

Mixed Convection Heat Transfer in 2D Lid Driven Filled with Newtonian Fluid

Siti Nurmaizatul Ain Nasran¹, Muhamad Ghazali Kamardan^{1*}

¹ Department of Mathematics and Statistics, Faculty of Applied Science and Technology, UTHM Kampus Cawangan Pagoh, Hab Pendidikan Tinggi Pagoh, KM 1, Jalan Panchor, 84600 Pagoh, Muar, Johor, MALAYSIA

*Corresponding Author: mghazali@uthm.edu.my
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Abstract

The study of mixed convection heat transfer in a 2D lid driven filled with Newtonian fluid and insulated horizontal walls has been performed. COMSOL Multiphysics used to solve the problem by visualizing the streamlines and isotherms. The study considered parameters such as Prandtl, Rayleigh, and Peclet numbers, along with Dirichlet boundary conditions. When the Peclet number increases, streamlines near the moving walls become less visible, showing that advection has replaced buoyancy as the main force. At the same time, isotherms start tilting towards the vertical walls because temperature differences near the boundaries are increasing.

1. Introduction

Mixed convection heat transfer involves both forced and natural convection, a phenomenon explored in the earliest study in 1984 [1]. This study explored energy recirculation, focusing on time-dependent flow and temperature fields. It identified the Reynolds and Grashof numbers as key factors. The introduction of the Grashof-Reynolds square ratio played a crucial role in finding a constant heat transfer for Richardson's numbers under 1, while higher Richardson numbers were associated with increased efficiency [2]. Further, a study on a lid-driven square cavity with heated walls showed enhanced convective heat transfer with rising values of Grashof numbers [3]. Another study on rectangular cavities with lids demonstrated that increasing the Peclet number influenced the streamlines and isotherms [4]. A study explored mixed convection heat transport in a lid-driven cavity with wavy heated walls and diamond-shaped obstacles. The investigation analyzed the impacts of dimensionless parameters, such as the Reynolds number, Richardson number, Hartman number, Prandtl number, undulations number, and inner diamond obstacle sizes [5].

In the exploration of fluid dynamics and heat transfer, several studies have provided valuable insights into the behavior of Newtonian and non-Newtonian fluids. A study was conducted to investigate heat transfer in coiled, laminar flow in different coiling pipes using aqueous glycerol solutions. The viscosity and shear stress were found to be oppositely related, and simple formulas based on Prandtl numbers were established to predict heat transfer rates in different coiled pipes [6]. In a different study, the flow of fluid through porous surfaces was compared between natural water and thick solutions, which are simple and complex, respectively. The thicker solution responded to pressure differently from the natural water, which flowed consistently even when pressure changed [7]. A separate investigation on heat transfer in a twisted coil with different fluids determined that simple fluids like glycerol showed less heat transfer compared to thicker solutions like xanthan gum, and the Nusselt number was lower for the mixtures than for simple fluids [8].

COMSOL Multiphysics is a flexible software for modelling and simulating physical systems involving several physics phenomena. It has been applied in several studies, each focusing on different aspects. An investigation

on buoyant laminar flow in a square lid-driven enclosure has been conducted using COMSOL Multiphysics. It demonstrated a significant impact of pressure work and viscous dissipation on mixed convection flow [9]. A study of mixed convection heat transfer from heated spheres in power-law fluids found that macroscopic features in terms of dimensionless factors and drag coefficients [10]. COMSOL Multiphysics was used to investigate mixed convection in a square cavity with a spinning cylinder and nanofluid-filled ports. The results indicated that heat transfer increased as the volume percentage of nanoparticles increased, proving that generalised neural networks are effective in assessing thermal performance [11]. A study employed COMSOL Multiphysics to investigate heat transfer from a spheroid to a Newtonian fluid via mixed convection, contributing to the understanding of mixed convection phenomena and its diverse applications [12]. Lastly, the investigation of mixed convection of Casson fluid in a trapezoidal cavity with a heated bottom and a wavy wall was conducted by using COMSOL Multiphysics. They aimed to examine how various non-dimensional properties, including the Richardson number, Casson fluid parameters, and the number of oscillations, influence temperature and velocity profiles [13]. Other previous studies were done in heat transfer and fluid in worldwide according to various fields of studies [14, 15].

