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An Exponential Growth Model for Gold Price Prediction with Levenberg-Marquardt Technique

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Abstract

Gold has been a valuable commodity for centuries, and its price exhibits considerable up-and-down movement over time. So, the prediction of gold prices becomes a critical issue in investment. This paper proposes an exponential growth model for predicting gold selling prices. We introduce a loss function to represent the model sum of squared errors and derive the first-order necessary condition. The Levenberg-Marquradt technique is applied to update the model parameter value during the iteration procedure. When convergence is achieved, the iterative solution gives the optimal parameter to the model. Thus, a best-fit curve to the actual gold selling prices is resulted. In addition, we modify the prediction solution by adding the previous actual price into the analytical solution. So, the fluctuation behaviour revealed by gold-selling prices can be well predicted. Moreover, the simple moving average and nonlinear regression are used to predict gold selling prices and their performance measures are compared with the performance measure of the exponential growth model. The simulation results show that the exponential growth model presents a high prediction accuracy. In conclusion, using the exponential growth model to predict gold selling prices demonstrates a highly satisfactory prediction result.

1. Introduction

In financial mathematics, investment is an activity that utilizes money resources, such as assets, portfolios and bonds, to generate wealth, like profits, incomes and returns. However, a financial investment is challenging since the stock market is uncertain and random [1]. Gold is one of the most popular investments besides investing in stocks because it is less correlated with stocks or bonds. Buying gold through future contacts and derivatives is a way of diversifying risk [2]. It is believed that owning gold is a hedge against inflation and deflation because the gold price tends to increase when the cost of living increases and verse vice. Gold is also claimed to be a good store of value when a currency loses value [3]. The gold price is followed in financial markets around the world. Over the past few decades, the prices of gold have been influenced by various factors, such as the central bank interest rates, government policies, inflation and monetary policy [4], causing the gold price to present significant up-and-down trends and dramatic changes. This fluctuation behaviour has been viewed over a long period through long-term historical gold prices. Investors need to predict the price of gold to make an appropriate decision on buying gold [5, 6]. From the literature, statistical methods, like simple moving averages, exponential smoothing and nonlinear regression, are commonly used to forecast the prices of gold over a period [7]. The accuracy of these methods can be evaluated by using performance measures, such as root mean square errors (RMSE) and mean absolute errors (MAE) [8].

For thousands of years, gold has been regarded as the most precious metal in the world, valued for its various properties and applications [9]. The purity of gold is measured in karats (K) and is commonly categorized into 14K, 18K, 22K, and 24K [10]. Karat (K) denotes the proportion of gold in parts per thousand or the percentage of gold content. As a valuable metal, gold has long been a favoured asset for property and finance investments, even in the face of its increasing prices. The fluctuations in gold prices are subject to the influence of various factors. Among these factors, the price of oil and the exchange rate of the US dollar play a significant role, exerting both positive and negative effects on gold prices [11]. Gold serves as an excellent tool for hedging against inflation. In periods of high inflation, gold prices typically increase, whereas lower inflation or deflation levels can negatively affect gold prices [12]. Gold price prediction involves using various techniques and methods to forecast the future price of gold. It is a field that attracts the attention of many investors who closely monitor price movements. By predicting gold prices, investors can gain insights into the dynamics of the financial markets and make informed investment decisions [13].

Gold prices cannot be calculated accordingly because of randomness and uncertainty [14]. The fluctuation of gold prices reveals the difficulty of knowing the movement of gold prices in future. From this, the volatility of gold prices can influence decision-making on buying and selling gold. Without knowing the actual gold price, investors might decide to buy gold at a high price or sell gold at a low price. In these decisions, investors will have a risk of loss. Thus, it is necessary to have gold price predictions through historical data on gold prices before deciding to buy and sell gold. Many statistical methods have been developed in the literature to predict gold prices. These methods include time series analysis, simple moving averages, exponential smoothing, and multivariate regression analysis. Moreover, using machine learning and deep learning to predict gold prices has recently attracted interest and attention from research communities [15]. Although the performance of these methods is demonstrated to give a satisfactory prediction result, the performance measure values should be further improved. It is claimed that the high value of a performance measure reveals the inaccuracy of the method used. This will affect a wrong investment strategy made on gold. Other previous studies were done in using forecasting method in worldwide according to various fields of studies [16, 17, 18, 19].

