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Deteriorating Inventory in Multi-Supplier System with Quantity Discount

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Abstract

In today's uncertain economy, businesses navigate strategies for growth amid challenges in managing perishable products, known for their short lifespan and declining value. Effective coordination among multiple suppliers and utilizing quantity discounts optimizes supply chain profits. Systematic inventory management is crucial for identifying efficient order quantities, optimal supplier combinations, and cost-effective discounts, ultimately reducing overall inventory costs. This study considers how quantity discounts impact deteriorating inventory and seeks suppliers to maximize profits while minimizing inventory expenses. The model focuses on integrating quantity discounts into inventory systems for items with fixed lifetimes following the Weibull distribution. An illustrative example with two suppliers offering distinct quantity discounts demonstrates model practicality. The findings aid decision-making in perishable product supply chains, shedding light on optimizing total costs amidst parameter variations.

1. Introduction

Revenue and inventory management play a crucial role in the operational efficiency of food supply chains [1]. Due to the physical nature of raw materials, fresh food, fruits, and chemicals degrade, they have a limited useful lifetime, which means they may start to deteriorate before a set date but must cease to be usable after that date [2]. As we can see, during the pandemic season, there were significant changes in supply and demand because at that time the movement control order was implemented. The customers are unable to buy raw materials as usual at the store, causing the quantity of raw materials to be sold out at an uncertain time [3]. Uncertain demand causes retailers' inventory to be unstable. When demand is high, retailers need to order the product immediately while if demand is low, retailers need to keep the raw material longer or reduce the price [4]. Effective inventory management will minimize costs associated with holding inventory and ensure that the right amount of inventory is available to meet customer demand.

In terms of supply chain coordination with deteriorating products, the Stackelberg-Nash equilibrium was used to analyze a supply chain with degrading goods where a manufacturer and retailer set wholesale and retail prices, respectively, to maximize bilateral profits [5]. In order to maintain the desired freshness of the product items and close the gap between supply and market demand, it is reasonable for the retailer to think about placing an additional or smaller order throughout the replenishment cycle. Therefore, quantity discounts are commonly used in inventory management to incentivize large orders, which can reduce unit costs and increase profits. Quantity discounts have recently been used to help multi-supplier issues in both multi-period and single-period scenarios. For example, quantity discounts in a three-stage supply chain to coordinate resupply and shipping times are used by [6].

Many businesses are motivated by the opportunity to hedge supply risks because a single supplier exposes them to disrupted supply if labour strikes, machine breakdowns, material shortages, or natural disasters occur [7]. The second problem is that a single supplier may be unable to satisfy orders, especially if client demand surges [8]. Thus, there are a lot of benefits to having resources from multiple suppliers. The previous study considered many suppliers, one warehouse, and numerous retailers with the main objective of reducing the total cost of the system, which comprises ordering cost, inventory holding cost, and delivery cost. [9] However, quantity discounts can also lead to increased holding costs and potentially higher inventory levels than necessary.

This study focuses on the inventory method that takes the demand rate, the rate of deterioration, the order quantity, and the size of the quantity discount into account. By using Economic Order Quantity (EOQ) with Weibull distribution, this study can find the optimal inventory that minimizes costs while meeting customer demand. This study considers how quantity discounts impact deteriorating inventory and seeks suppliers to maximize profits while minimizing inventory expenses. It also identifies suppliers that maximize retailer profit.

2. Methodology

This paper will discuss solving a two-echelon supply chain with two suppliers and one retailer, in which the suppliers offer a single product that deteriorates over time in inventory by using Economic Order Quantity with Weibull distribution. Model formulation and data used in this study were gathered from [10] and labelled as "Supplier 1". Another supplier called "Supplier 2" is being introduced. Data for Supplier 2 comes from a range of different purchase quantities. A comparison between two suppliers that provide two different quantity discounts was made to find the supplier that gives more benefit to the retailer.

2.1 Assumption

This study is based on the following assumptions:

- To increase the overall profitability of the supply chain, a quantity discount coordination mechanism is used.
- The retailer may face a shortage of goods, and the supplier has sufficient resources to restock the reordered goods.
- The retailer only receives one product from the supplier.
- The Weibull distribution is used to calculate the inventory deterioration rate.

