Thus, this study proposed to solve the mechanical systems with more than one method which are

Laplace transform and FDM. Therefore, the objectives of this study are to apply the Laplace transform and FDM to solve mechanical system with known equations of motion and conduct a comprehensive comparison of the results obtained from both proposed methods.

## 2. Methodology

This study will solve several mechanical system problems using Laplace transform and FDM. In this section, methods of Laplace transform and FDM are explained in solving second-order ODEs. The general equation of the mechanical systems is also discussed. The methods of solving have been referred to [7] and [8].

### 2.1 Solving with Laplace Transform

The Laplace transform can be applied to the differential equations as the following. Consider a general form of second-order ODE in the time domain,

$$f(t) = a \frac{d^2 y(t)}{dt^2} + b \frac{dy(t)}{dt} + cy(t), \quad (1)$$

where  $y$  is the unknown function,  $t$  is time and,  $a, b, c$  are constants. The Laplace transform of a derivative is,

$$L\left\{\frac{d^n}{dt^n}[f(t)]\right\} = s^n F(s) - s^{(n-1)} f(0) - \dots - f^{(n-1)}(0), \quad (2)$$

where  $L$  denotes the Laplace transform,  $s$  is the complex variable in the Laplace domain, and  $F(s)$  represents the Laplace transform of the  $f(t)$ . Applying the Laplace to each term in (3), we obtain,

$$Y(s) = \frac{asy(0) + (ay'(0) + by(0) + F(s))}{as^2 + bs + c}. \quad (3)$$

Based on the initial conditions provided, the value of  $y(0)$  and  $y'(0)$  is substituted into (3) before rearranging and isolating  $Y(s)$  on one side of the equation. Finally, apply the inverse Laplace transform to obtain the general solution of  $y(t)$  in the time domain.

### 2.2 Solving with Finite Difference Method

The FDM method is a numerical technique used to approximate derivatives and solve differential equations. The 2-point forward and backward difference, and 3-point central difference are common formulas applied in first order derivatives. Meanwhile, the second-order 3-point central difference is commonly applied in second order derivative.

Considering two neighbouring points  $x + h$  and  $x$ , the function values at these points can be expressed using Taylor series expansions as follows,

$$f(x+h) = f(x) + hf'(x) + \frac{f''(x)}{2!} h^2 + \frac{f'''(x)}{3!} h^3 + \dots + \frac{f^{(n)}(x)}{n!} h^n + \dots \quad (4)$$

The first order derivative formula, the 2-point forward derived from (4) is given by,

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

or can be written in,  $i$ , discrete points as follows,

$$f'(x) = \frac{x_{i+1} - x_i}{h}. \quad (5)$$

By a similar method, the 2-point backward difference can be obtained by taking neighbouring points at  $x$  and  $x - h$  and can be expressed as follows,

$$f'(x) = \frac{x_i - x_{i-1}}{h}. \quad (6)$$

Subtracting (6) from (5), the first-order 3-point central difference can also be shown as follows,

$$f'(x) = \frac{x_{i+1} - x_{i-1}}{2h}. \quad (7)$$

The formula of the second-order 3-point central difference can be derived using the in same method by taking three neighbouring points,  $x + h$ ,  $x$ , and  $x - h$  and the Taylor series expansion up to second order, yields,

$$f'(x) = \frac{x_{i+1} - 2x + x_{i-1}}{h^2}. \quad (8)$$

From (3), by substituting (7) and (8), the equation can be expressed as

$$f(x_i) = a \left( \frac{x_{i+1} - 2x_i + x_{i-1}}{h^2} \right) + b \left( \frac{x_{i+1} - x_{i-1}}{2h} \right) + cx_i. \quad (9)$$

Rearranging the equation, gives the following

$$x_{i+1} = \frac{f(x_i) + x_i(4a - 2h^2c) + x_{i-1}(bh - 2a)}{2a + bh}. \quad (10)$$

The  $h$  in the formulas represents the step size of the neighbouring point. Based on the initial conditions, (12) can be iterated over time until reaching the desired final time.

### 2.3 Mechanical Systems

In a mechanical system, various components such as rigid bodies, joints, linkages, gears, and actuators interact with each other to transmit forces and motion that can be shown in form of equations.

