ENHANCED KNOWLEDGE IN SCIENCES AND TECHNOLOGY
e-ISSN: 2773-6385

EKST

Vol. 4 No. 1 (2024) 120-126
https://publisher.uthm.edu.my/periodicals/index.php/ekst

Solving Arms Problem Using Differential Transform Method (DTM)

Muhammad Shafiq Aiman Muhammad Faizal[1], Noor Azliza Abd Latif[1]*,

[1] *Department of Mathematics and Statistics, Faculty of Applied Science and Technology, Universiti Tun Hussien Onn Malaysia (Pagoh Campus),84600 Pagoh, Muar, Johor, MALAYSIA*

*Corresponding Author: azliza@uthm.edu.my
DOI: https://doi.org/10.30880/ekst.2024.04.01.014

Article Info

Received: 27 December 2023
Accepted: 03 June 2024
Available online: 27 July 2024

Keywords

Differential Transform Method, Arms Race Model, System of Ordinary Differential Equation.

Abstract

This research, a semi analytical solution called differential transform method (DTM) used to solve arms race model problem. With the help of Maple 2023, the approximate solution of the arms race model equations was obtained. Using this method to the arms race model proved that the computation works can be reduced and save amount of time. The numerical results derived by DTM are compared to the exact solution which included with all the arms race model equation. The results indicate the efficiency of DTM in solving non-linear DTM on the specific characteristic for every equation. DTM success in solving two examples for arms race model, emphasizing its simplicity in tackling differential problems with minimal computational work.

1. Introduction

This research's primary objective is to investigate the potential applicability of the differential transform method (DTM) in solving the arms race model problem. The aim is to delve into the arms race model equation, employing the DTM as a tool for analysis to derive an analytical solution. Through rigorous examination and mathematical scrutiny, I aspire to elucidate the effectiveness of the DTM in comparison to other existing techniques, ultimately gauging its efficiency in providing precise solutions to the intricate dynamics inherent in the arms race model. This research endeavours to contribute valuable insights into the realm of mathematical modelling and solution methodologies, particularly focusing on the arms race model, thereby advancing our understanding of complex systems and optimization techniques.

The DTM, initially introduced by Zhou (1986), offers a distinct approach to address various differential and integral equations [1]. Unlike the conventional high-order Taylor series, DTM does not necessitate symbolic computations of derivative functions. Its unique advantage lies in providing an alternative to the Taylor series, demonstrating greater accuracy in certain scenarios. Over time, DTM has undergone significant development and adaptation. It proves effective for both linear and non-linear systems, encompassing a spectrum from low-order to higher-order ordinary differential equations. The method's capability to convert differential equations into algebraic ones, coupled with its computational efficiency and flexibility, positions DTM as a pivotal tool in scientific research and engineering domains.

This paper delves into solving the arms race model using the DTM, focusing on comparing military strengths between two or more countries, such as the historical rivalry between Germany and Great Britain in World War I. The model encapsulates a first-order ordinary differential system, predominantly employed for data analytics to assess military capabilities [3]. In this context, the analysis centers on two countries, labelled as 'x' and 'y'. The model evaluates the impact of incentives for arming and constraints against it, primarily gauged by changes in armament levels. This arms race framework has been instrumental in mathematical modelling, notably aiding data analysts in juxtaposing military might, as exemplified during the Cold War between the United States and the Soviet Union [4]. Utilizing this model, nations can strategize and optimize their military capabilities based on collected data and insights. Other previous studies were done in mathematic application in worldwide according to various fields of studies [5, 6].

2. Research Methodology

The DTM is used to solve two systems regarding the application of system of first-order ordinary differential equation which is arms race model.

2.1 Mathematical Model

2.1.1 Arms race model

The system of arms race model considers as below [7]

$$\frac{dx}{dt} = ky - \alpha x + g \tag{1}$$

$$\frac{dy}{dt} = lx - \beta y + h \tag{2}$$

where x and y represent the amount of weaponry at time, t for country 1 and 2 respectively. k and l represent the efficiency increasing the armaments of x and y respectively. α and β represents factor. g and h are grievance constant. The initial condition follows the problem value for each equation.

2.1.2 Differential Transform Method

The one-dimensional differential transform function is defined as follows:

$$C(k) = \frac{1}{k!}\left[\frac{\partial^k}{\partial x^k} c(x)\right]_{x=0} \tag{3}$$

where $c(x)$ is the original function and $C(k)$ is the transformed function.

The differential inverse transform of $C(k)$ is defined as follows:

$$c(x) = \sum_{k=0}^{\infty} C(k) x^k \tag{4}$$

The fundamental operation performed by differential transformation as follow in Table 1.