2.2 Model

Let $\boldsymbol{\alpha}$ is the scale parameter, $\boldsymbol{\beta}$ is the shape parameter and t is elapsed time. Then, the given equation is Weibull power law failure intensity.

$$Z(t) = \alpha\beta t^{\beta-1},$$
$$0 \leq \alpha,$$
$$0 \leq \beta,$$
$$t > 0,$$
$$0 \leq Z(t) < 1.$$

When $\mathbf{0} \leq \boldsymbol{t} \leq \boldsymbol{T_1}$, the following equation (1) considers the inventory level $\boldsymbol{I_1(t)}$ given the Weibull power law failure rate, $\boldsymbol{\alpha\beta t^{\beta-1}}$

$$\frac{dI_1(t)}{dt} = p - d - \alpha\beta t^{\beta-1} I_1(t), \tag{1}$$

where p is the average production rate of the supplier, and d is the average demand rate of the retailer. Equation (1) is solved with the initial condition,

$$I_1(0) = 0.$$

Let $\boldsymbol{u^\beta}$ is t, by reveals the inventory level $\boldsymbol{I_1(t)}$, which is given by

$$I_1(t) = \frac{(p-d)\int_0^t e^{\alpha u^\beta} du}{e^{\alpha t^\beta}}, 0 \leq t \leq T_1 \tag{2}$$

$\boldsymbol{T_1}$ is the period of the supplier produces to fulfil the demand of the retailer, $\boldsymbol{T_2}$ is the period a supplier uses its inventory to meet a retailer's demand, and $\boldsymbol{T = T_1 + T_2}$ is the production cycle time of the supplier, when $\boldsymbol{T_1 \leq t \leq T_1 + T_2}$, the following equation (3) considers the inventory level $\boldsymbol{I_2(t)}$

$$\frac{dI_2(t)}{dt} = -d - \alpha\beta t^{\beta-1} I_2(t) \tag{3}$$

Equation (3) is solved with the condition,

$$I_2(T_1 + T_2) = 0.$$

By revealing the inventory level $I_2(t)$, which is given

$$I_2(t) = \frac{d\int_t^{T_1+T_2} e^{\alpha u^\beta} du}{e^{\alpha t^\beta}}, T_1 \leq t \leq T_1 + T_2. \tag{4}$$

Therefore, during the time $T_1 + T_2$, the holding inventory of the supplier is given

$$\int_0^{T_1} I_1(t)\, dt + \int_{T_1}^{T_1+T_2} I_2(t)\, dt,$$

and the holding cost is given by

$$IC_s = c_{sh}\left\{\int_0^{T_1} I_1(t)\, dt + \int_{T_1}^{T_1+T_2} I_2(t)\, dt\right\} \tag{5}$$

where $\boldsymbol{c_{sh}}$ is the inventory holding cost of the supplier. By subtracting the demand quantity of the retailer from the production quantity, we can get the quantity for deterioration,

$$pT_1 - d(T_1 + T_2)$$

Given the deterioration cost of the supplier,

$$DC_s = c_{sd}[pT_1 - d(T_1 + T_2)] \tag{6}$$

where c_{sd} is the deterioration cost of the supplier and the total cost of the supplier includes,

$$TC_s = c_{ss} + IC_s + DC_s \tag{7}$$

where c_{ss} is the setup cost of the supplier. Thus, the expected profit of the supplier is

$$\pi_S = (w - c_{sp})d(T_1 + T_2) - TC_s \tag{8}$$

where w is the supplier's sales price per unit, and c_{sp} is the production cost of the supplier.
When $t = T_1$, $I_1(T_1) = I_2(T_1)$, we can obtain the following relationship:

$$T_1 = \frac{d}{p-d}\left(T_2 + \frac{\alpha}{\beta+1} T_2^{\beta+1}\right). \tag{9}$$

The inventory level of the retailer at each stage can also be obtained following the approach proposed by [11]. When $\boldsymbol{t_1 \leq t \leq t_1 + t_2}$, then the retailer encounters a product shortage where $\boldsymbol{t_1}$ is the product consumption time for the retailer and $\boldsymbol{t_2}$ is the product shortage time for the retailer. While, when $\boldsymbol{0 \leq t \leq t_1}$, the inventory level of the retailer $\boldsymbol{I_{b1}(t)}$is given by

$$\frac{dI_{b1}(t)}{dt} = -\alpha\beta t^{\beta-1} I_{b1}(t) - d. \tag{10}$$

By solving the differential equation (10) within the initial condition of

$$I_{b1}(t_1) = 0,$$

we can obtain the inventory level of the retailer,

$$I_{b1}(t) = de^{-\alpha t^{\beta}} \int_{t}^{t_1} e^{\alpha u^{\beta}}\, du, 0 \le t \le t_1. \tag{11}$$