In this study, we investigate 2D lid-driven mixed convection heat transfer in a cavity filled with Newtonian fluid, with both horizontal walls insulated. The right and left vertical walls are set to hot and cold temperatures, respectively. The key parameters studied include Prandtl, Rayleigh, and Peclet numbers. This study further the earlier study [4] by using COMSOL Multiphysics to analyse the influence of these parameters on streamlines and isotherms. However, previous study who used Neumann boundary conditions, we utilize Dirichlet boundary conditions because it provides a clear and practical way to simulate the behaviour of the fluid in the cavity. Thus, we want to explore and identify their effects on streamlines and isotherms.

| | |
|--------------|---|
| Nomenclature | |
| g | acceleration due to gravity |
| H' | height of the enclosure |
| L' | length of the rectangular enclosure |
| p | dimensionless pressure |
| Pe | Peclet number |
| Pr | Prandtl number |
| q' | constant heat flux density |
| Ra | Rayleigh number |
| T | dimensionless temperature |
| t | dimensionless time |
| T'_o | reference temperature |
| u'_o | lid-velocity |
| (u,v) | dimensionless axial and transverse velocities |
| (x,y) | dimensionless axial and transverse coordinates |
| Greek symbol | |
| α | thermal diffusivity of fluid at reference temperature |
| β | thermal expansion coefficient of fluid at reference temperature |
| λ | thermal conductivity of fluid at reference temperature |
| ρ | fluid density at reference temperature |

2. Methodology

Fig. 1 shows the physical problem with its boundary conditions in a rectangular cavity with a height and length filled with Newtonian fluid. While both horizontal walls are insulated, it also revealed a uniform density of heat flow at the left walls. The two horizontal walls move in opposing directions at a constant speed and two vertical walls remain stationary in the lid-driven cavity.

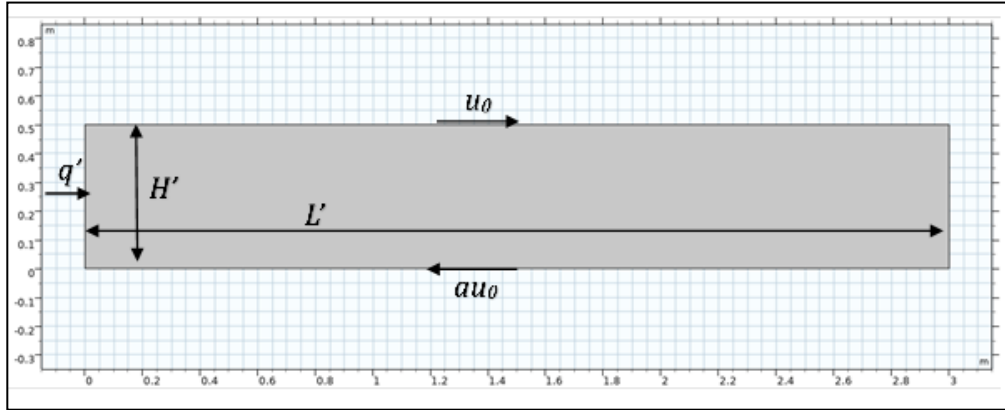


Fig. 1 Physical problem with its boundary conditions

When buoyancy is the only force driving motion, fluid circulation is slow because the value of fluid velocities is small which is considered as laminar. Since pressures is close to atmospheric, the fluid is incompressible. When compared to the imposed heat flux, the contribution of viscous dissipation is neglected except for the density in the buoyancy term. It follows the Boussinesq approximation, and the physical properties are thought to be temperature independent. Due to the size of the cavity's third dimension is big, the issue can be viewed as two-dimensional.