Table 1 *The fundamental operation performed by differential transform.*

Original functions	Transformed functions
$c(t) = u(t) \pm v(t)$	$C(k) = U(k) \pm V(k)$
$c(t) = \alpha u(t)$	$C(k) = \alpha U(k)$
$c(t) = \frac{\partial}{\partial t} u(t)$	$C(k) = (k+1)U(k+1)$
$c(t) = \frac{\partial^r}{\partial t^r} u(t)$	$C(k) = (k+1)(k+2)...(k+r)U(k+r)$
$c(t) = u(t)v(t)$	$C(k) = \sum_{r=0}^{k} U(r)V(k-r)$
$c(t) = e^{\lambda t}$	$C(k) = \frac{\lambda^k}{k!}$
$c(t) = (1+t)^m$	$C(k) = \frac{m(m-1)...(m-k+1)}{k!}$
$c(t) = \sin(\omega t + \alpha)$	$C(k) = \frac{\omega^k}{k!} \sin\left(\frac{\pi k}{2!} + \alpha\right)$

$c(t)=\cos(\omega t+\alpha)$	$C(k)=\frac{\omega^k}{k!}\cos\left(\frac{\pi k}{2!}+\alpha\right)$

3. Results

The application with initial condition is discussed to solve the application system. Arms race model denoted as example 1 and example 2 are the non-homogenous system. The exact solutions are solved by using Maple (2023). The application will be solved numerically using DTM. The results will also be calculated using the same software. The graph will be plotted by using Microsoft Excel.

3.1 Example 1

Consider the system [5],

$$\frac{dx}{dt}=4y-3x+2 \tag{5}$$

$$\frac{dy}{dt}=2x-y+2 \tag{6}$$

with initial condition $x=4, y=1$.
Given the exact solution

$$x(t)=4e^{t}+2e^{(-5t)}-2 \tag{7}$$

$$y(t)=4e^{t}+e^{(-5t)}-2 \tag{8}$$

By applying the DTM to the system, we get

$$x(k+1)=\frac{4y(k)-3x(k)+2}{k+1} \tag{9}$$

$$y(k+1)=\frac{2x(k)-y(k)+2}{k+1} \tag{10}$$

do the calculations using Maple (2023) by replacing $k=0$, $x(0)$, and $y(0)$,

$$x(0+1)=\frac{4y(0)-3x(0)+2}{0+1}$$
$$x(1)=-6 \tag{11}$$

$$y(0+1)=\frac{2x(0)-y(0)+2}{0+1}$$
$$y(1)=9 \tag{12}$$

continue the calculation using $k=1$, $k=2$, $k=3$, $k=4$ and $k=5$. The calculations stop at $k=5$ because for the $k=6$ and further make subtle change to the end results.

Table 2 *Numerical solution by using DTM*

K	X(k+1)	Y(k+1)
0	4	1
1	-6	9
2	28	$-\frac{19}{2}$
3	-40	$\frac{45}{2}$
4	53	$-\frac{201}{8}$
5	$-\frac{13}{2}$	$\frac{213}{8}$

6	$\frac{64}{3}$	$-\frac{301}{48}$

apply Taylor's series for *x*,

$$x(t) = 4 - 6t + 28t^2 - 40t^3 + 53t^4 - \frac{13}{2}t^5 + \frac{64}{3}t^6 \tag{13}$$

apply Taylor's series for *y*,

$$y = 1 + 9t - \frac{19}{2}t^2 + \frac{45}{2}t^3 - \frac{201}{8}t^4 + \frac{213}{8}t^5 - \frac{301}{48}t^6 \tag{14}$$

Evaluate both Taylor's series for both *x* and *y* to get numerical results (evaluate using maple). Plot the graph for comparison between the exact solution and DTM solution.

3.1.1 Numerical results

The DTM is used to find numerical results of example 1. The results are calculated using Maple (2023) software.