During time t_1, the retailer's inventory is given by

$$I_{b1}(t) = \int_{0}^{t_1} de^{-\alpha t^{\beta}} \int_{t}^{t_1} e^{\alpha u^{\beta}}\, dudt. \tag{12}$$

and the retailer's holding cost is given by

$$IC_b = c_{bh} \int_{0}^{t_1} de^{-\alpha t^{\beta}} \ + \int_{0}^{t_1+\, t_2} e^{\alpha u^{\beta}}\, dudt \tag{13}$$

where c_{bh} is the inventory holding cost of the retailer. When $t_1 \le t \le t_1 + \, t_2$, the retailer encounters a product shortage and thus, the inventory level of the retailer varies, as

$$\frac{dI_{b2}(t)}{dt} = -d. \tag{14}$$

Then, we will solve the differential equation (14) with the condition

$$I_{b2}(t_1) = 0.$$

By revealing the inventory level of the retailer $I_{b2}(t)$, which is given

$$I_{b2}(t) = d(t - t_1). \tag{15}$$

The shortage quantity of retailer is thus given

$$\int_{t_1}^{t_1+\, t_2} I_{b2}(t)\, dt = \int_{t_1}^{t_1+\, t_2} d(t - t_1)\, dt \tag{16}$$

and the shortage cost is given by

$$RC = c_{bs} \int_{0}^{t_1+\, t_2} d(t - t_1)\, dt. \tag{17}$$

where c_{bs} is the shortage cost of the retailer. By subtracting the demand quantity at t_1 from the order quantity, we can obtain the deterioration quantity,

$$RD_b = I_{b2}(0) - dt_1. \tag{18}$$

The deterioration cost of the retailer is

$$DC_b = c_{bd} RD_b \tag{19}$$

where $\boldsymbol{c_{bd}}$ is the deterioration cost of the retailer. We then add the demand quantity of the buyer at $\boldsymbol{t_1}$ and the deterioration quantity to obtain the order quantity of the retailer,

$$q = dt_1 + I_{b2}(0) - dt_1 = d\frac{\alpha t_1^{\beta+1}}{\beta+1} + dt_1 \tag{20}$$

Consequently, the purchasing cost of the retailer is

$$PC = c_{bp} q = c_{bp}\left(d\frac{\alpha t_1^{\beta+1}}{\beta+1} + dt_1\right) \tag{21}$$

where $\boldsymbol{c_{bp}}$ is the purchasing cost of the retailer. We divide the production cycle time of the supplier by the order cycle of the retailer to obtain the number of orders from the retailer,

$$n = \frac{T_1+T_2}{t_1+t_2} \tag{22}$$

with the order cycle time of the retailer is calculated as

$$t_1 + t_2 = \frac{T_1+T_2}{n} = \frac{T}{n}. \tag{23}$$

The retailer's total cost is given by

$$TC_B = PC + c_{bo} + IC_b + RC + DC_b. \tag{24}$$

for each ordering cycle, where c_{bo} is the ordering cost of the retailer. Therefore, the expected profit of the retailer is

$$\pi_B = sdt_1 + sRD_b - TC_B \tag{25}$$

where s is the sales price of retailer. This study examines how quantity discounts might be coordinated to maximize both parties' profitability and the overall advantages of the supply chain. The retailer will favor placing greater orders than in the past if coordination is successful if both sides are willing to bargain, exchange information, and work together to maximize their individual gains and the advantages of the overall supply chain. If suppliers and retailers properly exchange information, the supply chain can be profitable. Thus, the overall supply chain's profit is

$$\pi_{SC} = n\pi_B + \pi_s . \tag{26}$$

2.3 Data for Supplier 1 and Supplier 2

In this study, we illustrate practical applications through numerical examples, including a scenario involving a single-item multi-supplier setup with two suppliers and one retailer. The intention is to demonstrate the theoretical concepts discussed earlier. Our focus is to assess the impact of various quantity discounts and associated parameters on individual performance indicators. By comparing numerical data, we emphasize the differences in expected profit for each entity before and after the implementation of quantity discounts.