Therefore, in this study, an exponential growth model will be applied for gold selling price prediction. The dataset of daily trading prices (in Ringgit) of Malaysia's gold bullion coin was obtained from the Central Bank of Malaysia website [20]. The first-order linear ordinary differential equation (ODE) to the growth model and its analytical solution are covered. The Levenberg-Marquardt technique is applied to estimate the parameter in the exponential growth model during the calculation procedure. A simple computational algorithm proposed in our study combines the least squares optimization problem and parameter estimation for predicting gold prices. With the optimal parameter, the differences between the actual gold prices and predicted prices can be minimized. Hence, a satisfactory prediction of gold selling prices can be suggested.

The rest of the paper is organized as follows. In Section 2, the problem formulation is described, the analytical solution for an exponential growth model is discussed, and the Levenberg-Marquardt recursion equation is expressed. In Section 3, the simulation results for the model parameter are presented. Predictions of gold selling prices for Malaysia's gold bullion coin are discussed. Performance measure comparison between the method of growth exponential model, simple moving average and nonlinear regression are interpreted. At the end of the paper, a conclusion is made.

2. Materials and Methods

Consider a set of gold prices given by

$$y = \{y_1, y_2, \cdots, y_n\}\,, \tag{1}$$

where n is the number of gold prices in the set y. Then, the gradient of the actual data is obtained from

$$y_i' = \frac{y_{i+1} - y_i}{t_{i+1} - t_i}, i = 1, \cdots, n-1, \tag{2}$$

with t_i is the time concerned. Denote this gradient be a vector as follows,

$$y' = \{y_1', y_2', \cdots, y_{n-1}'\}. \tag{3}$$

Define a loss function,

$$J(\beta) = \frac{1}{2}(y' - \beta x)^{\mathrm{T}}(y' - \beta x), \tag{4}$$

where β is the unknown parameter to be determined such that the loss function (4) is minimized over the first-order differential equation,

$$\frac{dx}{dt} = \beta x\,. \tag{5}$$

The initial value $x(0) = x_0$ is given. This problem is called the gold prices prediction problem using an exponential growth model.

2.1 Levenberg-Marquardt Method

Now, write the loss function (4) into the Taylor series expansion in the second order,

$$J(\beta) \approx J(\beta^{(i)}) + J'(\beta^{(i)})(\beta - \beta^{(i)}) + \frac{1}{2} J''(\beta^{(i)})(\beta - \beta^{(i)})^2 \tag{6}$$

By considering the first-order necessary condition, (6) becomes

$$0 = J'(\beta) \approx J'(\beta^{(i)}) + J''(\beta^{(i)})(\beta - \beta^{(i)}) \tag{7}$$

The resulting normal equation is

$$J''(\beta^{(i)})(\beta - \beta^{(i)}) = -J'(\beta^{(i)}) \tag{8}$$

The parameter β is updated through the following equation,

$$\beta^{(i+1)} = \beta^{(i)} - (J''(\beta^{(i)}))^{-1} J'(\beta^{(i)}) \tag{9}$$

where

$$J'(\beta) = -(x)^{\mathrm{T}}(y' - \beta x) \tag{10}$$

$$J''(\beta) = x^{\mathrm{T}} x \tag{11}$$

To avoid the term $(J''(\beta^{(i)}))^{-1}$ being undefined, the damping parameter λ is added into (9), that is,

$$\beta^{(i+1)} = \beta^{(i)} - (J''(\beta^{(i)}) + \lambda I)^{-1} J'(\beta^{(i)}) \tag{12}$$

The recursion equation (12) is known as the Levenberg-Marquardt recursion equation [21]. Notice that the small value of the damping parameter λ gives the Gauss-Newton updates, while the large value of the damping parameter λ gives the steepest descent updates.

2.2 Prediction Solution

Referring to (5), the analytical solution [22] of the exponential growth model is given by

$$x(t) = x_0 e^{\beta(t - t_0)}. \tag{13}$$

Suppose the optimal parameter is estimated by the updating rule (12) to yield

$$\hat{\beta} = \beta^{(i+1)} \approx \beta^{(i)} \tag{14}$$

when convergence is achieved. Then, the analytical solution of the exponential growth model becomes

$$x(t) = x_0 e^{\hat{\beta}(t - t_0)}. \tag{15}$$

For the prediction purpose, we rewrite the solution (15) as follows,

$$\hat{x}(t_{i+1}) = x(t_i) e^{\hat{\beta}(t_{i+1} - t_i)}, \tag{16}$$

for $t_i < t < t_{i+1}$, $i = 0, 1, \ldots, n$. Here, $\hat{x}$ is the predicted solution at time t_{i+1} and x is the solution of the exponential growth model at time t_i.