By taking the above assumptions, the dimension equations formed and written in terms of the velocity (u' and v'), pressure (p') and temperature (T') as follows:

The conservation of mass equation:

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \quad (1)$$

Momentum equation:

$$\frac{\partial u'}{\partial t'} + u' \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial y'} = -\frac{1}{\rho} \frac{\partial p'}{\partial x'} + \nu \left[\frac{\partial^2 u'}{\partial x'^2} + \frac{\partial^2 u'}{\partial y'^2} \right] \quad (2)$$

$$\frac{\partial v'}{\partial t'} + u' \frac{\partial v'}{\partial x'} + v' \frac{\partial v'}{\partial y'} = -\frac{1}{\rho} \frac{\partial p'}{\partial y'} + \nu \left[\frac{\partial^2 v'}{\partial x'^2} + \frac{\partial^2 v'}{\partial y'^2} \right] + g\beta [T' - T'_0] \quad (3)$$

Energy equation:

$$\frac{\partial T'}{\partial t'} + u' \frac{\partial T'}{\partial x'} + v' \frac{\partial T'}{\partial y'} = \alpha \left[\frac{\partial^2 T'}{\partial x'^2} + \frac{\partial^2 T'}{\partial y'^2} \right] \quad (4)$$

where there are fluid density (ρ), kinematic viscosity (ν), gravitational acceleration (g), thermal expansion coefficient (β) and thermal diffusivity (α)

The thermal and kinematic boundary conditions associated to the governing equations are:

$$\text{Left wall: } u' = v' = 0, T = T_h \quad (5)$$

$$\text{Right wall: } u' = v' = 0, T = T_c \quad (6)$$

$$\text{Top wall and Bottom wall: } u' = u_0, v' = 0, \frac{\partial T'}{\partial x'} = 0 \quad (7)$$

Top wall and Bottom wall:

The following variables and parameters are applied to the equation (1)-(4) to get the dimensionless governing equations:

$$p = \frac{p'}{\rho u_0'^2}, t = \frac{t' u_0'}{H'}, T = \frac{(T' - T'_0) \lambda}{q' H'}, (u, v) = \frac{(u', v')}{u_0'}, (x, y) = \frac{(x', y')}{H'}$$

$$\text{Pr} = \frac{\nu}{\alpha}, \text{Pe} = \frac{u_0' H'}{\alpha}, \text{Ra} = \frac{g \beta q' H'^4}{\nu \lambda \alpha}$$

The dimensionless governing equations formed from (1) - (4) become:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (8)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\text{Pr}}{\text{Pe}} \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] \quad (9)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{\text{Pr}}{\text{Pe}} \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] + \frac{\text{Ra Pr}}{\text{Pe}^2} T \quad (10)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\text{Pe}} \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] \quad (11)$$

The dimensionless boundary equations of the problem are formed from (5)-(7) are shown below.

$$\text{Left wall:} \quad u = v = 0, T = 1 \quad (12)$$

$$\text{Right wall:} \quad u = v = 0, T = 0 \quad (13)$$

$$\text{Top wall:} \quad u = 1, v = 0, \frac{\partial T}{\partial x} = 0 \quad (14)$$

$$\text{Bottom wall:} \quad u = -1, v = 0, \frac{\partial T}{\partial x} = 0 \quad (15)$$

We applied dimensionless governing equations and boundary conditions to explore mixed convection heat transfer in a 2D lid driven cavity filled with Newtonian fluid in COMSOL Multiphysics. We specifically used the laminar flow solver for fluid dynamics, the heat transfer solver for thermal effects, and the Poisson equation solver for pressure fields. The approach of dimensionless equations enhanced the adaptability of our simulations, providing valuable insights into the dynamics of heat transfer in the lid-driven cavity.

3. Results and Discussion

Streamlines known as the flow of fluid that illustrate the flow of the velocity direction at each point. Isotherms are lines or curves that connect points at the same temperature. In this research, COMSOL Multiphysics has been used to show the results in streamlines and isotherms for the mixed convection heat transfer in 2D lid driven filled with Newtonian fluid. The parameters used for the research are the Rayleigh number (Ra), the Peclet number (Pe), and the Prandtl number (Pr).

A Rayleigh number of 10,000 was chosen to ensure laminar flow and the dominance of natural convection in heat transfer. The Prandtl number was set at 0.71, representing the thermal and momentum diffusivities of air at the given temperature, indicating comparable significance of heat and momentum transfer in the fluid. The Peclet number was varied with values of 0.1, 0.5, 5, 25, 50, 100, and 150 to explore the transition from forced convection dominated heat transfer to mixed convection and natural convection dominant heat transfer. Fig. 2 and Fig. 3 show the results of streamlines and isotherms of the study.