An example of mechanical system is a torsional pendulum. It consists of a mass suspended from a torsion wire that enable the mass to oscillate in a rotational motion. The behaviour of the mass can be understood with the principles of rotational dynamics and Hooke's law for torsion. Thus, the general formula for the period of a torsion pendulum can be derived based on those two. The second order ODE for a torsion pendulum is given by,

$$\theta''(t) + \left( \frac{c}{mL} \right) \theta'(t) + \left( \frac{k}{mL^2} \right) \theta = 0, \quad (11)$$

where  $\theta$  is the angular displacement,  $m$  is the mass,  $L$  is the length of torsion wire,  $c$  is the damping coefficient, and  $k$  is the stiffness of the torsion wire.

Another example of a mechanical system is the single degree-of-freedom (SDOF) system which models the behaviour and response of various physical systems. The motion can typically be written as,

$$mx''(t) + cx'(t) + kx(t) = F(t), \quad (12)$$

where  $m$  is the mass,  $x$  is the displacement of the system,  $c$  is the damping coefficient,  $k$  is the stiffness of the system, and  $F(t)$  is the external force acting on it.

### 3. Result and Discussion

This section demonstrates the solutions of several mechanical equations using the proposed method. Next, the solutions obtained from the FDM will be compared with the exact solution, which is obtained from the Laplace transform. Then, the absolute error of the FDM will be computed and analyzed. The test problems used in this study are retrieved from [7].

### 3.1 Test Problem (a)

A simple pendulum with a length of 1m with a bob is released from an initial angular displacement of 0.2 rad with an initial angular velocity of -0.3 rad/s. Determine the expression that depicts the subsequent angular displacement of the pendulum bob and the acceleration due to gravity is 9.8m/s<sup>2</sup>.

#### Solution

The pendulum can be described through the following equation,

$$\frac{d^2\theta}{dt^2} + 9.8\theta = 0 \tag{13}$$

with the initial conditions  $\theta(0) = 0.2$  and  $\theta'(0) = -0.3$ . For this problem, consider the step size is 0.01 and 0.1. Laplace transform can be applied on (13) and substitute the initial values to obtain,

$$\begin{aligned} (s^2\Phi(s) - s\theta(0) - \theta'(0)) + 9.8\Phi(s) &= 0 \\ (s^2\Phi(s) - 0.2s - 0.3) + 9.8\Phi(s) &= 0 \\ \Phi(s) &= \frac{0.2s - 0.3}{s^2 + 9.8}. \end{aligned} \tag{14}$$

Applying inverse Laplace transform on (14) to get  $\theta(t)$ , the exact solution for this test problem.

$$\theta(t) = \frac{1}{5} \cos\left(\frac{7\sqrt{5}}{5}t\right) - \frac{3\sqrt{5}}{70} \sin\left(\frac{7\sqrt{5}}{5}t\right). \tag{15}$$

For solution using FDM, substitutes (8) into (13), the equation can be expressed in discretized form

$$\begin{aligned} \frac{\theta_{i+1} - 2\theta_i + \theta_{i-1}}{h^2} + 9.8\theta_i &= 0 \\ \theta_{i+1} &= \theta_i(2 - 9.8h^2) - \theta_{i-1}. \end{aligned} \tag{16}$$

Then, using MATLAB, the numerical values of (15) and (16) are computed from  $0 < t < 1$ , with step size of 0.1 and 0.01.

**Table 1** Numerical results of Laplace transform and FDM for Test Problem (a) with  $h = 0.1$

| t   | Laplace transform<br>Y | FDM                     |                          |
|-----|------------------------|-------------------------|--------------------------|
|     |                        | Numerical solution<br>y | Absolute error<br> Y - y |
| 0   | 0.2000000              | 0.2000000               | 0                        |
| 0.1 | 0.1607674              | 0.1700000               | 9.232630E-03             |
| 0.2 | 0.1059078              | 0.1233400               | 1.743220E-02             |
| 0.3 | 0.0407537              | 0.0645927               | 2.383894E-02             |
| 0.4 | -0.0283617             | -0.0004847              | 2.787695E-02             |
| 0.5 | -0.0947203             | -0.0655146              | 2.920564E-02             |
| 0.6 | -0.1518718             | -0.1241241              | 2.774775E-02             |
| 0.7 | -0.1942611             | -0.1705694              | 2.369172E-02             |
| 0.8 | -0.2177678             | -0.2002989              | 1.746887E-02             |
| 0.9 | -0.2201069             | -0.2103991              | 9.707795E-03             |
| 1   | -0.2010512             | -0.1998802              | 1.170938E-03             |