Since the gold selling prices present the fluctuation behaviour, to improve the accuracy of the prediction, we modify the predicted solution (16) to be

$$\hat{x}(t_{i+1}) = y(t_i) e^{\hat{\beta}(t_{i+1} - t_i)}, \tag{17}$$

for $t_i < t < t_{i+1}$, $i = 0, 1, \ldots, n$, and y represents the actual gold selling price.

3. Results and Discussion

From the actual trading prices (in Ringgit) of Malaysia's gold bullion coin, the gold selling prices were collected from 01 January 2020 to 31 December 2020, which are 250 data in our study [20].

3.1 Simulation Results for Model Parameter

The simulation results of applying the LM method to determine the optimal parameter β are shown in Table 1. Here, two initial values of the parameter β_0 are considered, which $\beta_0 = 0.01$ is for the exponential growth model and $\beta_0 = -0.1$ is for the exponential decay model. These two initial values are randomly selected to start the iterative procedure, and their final results are observed.

Table 1 *Simulation results using exponential growth model*

Initial Parameter β_0	Final Parameter β^*	Iteration Number	Elapsed Time (s)	Loss Function	Prediction Error
0.01	0.194	13	0.3149	4.4683	3.2917×10^{-3}
-0.1	0.194	14	0.3178	4.4683	3.2917×10^{-3}

From the simulation results, we notice that using the exponential growth model as the initial model, the LM method takes 13 iterations to converge in 0.32 seconds. While applying the exponential decay model as the initial model, there are 14 iterations to achieve convergence within 0.32 seconds. The value of the loss functions when using both initial models is 4.47 units, and the prediction error is 3.29×10^{-3} for the final model parameters. With these minor prediction errors, the accuracy of the prediction results in our study is acceptable.

Referring to Table 1, the initial models with the initial value of the parameter β are written as below,

•Exponential growth model
$$\frac{dx}{dt} = 0.01x \tag{18}$$

•Exponential decay model
$$\frac{dx}{dt} = -0.1x \tag{19}$$

with the initial value $x(0) = 0.7172.$

The final model refers to the final value of the parameter β after running the LM method when convergence is achieved, which is the optimal parameter. From the simulation results, we write the final model and the respective solution as follows,

$$\frac{dx}{dt} = 0.194x \text{ and } x(t_{i+1}) = x(t_i)e^{0.194(t_{i+1}-t_i)} \tag{20}$$

with the initial value $x(0) = 0.7172.$

We also improve the accuracy of the prediction of gold selling prices by modifying the solution given in (20), where the actual prices are employed into the solution. Thus, this improved solution is provided below,

$$x(t_{i+1}) = y(t_i)e^{0.194(t_{i+1}-t_i)}, \tag{21}$$

with the initial value $x(0) = 0.7172.$

3.2 Prediction Solution of Gold Selling Prices

By using the exponential growth model (18) as the initial model, the solution is shown in Figure 1. The solution curve is upward and increasing exponentially from 0.7 to 9. Figure 2 shows the initial solution of the exponential decay model (19), where the value of the model solution decreases rapidly in a downward trend from 0.7 towards 0 and stays at zero after 50 days.

