

# Mixed Convection in a Lid Driven Cavity with Inclined Magnetic Field using COMSOL

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## Abstract

This study investigates the fluid flow influenced by the magnetic field's strength with inclination angle in a two-dimensional square cavity. The top wall was retained at a heated temperature, while the lower wall was retained cold and two vertical walls were well insulated. This analysis of the streamlines and isotherms plot was performed by using COMSOL software. It was found that a stronger magnetic field resulting in the streamlines circulation has slowed the flow of fluids in the cavity and isotherms contour almost vertically stratified. Furthermore, the presence with different strength and inclination angle of magnetic field slowed down the flow of mixed convection.

## 1. Introduction

Researchers have shown interest in the interaction between mixed convection and magnetic fields. The study explored the effects of different heating sites and magnetic fields on heat transfer rates, emphasizing their substantial impact [1]. Furthermore, the study focused on hydromagnetic fields in lid-driven cavities, observing their influence on heat transfer and fluid flow characteristics [2]. Investigate the influence of magnetic fields on triple convection, highlighting their significant impact on Marangoni convection circulation and it showed that magnetic fields significantly impact circulation from Marangoni convection [3].

Next is the investigation of unstable dual-diffusive natural conduction in a sloped rectangular cavity with various angles of magnetic force appears. It was discovered that raising the Hartmann number or inclination angle increased the rate at which heat was transmitted along the heated walls [4]. They also investigated into how flow and heat transfer in mixed convection are affected by the presence of magnets and cavity inclination angles, and they discovered that larger Hartmann numbers or inclination angles result in greater heat transmission [5].

Additionally, the research discussed the consequences of a sequential magnetic field on mixed convection and creation of entropy in a cavity driven by twin lids and filled with nanofluid that suggested that an increase in the magnetic field's average rate of entropy development with higher intensity [6]. Besides, another study evaluates the influence of various flow parameters on dimensionless profiles through plots and summarizes the nature of physical parameters in table form. The findings emphasize a significant impact of the inclined magnetic field on fluid flow in the studied conditions [7]. In addition, the presence of a magnetic field slows mixed convection flow, and the average Nusselt number increases with increasing magnetic field angle, according to numerical analysis [8]. A numerical study was conducted to examine the effects of the size of the internal heat source, the concentration of solid nanoparticles, and the magnetic field. The findings indicate that the presence and increasing values of solid nanoparticle concentration led to an enhancement of heat transfer, while the magnetic field exerts a negative influence [9]. Thus, the goal of this research is to use COMSOL software for exploring mixed convection flow in a lid-driven cavity. This is important to analyse isotherms and streamlines

plot for understanding the effect of different magnetic field strengths and inclination angle in the fluid flow patterns.

Nomenclature	
$B$	Uniform magnetic field of strength
$L$	Length and height of cavity
$P$	Pressure
$\rho$	Density
$T$	Temperature
$T_c$	Cold temperature
$T_h$	Hot temperature
$u, v$	Components of velocity in the $x$ - and $y$ -axes
$U_o$	Constant velocity
$Ha$	Hartmann number
$Pr$	Prandtl number
$Ri$	Richardson number
$Re$	Reynolds number
$Gr$	Grashof number
Greek symbols	
$\alpha$	Thermal diffusivity
$\beta$	Coefficient of thermal expansion
$\mathcal{G}$	Gravity
$\nu$	Kinematic viscosity
$\gamma$	Inclination angle
$\sigma$	Electrical conductivity