**Table 2** Numerical results of Laplace transform and FDM for Test Problem (a) with  $h = 0.01$

| t | Laplace | FDM |
|---|---------|-----|
|---|---------|-----|

| transform |            | Numerical solution | Absolute error |
|-----------|------------|--------------------|----------------|
|           | $Y$        | $y$                | $ Y - y $      |
| 0         | 0.2000000  | 0.2000000          | 0              |
| 0.1       | 0.1607674  | 0.1617260          | 9.586656E-04   |
| 0.2       | 0.1059078  | 0.1077307          | 1.822860E-03   |
| 0.3       | 0.0407537  | 0.0432628          | 2.509027E-03   |
| 0.4       | -0.0283617 | -0.0254107         | 2.950968E-03   |
| 0.5       | -0.0947203 | -0.0916140         | 3.106272E-03   |
| 0.6       | -0.1518718 | -0.1489115         | 2.960359E-03   |
| 0.7       | -0.1942611 | -0.1917333         | 2.527865E-03   |
| 0.8       | -0.2177678 | -0.2159166         | 1.851166E-03   |
| 0.9       | -0.2201069 | -0.2191107         | 9.962299E-04   |
| 1         | -0.2010512 | -0.2010050         | 4.618358E-05   |

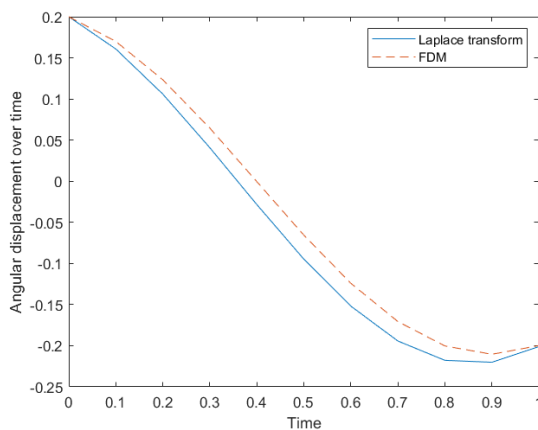


Fig. 1 Laplace transform and FDM for Test Problem (a) with  $h = 0.1$

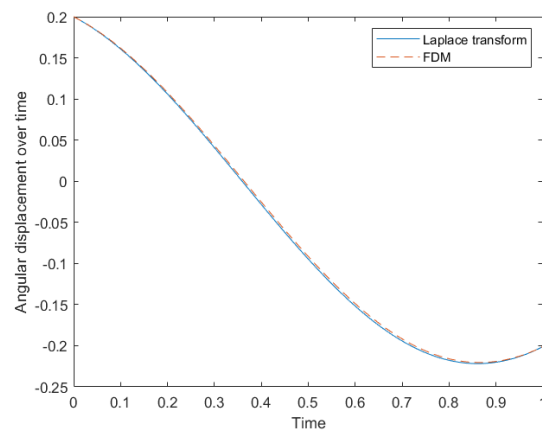


Fig. 2 Laplace transform and FDM for Test Problem (a) with  $h = 0.01$

### 3.2 Test Problem (b)

Suppose that a car with a mass of  $1500kg$  and a suspension system. The combined effect of the springs in the suspension system has a damping coefficient of  $500Ns/m$ , and the stiffness of the suspension springs is  $30000N/m$ . When the car is travelling, it encounters a sudden bump in on the road, resulting in an external force  $F(t) = e^{-t} N$ . Determine the vertical displacement of the car body over time given that the initial vertical displacement and acceleration is 0.