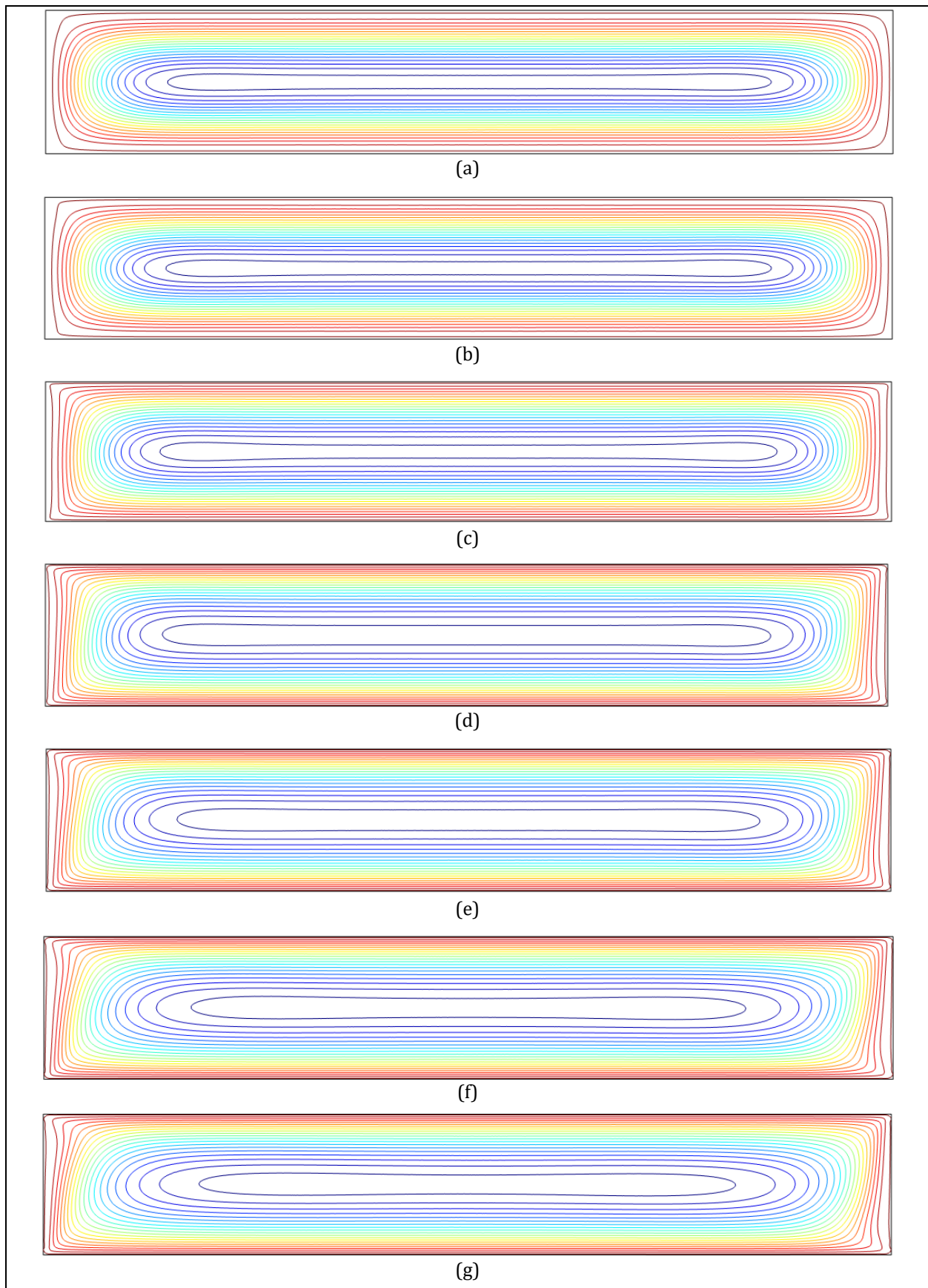


Fig. 2 Streamlines for $Ra = 10,000$, $Pr = 0.71$ and various values of Pe : (a) $Pe = 0.1$, (b) $Pe = 0.5$, (c) $Pe = 5$, (d) $Pe = 25$, (e) $Pe = 50$, (f) $Pe = 100$ and (g) $Pe = 150$