## 2. Methodology

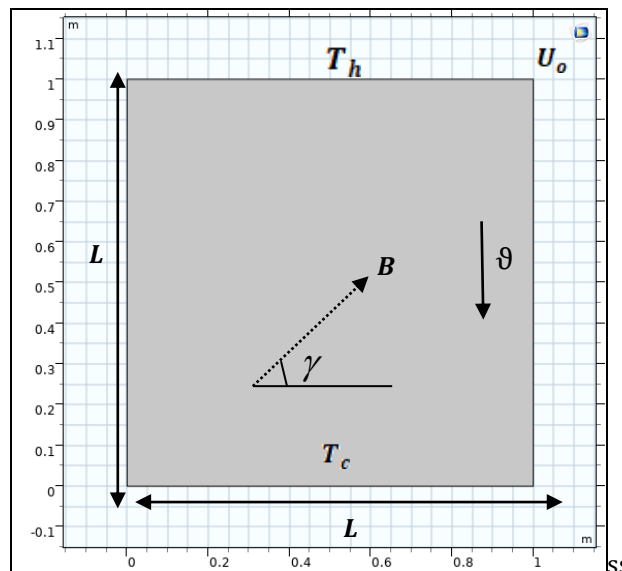


Fig. 1 The physical model configuration

Fig. 1 illustrating a square cavity with thermally insulated vertical side walls. The top wall is maintained at a constant rightward velocity,  $U_o$  and is sustained at a high temperature,  $T_h$ . The bottom wall is consistently kept at a lower temperature,  $T_c$  where a strength of magnetic field,  $B$  with different inclination angles  $\gamma$  enters the enclosure. The operational fluid is characterized as incompressible and Newtonian. Internal circulation inside the enclosure is assumed to be laminar in two dimensions, and constant. Other fluid thermophysical characteristics are regarded as constants, and the approximation of Boussinesq is applied to system of equations and boundary conditions, based on the assumptions by the authors [10] is expressed as follows:

Dimensional governing equations are shown below:

The conservation of mass equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum equations:

The velocity component ( $u$ ) in  $x$  direction:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \frac{B^2 \sigma}{\rho} (v \sin \gamma \cos \gamma - u \sin^2 \gamma) \quad (2)$$

The velocity component ( $v$ ) in  $y$  direction:

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \frac{B^2 \sigma}{\rho} (u \sin \gamma \cos \gamma - v \cos^2 \gamma) + \mathcal{G} \beta (T - T_c) \quad (3)$$

Energy equation:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (4)$$

where  $\nu$  is kinematic viscosity, magnetic field ( $B$ ), electrical conductivity ( $\sigma$ ), density ( $\rho$ ), inclination angle of magnetic field ( $\gamma$ ), the velocity components ( $u, v$ ) in  $x$  and  $y$  direction, pressure ( $p$ ), gravity ( $\mathcal{G}$ ) in  $y$  direction, the coefficient of thermal expansion ( $\beta$ ), temperature ( $T$ ) and thermal diffusivity ( $\alpha$ ).

The requirements for boundary conditions before dimensionless are:

Upper wall:

$$u = U_o, v = 0, T = T_h \quad (5)$$

Bottom wall:

$$u = v = 0, T = T_c \quad (6)$$

Left and right wall:

$$u = v = 0, \frac{\partial T}{\partial x} = 0 \quad (7)$$

The dimensionless variables are stated as below:

$$X = \frac{x}{L}, L = \frac{y}{L}, U = \frac{u}{U_o}, V = \frac{v}{U_o}, \theta = \frac{T - T_c}{T_h - T_c}$$

$$Ha = BL \sqrt{\frac{\sigma}{\rho \nu}}, Gr = \frac{\mathcal{G} \beta (T_h - T_c)}{\nu^2}, Pr = \frac{\nu}{\alpha},$$

$$P = \frac{p}{\rho U_o^2}, Re = \frac{U_o L}{\nu}$$

where  $P$  and  $\theta$  are the dimensionless pressure and temperature.

Substitute variables above into equation (1-4) to get dimensionless governing equations

The dimensionless governing equation are formed as below:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (8)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) + \frac{Ha^2}{Re} (V \sin \gamma \cos \gamma - U \sin^2 \gamma) \quad (9)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{\text{Re}} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{Ha^2}{\text{Re}} (U \sin \gamma \cos \gamma - V \cos^2 \gamma) + \frac{Gr}{\text{Re}^2} \theta \tag{10}$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = -\frac{1}{\text{Re Pr}} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \tag{11}$$

The dimensionless boundary conditions for the study are formed as below:

Upper wall:

$$U = 1, V = 0, \theta = 1 \tag{12}$$

Bottom wall:

$$U = V = 0, \theta = 0 \tag{13}$$

Left and right wall:

$$U = V = 0, \frac{\partial \theta}{\partial X} = 0 \tag{14}$$

Other previous studies were done in flow application in worldwide according to various fields of studies [11, 12].