The data used in this study were gathered from [10] and labelled as "Supplier 1". Another supplier called "Supplier 2" is being introduced. Data for Supplier 2 comes from a range of different purchase quantities. The data is about a small-scale company that supplies perishable cold food items to a single retailer. Once these cold food items deteriorate over time, they hold no value.

- The retailer's average daily demand rate is 200 units.
- The retail price for the product is $10.
- The retailer's ordering cost is $50.
- The retailer's inventory storage cost is $1.
- The retailer's shortage cost is $25.
- The supplier's average daily production rate is 220 units.
- The product's wholesale price is $7.
- The supplier's deterioration cost is $12.

We assume that the inventory deterioration rates for the retailer and suppliers are the same, with the scale parameter α = 0.05 and shape parameter β = 1.5. The different parameters of the suppliers are summarized in Table 1.

Table 1 *The Related Parameters Settings*

Parameter	Description	Supplier 1		Supplier 2	
c_{bp}	The buyer's purchasing cost	$7	$8	$9	$10
c_{bd}	The buyer's deterioration cost	$10	$15	$16	$17

c_{ss}	The supplier's setup cost	\$150	\$200	\$210	\$220
c_{sp}	The supplier's production cost per unit	\$3	\$4	\$5	\$6

A quantity can be purchased from suppliers 1 and 2 using the discount schedules in Tables 2 and 3, respectively.

Table 2 *Quantity Discount for Supplier 1*

Purchase quantity	Price per unit (\$)
0-200	7
201-220	6
> 220	5

Table 3 *Quantity Discount for Supplier 2*

Purchase quantity	Price per unit (\$)
0-200	8
201-250	6
>250	4

To determine a strategy to maximize profit and minimize total cost, the parameters in Table 1, 2 and 3 are applied to the model.

2.4 Optimal Order Quantity for Cost Minimization

Supplier 2 provides a range of sales prices, offering flexibility to the retailer. This strategic approach ensures that the retailer can choose the best purchasing cost to minimize costs and maximize profits. When determining the sales price, \$8 proves to be the optimal choice for both the retailer and the entire supply chain. Table 4 shows the price point maximizes the retailer's profit to nearly \$95,000, ensures favourable margins for the supplier, and results in the highest combined profit for all parties. Although opting for higher prices may be tempting, it ultimately jeopardizes both individual and collective profits in the long run.

Table 4 *Comparison of Profitability by Sales Price*

	Supplier 2		
Sales price	8	9	10
Supplier(profit)	27,590.00	21,580.00	15,570.00
Buyer (Profit)	94,620.82	93,573.61	93,750.00
Supply chain(Profit)	122,210.82	115,153.61	109,320.00

To minimize more costs, the retailer is seeking a quantity discount from Supplier 1 and Supplier 2. Optimizing order quantities with quantity discounts improved cost efficiency and minimized total purchasing costs. Table 5 shows the result when the retailer deals with Supplier 1. The optimal quantity discount size is above 220 units, the number of orders, n is 4 and the unit purchasing cost is \$5. The number of orders is calculated using Equation (22). The corresponding total cost is \$ 2,297.40. Table 5 indicates that the retailer can earn more profit after a quantity discount than they can without a quantity discount.

Table 5 *Result Before and After Quantity Discount of Supplier 1*

Supplier 1	Before	After
Sales price (\$)	7	5
Supplier (Profit) (\$)	33,640.00	44,297.40

Buyer(Profit) ($)	87,620.82	90,068.03
Supply chain (Profit) ($)	121,260.82	134,365.43
Number of order, *n*	5	4

Furthermore, it can be seen from Table 6 shows when a retailer deals with Supplier 2, the optimal quantity discount size is 250 units, the number of orders, *n* is 4 and the unit purchasing cost is $4, and the corresponding total cost is $ 2,247.40. Table 6 indicates that the retailer can earn more profit after a quantity discount than they can without a quantity discount. Moreover, the profit of the supplier under a quantity discount is increased compared to the supplier's profit with no quantity discount. The threshold of the supplier's selling price is smaller after the retailers engage in quantity discounts.