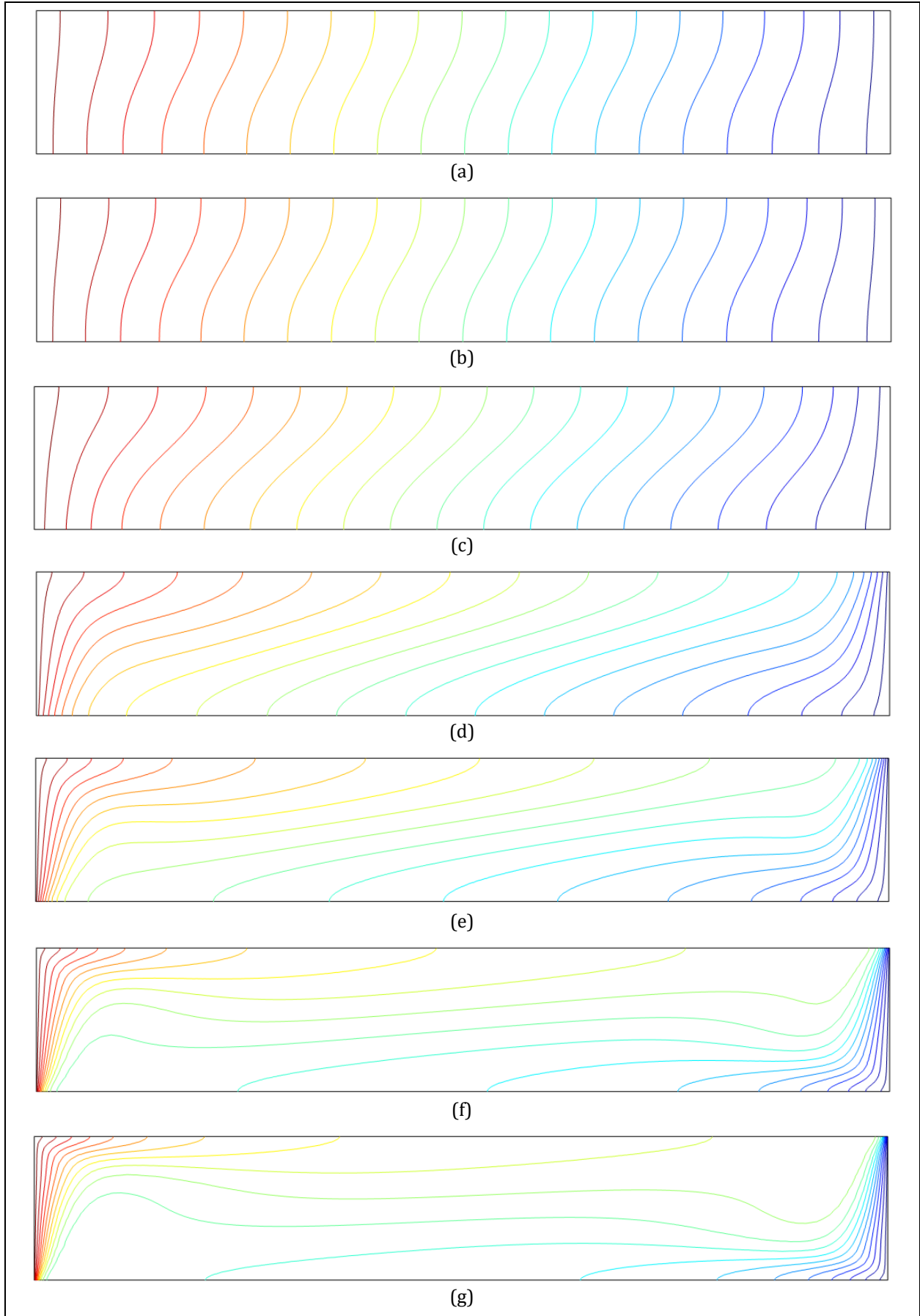


Fig. 3 Isotherms for $Ra = 104$, $Pr = 0.71$ and various values of Pe : (a) $Pe = 0.1$, (b) $Pe = 0.5$, (c) $Pe = 5$, (d) $Pe = 25$, (e) $Pe = 50$, (f) $Pe = 100$ and (g) $Pe = 150$