### 3. Results and Discussion

In this paper, we study the plots with fixed prandlt number (Pr), grashof number (Gr), richardson number (Ri), reynolds number (Re). In addition, the magnetic field parameter and the magnetic field inclination angle are the parameters in this research. Hartman number (Ha) is defined as 10, 30, and 60, while  $\gamma$  varies from  $0^\circ$  to  $90^\circ$  when the magnetic field is tilted counterclockwise. Hartmann number and inclination angle are the controlling parameters on the fluid flow. Hartmann numbers which are the strength of magnetic field is important to study the fluid flow pattern.

a) Streamlines for different Hartmann number,  $Ha$  with inclination angle,  $\gamma = 0$

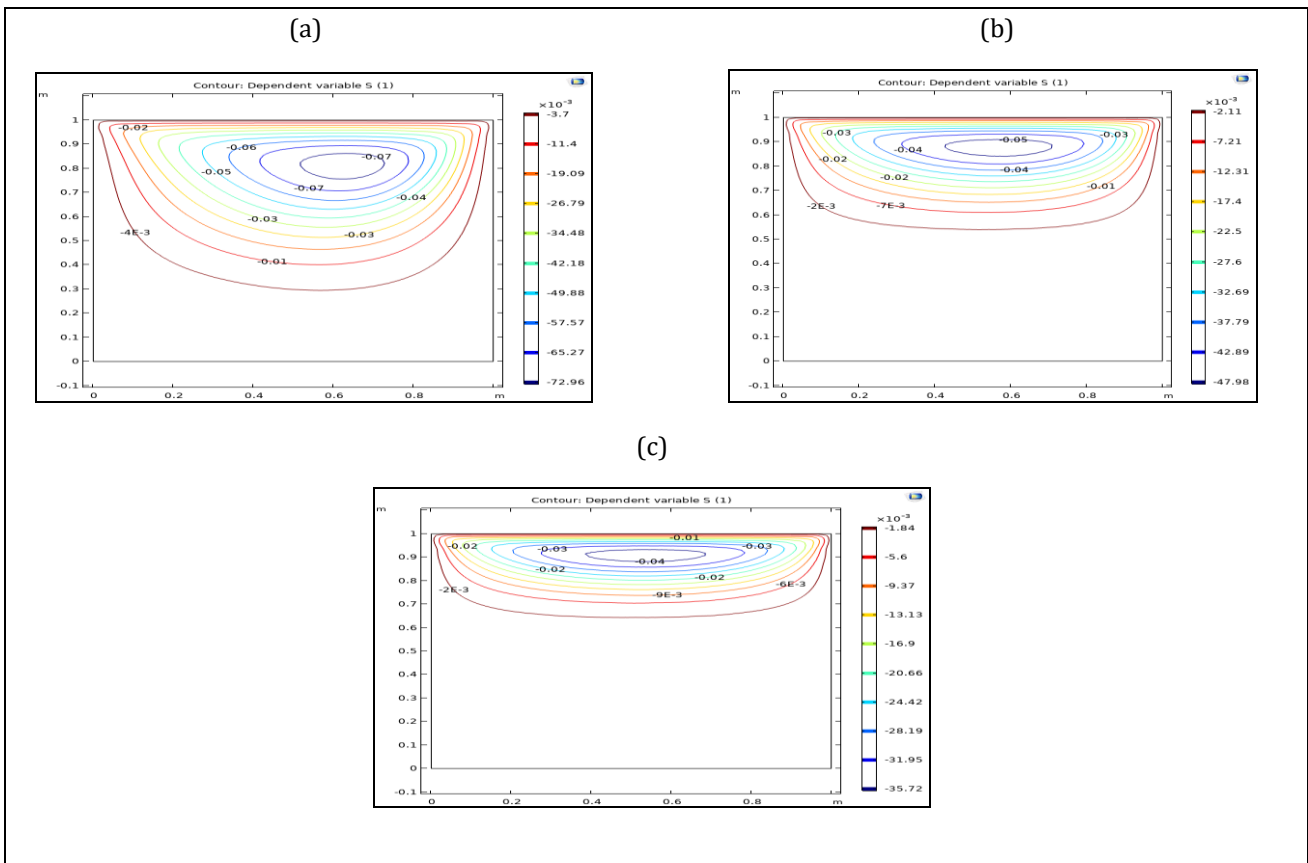
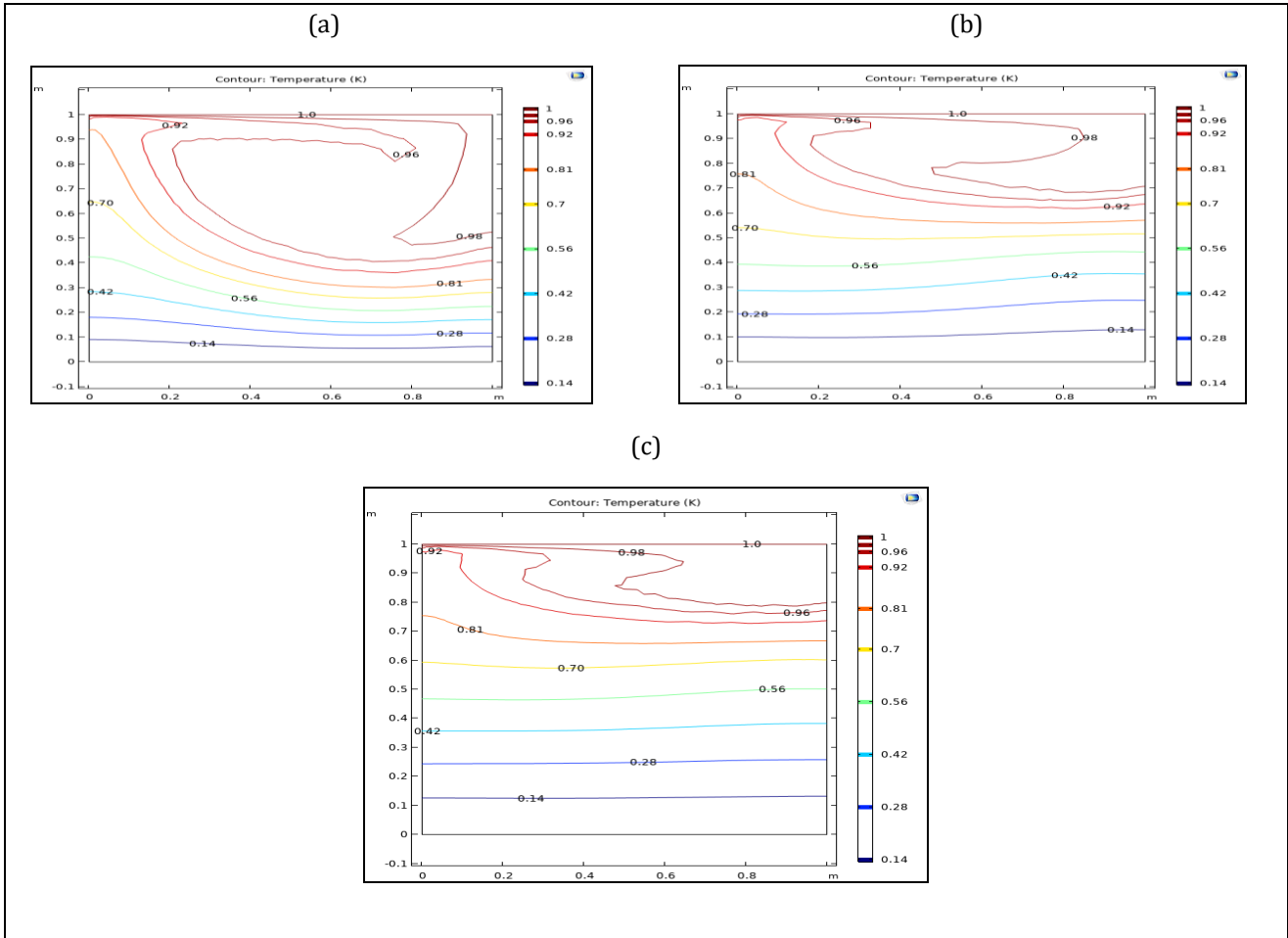


Fig. 2: Streamlines in the cavity with  $Ri=0.1$ ,  $Re = 100.0$ ,  $Gr = 10^3$ ,  $Pr = 7.0$ ,  $\gamma = 0^\circ$  and different Hartmann number  $Ha$ : (a)  $Ha = 10$ , (b)  $Ha = 30$ , (c)  $Ha = 60$ .