Table 3 *Numerical results of example 1 for x*

x	Exact Value (x)	DTM Value (x)	/Exact – DTM/ (x)
0	4	4	0
0.1	3.64526	3.64526	0
0.2	3.68409	3.68409	0
0.3	4.06906	4.06906	0
0.4	4.89762	4.89762	0

Table 4 *Numerical results of example 1 for y*

y	Exact Value (y)	DTM Value (y)	|Exact – DTM| (y)
0	1	1	0
0.1	1.81415	1.82525	0.01109
0.2	2.51773	2.56792	0.05019
0.3	3.17631	3.30911	0.13281
0.4	3.83196	4.12375	0.29179

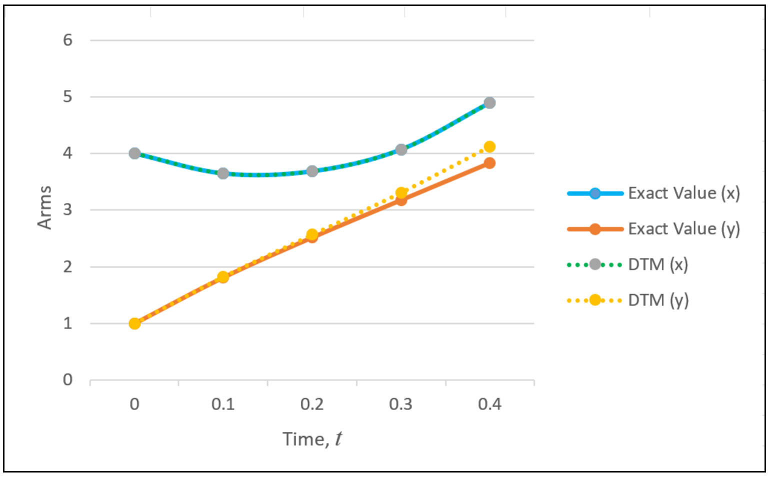

Fig. 1 *Comparison DTM and exact solution*

3.1.2 Discussion

In example 1, there is no error for x which represent country 1. Whereas for y there was error at y = 0.1 with 0.01109 difference, y = 0.2 with 0.05019 difference, y = 0.3 with 0.13281 difference and at point y = 0.4 with 0.29179 difference with the highest error at point y = 0 .4. We can clearly see that the DTM can produce results that closely approximate, or even match, the exact solution, since there is only a slight difference, less than 1.0 for all the absolute errors.

Example 2

Consider the system [8],

$$\frac{dx}{dt} = y - 3x + 3 \tag{15}$$

$$\frac{dy}{dt} = 2x - 4y + 8 \tag{16}$$

with initial condition $x = 12, y = 15$.
given the exact solution

$$x(t) = \frac{32}{3}e^{-2t} - \frac{2}{3}e^{-5t} + 2 \tag{17}$$

$$y(t) = \frac{32}{3}e^{-2t} - \frac{4}{3}e^{-5t} + 3 \tag{18}$$

by applying the DTM to the system, we get

$$x(k+1) = \frac{y(k) - 3x(k) + 3}{k+1} \tag{19}$$

$$y(k+1) = \frac{2x(k) - 4y(k) + 8}{k+1} \tag{20}$$

do the calculations using Maple (2023) by replacing $k = 0$, $x(0)$, and $y(0)$,

$$x(0+1) = \frac{y(0) - 3x(0) + 3}{0+1}$$
$$x(1) = -18 \tag{21}$$

$$y(0+1) = \frac{2x(0) - 4y(0) + 8}{0+1}$$
$$y(1) = -28 \tag{22}$$

continue the calculation using $k = 1$, $k = 2$, $k = 3$, $k = 4$ and $k = 5$. The calculations stop $k = 5$ at because for the $k = 6$ and further make subtle change to the end results.

Table 5 *Numerical solution by using DTM*

k	*X(k+1)*	*Y(k+1)*
0	*12*	*15*
1	*-18*	*-28*
2	$\frac{29}{2}$	*42*
3	$\frac{1}{2}$	$-\frac{131}{3}$
4	$-\frac{253}{24}$	$\frac{551}{12}$
5	$\frac{1933}{120}$	$-\frac{787}{20}$
6	$-\frac{1129}{80}$	$\frac{11857}{360}$

apply Taylor's series for x,

$$x(t)=12-18t+\frac{29}{2}t^2+\frac{1}{2}t^3-\frac{253}{24}t^4+\frac{1933}{120}t^5-\frac{1129}{80}t^6 \tag{23}$$

apply Taylor's series for y,

$$x(t)=15-28t+42t^2-\frac{131}{3}t^3+\frac{551}{12}t^4-\frac{787}{20}t^5+\frac{11857}{360}t^6 \tag{24}$$

Evaluate both Taylor's series for both x and y to get numerical results (evaluate using maple). Plot the graph for comparison between the exact solution and DTM solution.

3.2 Numerical results

The DTM is used to find numerical results of example 1. The results are calculated using Maple (2023) software.