#### Solution

The system's equation of motion can be shown through the following equation,

$$1500 \frac{d^2x}{dt^2} + 500 \frac{dx}{dt} + 30000 x = e^{-t} \tag{17}$$

with the initial conditions  $x(0) = 0$  and  $x'(0) = 0$ . For this problem, assume the step size is 0.01 and 0.1. Apply the Laplace on (17) and then substitute the initial values to get

$$1500 (s^2 X(s) - sx(0) - x'(0)) + 500 (sX(s) - x(0)) + 30000 X(s) = e^{-t}$$

$$1500s^2 X(s) + 500sX(s) + 30000X(s) = \frac{1}{s+1},$$

$$X(s) = \frac{1}{(s+1)(1500s^2 + 500s + 30000)}. \tag{18}$$

Taking the inverse Laplace transform on (20) yields the following equation, where  $x(t)$  is the exact solution for this test problem

$$x(t) = \frac{e^{-t}}{31000} - \frac{e^{-\frac{t}{6}} \left[ \cos\left(\frac{\sqrt{719}}{6}t\right) - \frac{5\sqrt{719}}{719} \sin\left(\frac{\sqrt{719}}{6}t\right) \right]}{31000} \tag{19}$$

For FDM method, substitutes (7) and (8) into (17), the equation can be expressed in discretized form

$$1500 \left( \frac{x_{i+1} - 2x_i + x_{i-1}}{h^2} \right) + 500 \left( \frac{x_{i+1} - x_{i-1}}{2h} \right) + 30000x_i = e^{-t}$$

$$x_{i+1} = \frac{2h^2 e^{-t} + x_i(6000 - 60000 h^2) + x_{i-1}(500 h - 3000)}{3000 + 500 h} \tag{20}$$

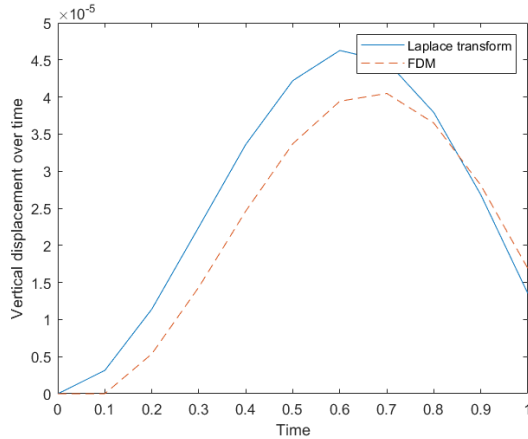
Next, use MATLAB to determine the numerical values of (19) and (20) within the interval  $0 < t < 1$ , using step sizes of 0.1 and 0.01.

**Table 3** Numerical results of Laplace transform and FDM for Test Problem (b) with  $h = 0.1$

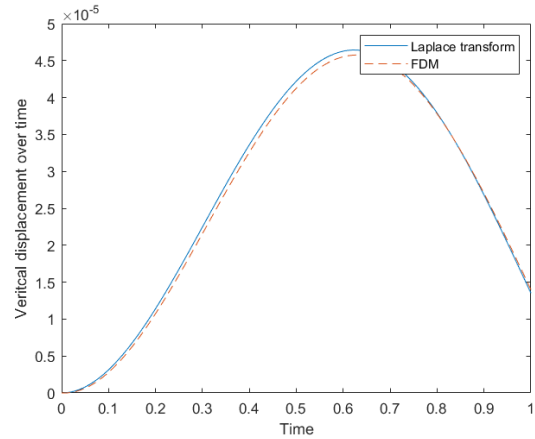
| $t$ | Laplace transform<br>$Y$ | FDM                       |                             |
|-----|--------------------------|---------------------------|-----------------------------|
|     |                          | Numerical solution<br>$y$ | Absolute error<br>$ Y - y $ |
| 0   | 0                        | 0                         | 0                           |
| 0.1 | 3.135735E-06             | 0                         | 3.135735E-06                |
| 0.2 | 1.139994E-05             | 5.368726E-06              | 6.031217E-06                |
| 0.3 | 2.247026E-05             | 1.436311E-05              | 8.107151E-06                |
| 0.4 | 3.361050E-05             | 2.463261E-05              | 8.977889E-06                |
| 0.5 | 4.220847E-05             | 3.369689E-05              | 8.511577E-06                |
| 0.6 | 4.627271E-05             | 3.943385E-05              | 6.838860E-06                |
| 0.7 | 4.479326E-05             | 4.048154E-05              | 4.311719E-06                |
| 0.8 | 3.790108E-05             | 3.647771E-05              | 1.423371E-06                |
| 0.9 | 2.680201E-05             | 2.809524E-05              | 1.293228E-06                |
| 1   | 1.350518E-05             | 1.687300E-05              | 3.367822E-06                |