Streamlines for Fig. 2 indicate that most of the upper cavity was occupied by a clockwise recirculating vortex that can be seen in (a). When  $Ha$  goes to 30 which is seen in Fig. 2 (b), the clockwise recirculating vortex for the top part becomes smaller than Figure 2 (a) and the top part becomes smaller again when adding up of  $Ha$  to 60 that showed in (c). This is a result of the streamlines circulation has slowed the flow of fluids in the cavity gradually due to the magnetic field strength increased.

b) Isotherms for different Hartmann number,  $Ha$  for inclination angle  $\gamma = 0^\circ$

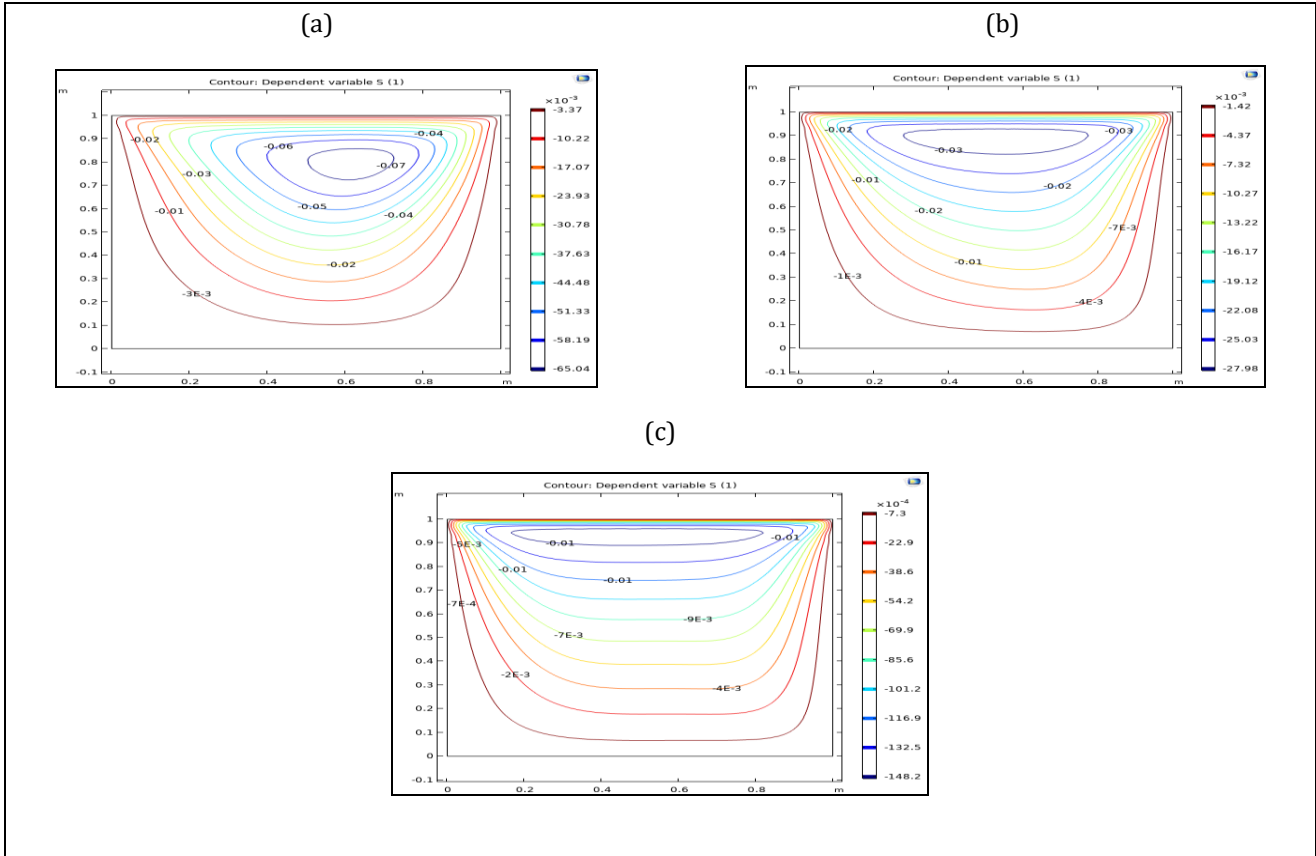


**Fig. 3:** Isotherms in the cavity with  $Ri=0.1$ ,  $Re = 100.0$ ,  $Gr = 10^3$ ,  $\gamma = 0^\circ$  and different Hartmann number  $Ha$ : (a)  $Ha = 10$ , (b)  $Ha = 30$ , (c)  $Ha = 60$ .