Table 6 *Result Before and After Quantity Discount of Supplier 2*

Supplier 2	Before	After
Sales price ($)	8	4
Supplier (Profit) ($)	27,590.00	38,247.40
Buyer (Profit) ($)	94,620.82	98,291.64
Supply chain (Profit) ($)	122,210.82	136,539.04
Number of order, *n*	5	4

2.5 Impact of Quantity Discounts on Total Costs

Changes in costs associated with both the retailer and supplier have a direct impact on the overall profitability of the supply chain. Specifically focusing on the retailer's costs, fluctuations in these costs proportionately affect the retailer's profit. An increase in the retailer's expenses results in a corresponding reduction in profit, particularly evident in purchasing costs, which wield the most significant influence on the retailer's bottom line. Elevated purchasing costs notably diminish the retailer's profit, highlighting the substantial impact of these expenses. Similarly, a rise in the number of orders escalates ordering costs, leading to a considerable decrease in the retailer's profit. Thus, effective management of fluctuations in purchasing and ordering expenses emerges as critical for enhancing the retailer's profitability.

Fig. 1 shows that the total cost of the supplier decreases while the total cost of the retailer increases. This is caused by the decrease in supplier holding costs and the increase in the purchasing costs of the retailer after quantity discounts. The total cost for Supplier 1 before quantity discount with a unit price sale of $7 is $8,360.00, and the total cost of retailer is $65,384.75, while after the quantity discount with a unit price sale of $5, the total cost is $2,297.40 and $67,831.97 respectively. For the total cost for Supplier 2 before the quantity discount with unit price sale of $8 is $8,410.00, and the retailer is $72,384.75, while after quantity discount with unit price sale of $4 is $2,247.40 and $76,055.57 respectively. The difference in retailer's total cost after coordination when dealing with Supplier 2 is higher, which is $3,670.82, while with Supplier 1 is $2,447.22.

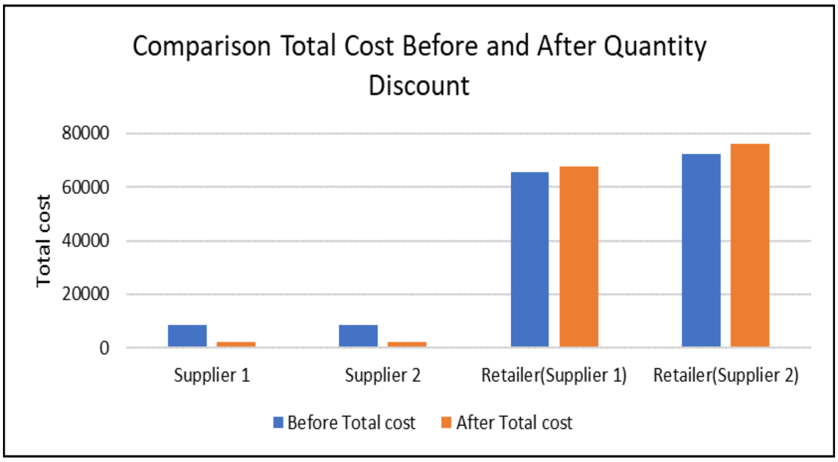

Fig. 1 *Total Cost Before and After Quantity Discount*

2.6 Comparison of Profit Gained

Evaluating suppliers with the aim of maximizing profits and minimizing inventory costs remains a critical function within business operations. The adept management of inventory and strategic procurement significantly shapes a retailer's financial landscape, driving increased profitability. By skilfully harmonizing cost reduction with profit maximization, retailers can secure financial stability, fortify competitive advantages, and capitalize on avenues for future growth. Managing inventory also involves smart decisions on order quantities and supply chain efficiency to balance meeting demand without excess stock that increases holding costs. Selecting suppliers who offer quality products at competitive prices. This not only impacts immediate profits but also strengthens a retailer's market position and customer satisfaction.