Table 6 *Numerical results of example 2 for x*

x	Exact Value (x)	DTM Value (x)	|Exact – DTM| (x)
0	12	12	0
0.1	10.32877	10.34459	0.01582
0.2	8.90483	8.97138	0.06656
0.3	7.70524	7.86197	0.15673
0.4	6.70262	6.98928	0.28666

Table 7 *Numerical results of example 2 for y*

y	Exact Value (y)	DTM Value (y)	|Exact - DTM |(y)
0	15	15	0
0.1	12.54184	12.58056	0.03873
0.2	10.64059	10.79365	0.15306
0.3	9.15150	9.50131	0.34982
0.4	7.97329	8.63276	0.65947

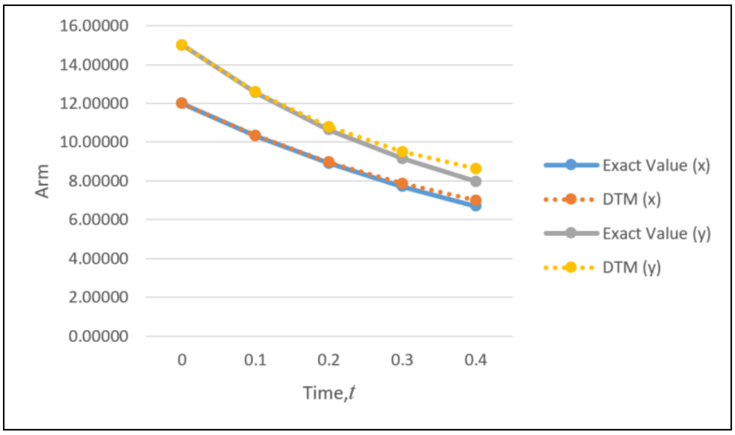

Fig. 2 *Comparison DTM and exact solution*

3.3 Discussion

In example 2, there is slightly error for x at point $x = 0.1$ with 0.01582 difference, $x = 0.2$ with 0.06656 difference, $x = 0.3$ with 0.15673 difference and the highest error at point $x = 0.4$ with 0.28666 difference. Meanwhile for y there was error at $y = 0.1$ with 0.03873 difference, $y = 0.2$ with 0.15306 difference, $y = 0.3$ with 0.34982 difference and at point $y = 0.4$ with 0.65947 difference with the highest error at point $y = 0.4$. We can clearly see that the DTM can produce results that closely approximate, or even match, the exact solution, since there is only a slight difference, less than 1.0 for all the absolute error.

4. Conclusion

In conclusion the DTM successfully solved the system of ordinary differential equation. In this research, it has been proven that DTM indeed can solve the Arms race model equations. Since approximate solution by DTM is close to the exact solutions, therefore the application of the DTM proves to be a valuable and efficient approach in solving complex mathematical and engineering problems, offering a reliable alternative for obtaining accurate results in situations where exact solutions may be challenging to derive. Several recommendations can be made to improve the accuracy and efficiency of DTM for further study. Make an improvement of the algorithm and coding of DTM to obtain more accurate solution and compare the end results with other methods to improve the DTM to saving differential equations.

Acknowledgement

The authors would thank the Faculty of Applied Sciences and Technology, Universiti Tun Hussein Onn Malaysia for its support.

Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design:*** *Muhammad Shafiq Aiman Muhammad Faizal:* ***Analysed and interpreted data.*** *Muhammad Shafiq Aiman Muhammad Faizal and Noor Azliza Abd Latif. All authors reviewed the results and approved the final version of the manuscript.*

References

[1] Zhou, J.X. (1986). Differential Transform Method and Its Application for Electric Circuits, Huazhong University Press Wuhan.
[2] He, J. H. (1997). A New Approach to Nonlinear Partial Differential Equations. Communications in Nonlinear and Numerical Simulations, 2, 230-235.
[3] Muller, H. (1946). The Arms Race, American Journal of Sociology, 51(3),247-259.
[4] Jervis, R. (1978). Cooperation Under the Security Dilemma. World Politics, 30(2). 167-214.
[5] Ibrahim, M. H. R., & Aman, F. (2022). Analysis on Hiemenz flow over a shrinking sheet in hybrid nanofluid. Enhanced Knowledge in Sciences and Technology, 2(1), 221-230.

[6] Azmi, M. A. & Aman, F. (2022). Analysis on hybrid nanofluid over a shrinking sheet with transpiration and uniform shear flow. Enhanced Knowledge in Sciences and Technology, 2(1), 210-220.
[7] Razali, M. A. (2022). Forth Order Runge Kutta Method for Solving Application of System of First Order Ordinary Differential Equations, University Tun Hussien Onn Malaysia
[8] Dennis G Zill (2013). A First Course in Differential Equations with Modelling Applications< Brooks/Cole.