Fig. 3 shows that isotherms show that the cavity's upper left and middle corners have a sharp temperature differential. The isotherms demonstrated that the fluid has a thermal stratification along the bottom cavity that begins at the bottom wall and progresses to the upper part. With an increase in magnetic field, the steep temperature gradient progressively decreased that was shown in Fig. 3 (b) and Fig. 3 (c). Thus, the fluid flow of the cavity has slowed down due to the increment of the strength Hartmann number.

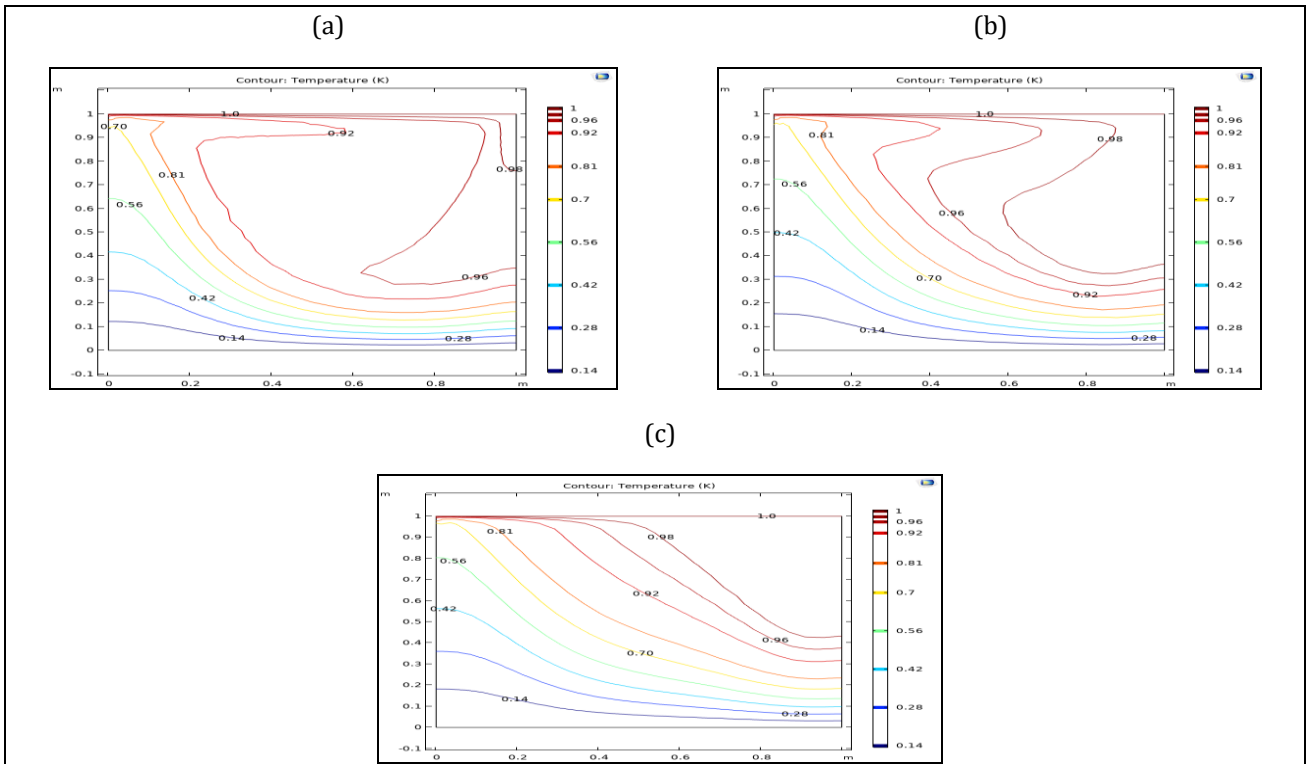
a) Streamlines for different Hartmann number,  $Ha$  for inclination angle,  $\gamma = 30^\circ$