**Table 4** Numerical results of Laplace transform and FDM for Test Problem (b) with  $h = 0.01$

| $t$ | Laplace transform<br>$Y$ | FDM                       |                             |
|-----|--------------------------|---------------------------|-----------------------------|
|     |                          | Numerical solution<br>$y$ | Absolute error<br>$ Y - y $ |
| 0   | 0                        | 0                         | 0                           |
| 0.1 | 3.135735E-06             | 2.791932E-06              | 3.438033E-07                |
| 0.2 | 1.139994E-05             | 1.073286E-05              | 6.670841E-07                |
| 0.3 | 2.247026E-05             | 2.156747E-05              | 9.027908E-07                |
| 0.4 | 3.361050E-05             | 3.260688E-05              | 1.003621E-06                |
| 0.5 | 4.220847E-05             | 4.125808E-05              | 9.503882E-07                |
| 0.6 | 4.627271E-05             | 4.551803E-05              | 7.546874E-07                |
| 0.7 | 4.479326E-05             | 4.433772E-05              | 4.555357E-07                |
| 0.8 | 3.790108E-05             | 3.779022E-05              | 1.108571E-07                |
| 0.9 | 2.680201E-05             | 2.701639E-05              | 2.143734E-07                |
| 1   | 1.350518E-05             | 1.396610E-05              | 4.609238E-07                |



**Fig. 3** Laplace transform and FDM for Test Problem (b) with  $h = 0.1$



**Fig. 4** Laplace transform and FDM for Test Problem (b) with  $h = 0.01$

### 3.3 Test Problem (c)

A torsional pendulum that involves twisting motion can be described by the following equation,

$$I\theta''(t) + c\theta'(t) + k \sin(\theta(t)) = 0, \tag{21}$$

where  $\theta$  represents the angular displacement. Suppose that the system has a moment of inertia,  $I = 0.1kgm^2$ , a damping coefficient,  $c = 0.01kgm^2$  and a torsional stiffness coefficient  $k = 0.5Nm/rad$ . The pendulum is released from an initial angular displacement of  $0 \text{ rad}$  with an initial angular velocity of  $0.5 \text{ rad/s}$ . Determine the angular displacement of the system body over time.

#### Solution

The system of motion can be rewrite into,

$$0.1 \frac{d^2\theta}{dt^2} + 0.01 \frac{d\theta}{dt} + 0.5 \sin(\theta(t)) = 0, \tag{22}$$

with the initial conditions  $\theta(0)=0$  and  $\theta'(0)=0.5$  and assuming the step size is 0.01 and 0.1 in this problem. Since Laplace transform cannot be applied directly to (22) due to the nonlinearity introduced into the system, linearisation using Taylor series is carried out first by considering the equilibrium point,  $a = 0$ .

$$f(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2!} (x - a)^2 + \dots \tag{23}$$

$$\sin(\theta) = \sin(a) + \cos(a)(\theta - a) + \frac{\sin(a)}{2!} (\theta - a)^2 + \dots \tag{24}$$

$$\sin(\theta) = \sin(0) + \cos(0)(\theta - 0) + \frac{\sin(0)}{2!} (\theta - 0)^2 + \dots \tag{25}$$

$$\sin(\theta) = \theta. \tag{26}$$

Substituting (26) into (22), a linearised second order ODE is obtained, and the Laplace transform can be applied as follows,

$$\begin{aligned} 0.1 \frac{d^2\theta}{dt^2} + 0.01 \frac{d\theta}{dt} + 0.5\theta &= 0 \\ 0.1(s^2\Phi(s) - s\theta(0) - s'\theta'(0)) + 0.01(s\Phi(s) - \theta(0)) + 0.5\Phi(s) &= 0 \\ 0.1(s^2\Phi(s) - 0.5) + 0.01s\Phi(s) + 0.5\Phi(s) &= 0 \end{aligned}$$

$$\Phi(s) = \frac{0.05}{0.15s^2 + 0.01s + 0.5} \tag{27}$$

Apply the inverse Laplace transform on (27) gets the following equation, where  $\theta(t)$  is the exact solution for this particular problem

$$\theta(t) = \frac{10\sqrt{1999}e^{-\frac{t}{20}} \sin\left(\frac{\sqrt{1999}}{20}t\right)}{1999} \tag{28}$$