Fig. 2 (a)-(g) show the streamlines of mixed convection heat transfer in 2D lid driven filled with Newtonian fluid when Ra is fixed to 10,000 and Pr is fixed to 0.71. The value of Pe is set to various values, from 0.1 to 150. Based on the combined effects of buoyancy and shear, the streamlines in the COMSOL Multiphysics findings show a uniform motion with a clockwise rotation. When Pe = 0.1 in Fig. 2(a) and Pe = 0.5 in Fig. 2(b), the streamlines are noticeably distant from the moving walls. When the Pe is low, conduction becomes the dominant mode of heat transfer, as the convective heat transfer rate is relatively small. The influence of Ra is limited, and the effects driven by buoyancy are not significant.

As we progress to Pe = 5 in Fig. 2(c), the streamlines gradually approach the moving walls because convective effects become more noticeable. Significantly, when Pe = 25 in Fig. 2(d), the streamlines densely pack against the moving walls, making them almost invisible. This trend persists and becomes more pronounced as the Pe increases from 50 to 150, as shown in Fig. 2(e) to Fig. 2(g). Therefore, when Pe increases, convective heat transfer dominates the system, while Ra contributes to buoyancy-driven flow, influencing the development of vortices and fluid motion. Pr maintains its role in determining the balance between momentum and heat transfer, affecting the overall fluid behaviour and streamline patterns.

Fig. 3 (a)-(g) exhibits the isotherms of the study when Ra is fixed to 10,000 and Pr is fixed to 0.71. The value of Pe is set to various values from 0.1 to 150. In the context of a clockwise flow, the left vertical wall exhibits hot temperatures, while the right vertical wall remains cooler. Based on Fig. 3(a) at Pe = 0.1 and Fig. 3(b) at Pe = 0.5, the isotherms are more parallel and evenly spaced, which indicates efficient thermal conduction. Temperature gradients are predominantly influenced by conduction, resulting in a uniform temperature distribution, and Ra has a limited impact on buoyancy-driven flow.

When Pe was increased to 5, as shown in Fig. 3(c), a noticeable increase in the tilt of the isotherms became apparent. Isotherms may begin to exhibit deformations, especially in areas affected by fluid motion. Furthermore, when Pe = 25 in Fig. 3(d), a tightening of the isotherms emerges, particularly at the vertical walls. This trend continues when Pe increases from 50 to 150, as seen in Fig. 3(e) to Fig. 3(g), where the isotherms visibly tighten, approaching the right and left vertical walls as regions with vigorous fluid movement experience significant temperature variations. Ra continues to play a role in buoyancy-driven effects, influencing the deformations of isotherms, while Pr maintains its role in determining thermal boundary layer thickness and heat transfer characteristics, affecting the shape and distribution of isotherms.

4. Conclusion

The analysis revealed that as the Pe increased, the streamlines gradually converged on the walls, making them almost invisible. Consequently, convective heat transfer dominates the system, leading to complex fluid motion patterns. This shift signifies a crucial change in the heat transfer dynamics within the 2D lid-driven cavity filled with a Newtonian fluid.

Regarding the isotherms, an increase in Pe caused them to tilt, approaching closely to the right and left vertical walls. This phenomenon signifies intense temperature gradients near the boundaries. The findings indicate that changes in the Pe have a major impact on the heat distribution and flow dynamics in lid-driven cavities. This realization has applications, especially in heat exchanger design optimization and thermal system efficiency enhancement.

Future studies could examine more complex elements of mixed convection heat transfer while considering real-world situations, complicated geometries, and other factors. The effects of various fluid characteristics, boundary conditions, transient analyses, and turbulence in lid-driven cavities may provide new lines of inquiry that increase our general understanding and advance related applications.

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Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: study conception and design, data collection: Siti Nurmaizatul Ain Nasran; analysis and interpretation of results, draft manuscript preparation: Siti Nurmaizatul Ain Nasran and Muhamad Ghazali Kamardan. All authors reviewed the results and approved the final version of the manuscript.

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