The streamline obtained by adding the magnetic force with angle for  $\gamma = 30^\circ$  are displayed in Figure 4 that indicate that the streamline contour demonstrates that a major recirculating vortex filled in a counterclockwise direction for the cavity in Fig. 4 (a). The streamline falls somewhat to the bottom right corner when the magnetic field parameter,  $Ha = 30$  was added to the cavity which was displayed in Fig. 4 (b). The streamlines fall in the same direction and appear to be compressed against the moving wall when the magnetic field strength was increased to 60 that was shown in Fig. 4(c). It showed that the streamline circulation had shifted to the bottom right wall due to an increase of the inclined magnetic field.



**Fig. 4:** Streamlines in the cavity with  $Ri=0.1$ ,  $Re = 100.0$ ,  $Gr = 10^3$ ,  $\gamma = 30^\circ$  and different Hartmann number  $Ha$ : (a)  $Ha = 10$ , (b)  $Ha = 30$ , (c)  $Ha = 60$ .

b) Isotherms for different Hartmann number,  $Ha$  for inclination angle,  $\gamma = 30^\circ$



**Fig. 5:** Isotherms in the cavity with  $Ri=0.1$ ,  $Re = 100.0$ ,  $Gr = 10^3$ ,  $\gamma = 30^\circ$  and different Hartmann number  $Ha$ : (a)  $Ha = 10$ , (b)  $Ha = 30$ , (c)  $Ha = 60$ .

The isotherms obtained by adding the magnetic field inclination angle for  $\gamma = 30^\circ$  was displayed in Fig. 5. The temperature contours indicated that a sharp temperature gradient was seen in the upper left corner when  $Ha = 10$  was demonstrated in Fig. 5 (a). The steep temperature decreased as the magnetic field increased. Moreover, there is less circulation in the upper part of the cavity. When  $Ha = 60$  that was seen in Fig. 5 (c), it was obvious that the fluid had stratified throughout most of the cavity.

For this investigation, the changes in streamlines and isotherms with different  $Ha$  with the subsequent values of  $Ri = 0.1$ ,  $Re = 100.0$ ,  $Gr = 10^3$ ,  $\gamma = 60^\circ$  and  $\gamma = 90^\circ$  was showed to be the same as they are at  $\gamma = 30^\circ$ . However, when the strength of magnetic field increases, the isotherms contour almost vertically stratified. For all values of  $Ha$  for all Figure in (a), (b) and (c) for both streamlines and isotherms, the fluid flow was seen to be portrayed by a dominating recirculating vortex spinning counterclockwise. Additionally, when the magnetic intensity increases, the moving wall compresses the streamlines. As the magnetic intensity rises, the isotherms demonstrate that the fluid exhibits almost vertically stratified. In conclusion, the streamlines demonstrate that a new vortex split from the top vortex close to the bottom wall as the  $Ha$  increased that make the top part becomes smaller in the cavity. This demonstrated that mixed convection flow slowed down the flow of fluid as the strength and inclination angle of the magnetic field increases.

#### 4. Conclusion

In conclusion, this research shows how the strength of magnetic fields and inclination angles affect the mixed convection flow. Based on the analysis of the results, it was determined that a higher strength of magnetic field produced lower streamline flow of fluids in the cavity due to forced convection. In addition, imposing an angled magnetic field on the flow also slows and affects its streamline circulation. Furthermore, when the strength of magnetic field with inclination angle increases, the isotherms contours become nearly vertically stratified. Thus, we can conclude that, the increase of the magnetic field strength plus the increase of inclination angle for the magnetic field affecting of slowing the streamline flow.

Future research is suggested to conduct more in resolving other problems that involve mixed convection flow such as in different temperatures and different geometrical enclosures. This is to view the outcomes of the COMSOL-calculated problem's velocity streamlines, and isotherm plots, mixed convection flow using inclined magnetic field. Thus, this can increase our knowledge and understanding in mixed convection flow.

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#### Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of papers.

#### Author Contribution

*The authors confirm contribution to the paper as follows: study conception and design, data collection: Nur Athirah Amir; analysis and interpretation of results, draft manuscript preparation: Nur Athirah Amir and Muhamad Ghazali Kamardan. All authors reviewed the results and approved the final version of the manuscript.*

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