For the FDM, substitutes (7) and (8) into (22), the equation can be expressed in discretized form

$$0.1\left(\frac{\theta_{i+1} - 2\theta_i + \theta_{i-1}}{h^2}\right) + 0.01\left(\frac{\theta_{i+1} - \theta_{i-1}}{2h}\right) + 0.5 \sin(\theta_i) = 0$$

$$\theta_{i+1} = \frac{-h^2 \sin(\theta_i) + 0.4\theta_i + \theta_{i-1}(0.01h - 0.2)}{0.2 + 0.01h} \tag{29}$$

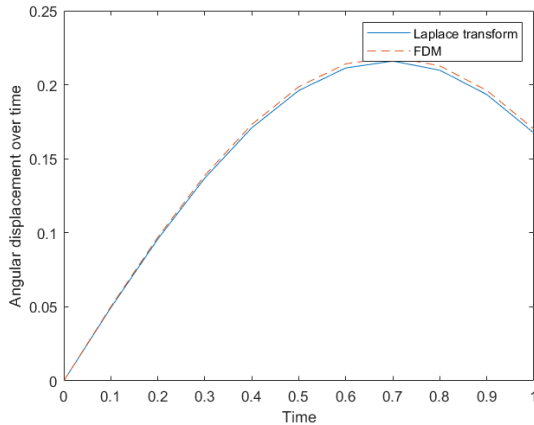
Afterwards, use MATLAB to evaluate the numerical solutions of (28) and (29) over the range of  $0 < t < 1$ , with step sizes of 0.1 and 0.01.

**Table 5** Numerical results of Laplace transform and FDM for Test Problem (c) with  $h = 0.1$

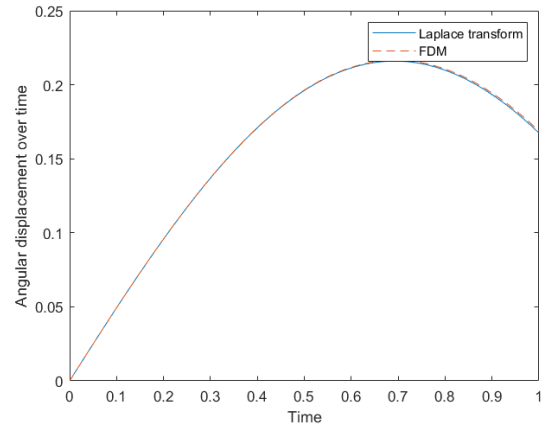
| t   | Laplace transform | FDM                  |                       |
|-----|-------------------|----------------------|-----------------------|
|     | Y                 | Numerical solution y | Absolute error  Y - y |
| 0   | 0                 | 0                    | 0                     |
| 0.1 | 0.0330382         | 0.0500000            | 1.696179E-02          |
| 0.2 | 0.0647623         | 0.0970160            | 3.225364E-02          |
| 0.3 | 0.0941301         | 0.1387450            | 4.461495E-02          |
| 0.4 | 0.1201841         | 0.1731782            | 5.299412E-02          |
| 0.5 | 0.1420833         | 0.1986960            | 5.661269E-02          |
| 0.6 | 0.1591297         | 0.2141394            | 5.500967E-02          |
| 0.7 | 0.1707909         | 0.2188567            | 4.806580E-02          |
| 0.8 | 0.1767162         | 0.2127253            | 3.600914E-02          |
| 0.9 | 0.1767475         | 0.1961513            | 1.940382E-02          |
| 1   | 0.1709229         | 0.1700458            | 8.770375E-04          |