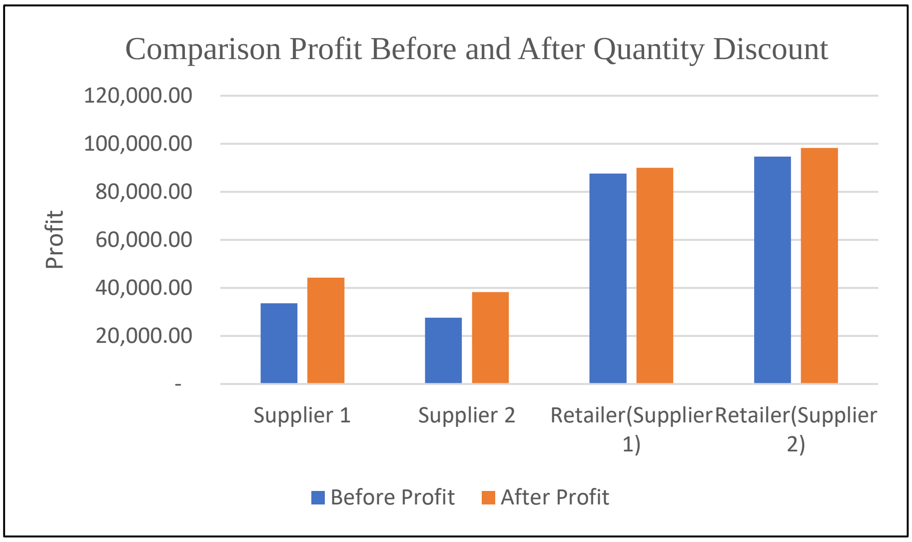

Fig. 2 *Comparison of Profit Before and After Quantity Discount*

Based on Fig. 2, we can see that the profit before and after quantity discount are changed. The results of numerical calculations considering the quantity discount obtain higher profits. The profits gained after the quantity discount for Supplier 1 and Supplier 2 are $44,297.40 and $38,247.40, respectively. The profit gained by the retailer when dealing with Supplier 1 is $90,068.03, while when dealing with Supplier 2, the profit made by the retailer is $98,291.64. The profit of the retailer made when dealing with Supplier 1 is smaller than the Supplier 2, with a difference of $8,223.61.

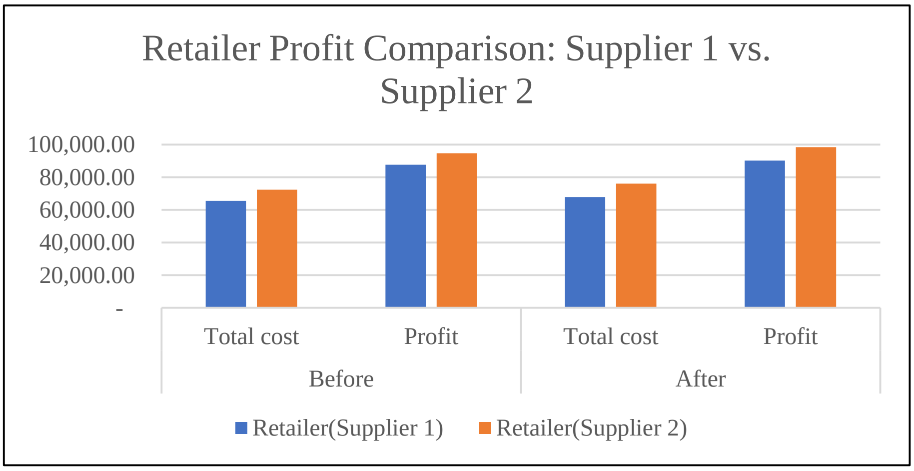

Fig. 3 *Retailer Profit Comparison: Supplier 1 vs. Supplier 2*

Fig. 3 shows that the retailer profits are higher when dealing with Supplier 2. In this case, the retailer is unwilling to cooperate with Supplier 1 because the retailer's profit is lower because of the higher selling price and lower order quantity. Despite the lower total cost incurred by the retailer when dealing with Supplier 1, opting for Supplier 2 results in higher profits, which effectively offset the relatively higher total cost. The difference in total cost between Supplier 1 and Supplier 2 is not substantial, allowing the increased profit from Supplier 2 to cover this margin.

3. Conclusions

This research seeks to enhance the overall profitability of an entire supply chain by exploring the benefits of a quantity discount strategy. This strategy involves suppliers offering varying discounts for different order quantities, aiming to minimize inventory costs and drive increased profits for all involved. This study used MATLAB to evaluate the integration formula under the Deteriorating Inventory and then used Microsoft Excel for the overall solution. The results indicate that the retailer's purchasing cost significantly impacts on its profit. Therefore, emphasizing the optimization of purchasing costs could significantly increase the retailer's profitability. In addition, the result shows that to maximize the profit of both suppliers and retailers, the purchasing cost and holding cost must be reduced. According to the findings, Supplier 2 will be selected by the retailer to maximize profits.

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Conflict of Interest

There is no conflict of interest regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception, data collection, analysis and interpretation of results, and manuscript preparation:*** *Tengku Sofia Zulaikha Tengku Mohamad Rozi;* ***guidance and made corrections:*** *Cik Sri Mazzura Muhammad Basri.*

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