**Table 6** Numerical results of Laplace transform and FDM for Test Problem (c) with  $h = 0.01$

| t   | Laplace transform | FDM                  |                       |
|-----|-------------------|----------------------|-----------------------|
|     | Y                 | Numerical solution y | Absolute error  Y - y |
| 0   | 0                 | 0                    | 0                     |
| 0.1 | 0.0330382         | 0.0493661            | 1.632789E-02          |
| 0.2 | 0.0647623         | 0.0957965            | 3.103422E-02          |
| 0.3 | 0.0941301         | 0.1370266            | 4.289654E-02          |
| 0.4 | 0.1201841         | 0.1710790            | 5.089494E-02          |
| 0.5 | 0.1420833         | 0.1963578            | 5.427449E-02          |
| 0.6 | 0.1591297         | 0.2117182            | 5.258852E-02          |
| 0.7 | 0.1707909         | 0.2165134            | 4.572254E-02          |
| 0.8 | 0.1767162         | 0.2106161            | 3.389992E-02          |
| 0.9 | 0.1767475         | 0.1944185            | 1.767098E-02          |
| 1   | 0.1709229         | 0.1688095            | 2.113389E-03          |



**Fig. 5** Laplace transform and FDM for Test Problem (c) with  $h = 0.1$



**Fig. 6** Laplace transform and FDM for Test Problem (c) with  $h = 0.01$

For Test Problem (a), the numerical results in Table 1 and Table 2 solved using Laplace transform and FDM are represented in the graph as shown in Fig. 1 and Fig 2 with step size of 0.1 and 0.01 respectively. According to Fig 1, the FDM deviates slightly from the Laplace transform which can be seen in the absolute errors of Table 1. However, in Fig 2, the FDM overlaps slightly with the Laplace transform as the absolute errors in Table 2 are minor. Based on Fig 1 and Fig 2, step size 0.01 is much more accurate than 0.1.

In Test Problem (b), the numerical results of the Laplace transform and FDM are illustrated in Fig. 3 and Fig. 4 with step sizes of 0.1 and 0.01 respectively. Fig 3 shows the results of FDM deviate quite much from the Laplace transform even though the absolute errors in Table 3 show better accuracy than the Test Problem (a). Similarly, Fig 4 also shows that FDM deviate slightly from the Laplace transform even though the absolute errors in Table 4 show better accuracy than the Test Problem (a). Comparing Fig 3 and Fig 4, it is evident that Fig 4 with step size of 0.01 is more accurate.

Lastly, for Test Problem (c), the numerical outcomes from both Laplace transform and FDM with step sizes of 0.1 and 0.01 are represented in Table 5 and Table 6 respectively, and their graphical representation is shown in Fig. 5 and Fig 6 distinctly indicates a minimal difference between FDM and Laplace transform, even traces of overlapping. However, the absolute errors detailed in Table 5 and Table 6 do not support this as their values are quite huge. From Fig 5 and Fig 6, it can be deduced that both step sizes, 0.1 and 0.01, are rather accurate. The reason why Fig 5 and Fig 6 showed similar graph trends suggest that equation exhibits a relatively smooth behaviour. If the angular displacement of the pendulum changes gradually over time, a larger step size might be sufficient to capture the general trend of the solution despite the change in step size.

Results for Test Problems (a), (b), and (c) are compared in terms of absolute errors. However, the disadvantage of absolute error is that it heavily relies on the scale of the data. Larger values will result in larger absolute errors. Therefore, this is also why the graphs for Test Problems (a), (b), and (c) show the Laplace transform and FDM coincide with each other, even though Test Problem (b) has a smaller absolute error than Test Problems (a) and (c).

#### 4. Conclusion

As a conclusion, the results of solving several mechanical equations by using FDM are comparable to the results using the Laplace transform since the absolute errors between them are minimal. Therefore, Laplace transform and FDM are comparable to each other.

Several recommendations can be used to improve this research. One of them is using smaller step sizes when computing numerical approximations using FDM. Secondly, Laplace transform and FDM can be compared with other methods to increase the accuracy of the results. Exploring the application of proposed method to more complex mechanical equations with nonlinearities, coupled components, or varying parameters is suggested.

#### Acknowledgement

The authors would thank the Faculty of Applied Sciences and Technology, Universiti Tun Hussein Onn Malaysia for its support.

## Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

## Author Contribution

*The authors confirm contribution to the paper as follows: **study conception and design, analysis and interpretation of results, and draft manuscript preparation:** Keng You Seng; **supervision and advisory:** Syahirbanun Isa. All authors reviewed the results and approved the final version of the manuscript.*

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