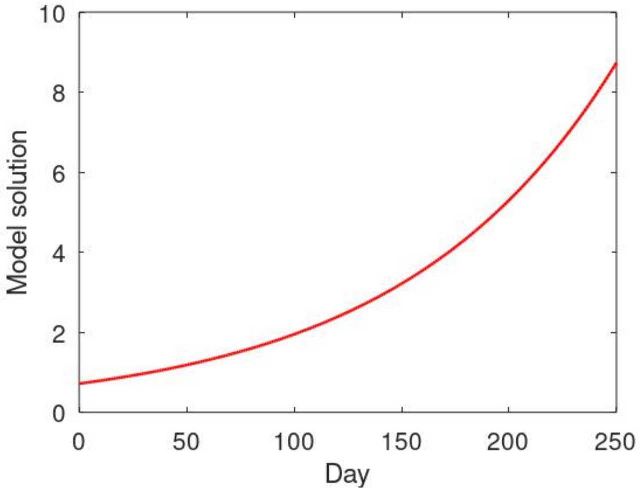

Fig. 1 *Solution with initial growth model*

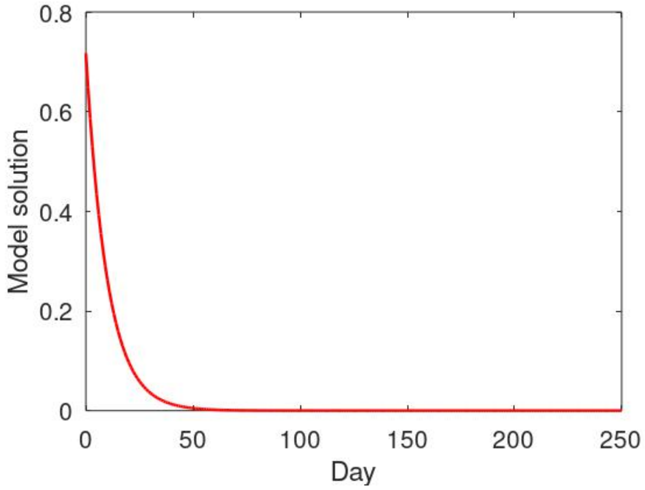

Fig. 2 *Solution with initial decay model*

Figure 3 shows the prediction solution to gold selling prices using the final solution (20), where the blue line represents the actual gold selling prices, and the red line represents the prediction solution using the final model. The prediction solution presents a linear trend fitting the actual gold selling prices. The prediction error, which is the difference between the actual gold selling prices and the prediction solution, is shown in Figure 4, and this error expresses the random behaviour.

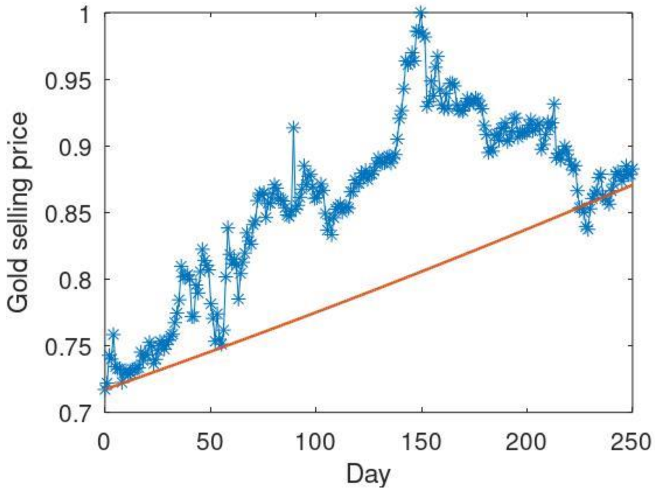

Fig. 3 *Prediction solution with final model and gold selling prices*

The loss function represents the mean square error (MSE) for the differences between the derivative of gold selling prices and the exponential function, as shown in Figure 5. The initial loss function is 4.5 units, which reduces to 4.468 units at the third iteration and stays at this value until convergence. This reveals a constant MSE between the derivative of the gold selling prices and the exponential growth function.

Figure 6 shows the prediction result using the improved solution (21), where the blue line represents the actual gold selling prices, and the red line represents the prediction result using the improved solution. Notice that the prediction result of this improved solution fits the gold selling prices closely, and the prediction error, which ranges from -0.06 to 0.06, is shown in Figure 7. Thus, the accuracy of prediction results using the improved solution (21) is proven.

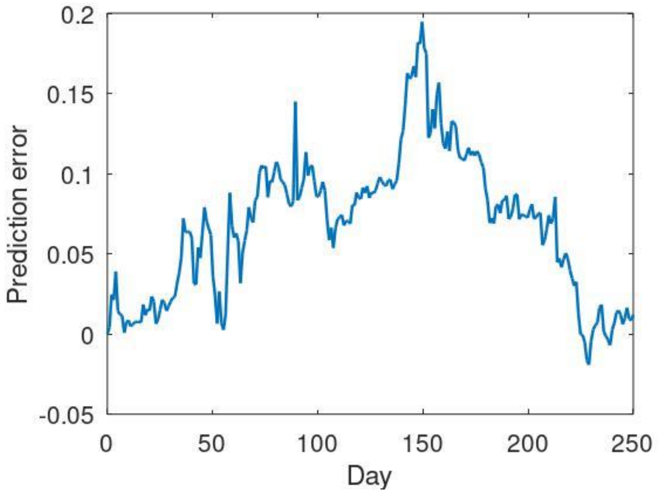

Fig. 4 *Prediction error for final model*

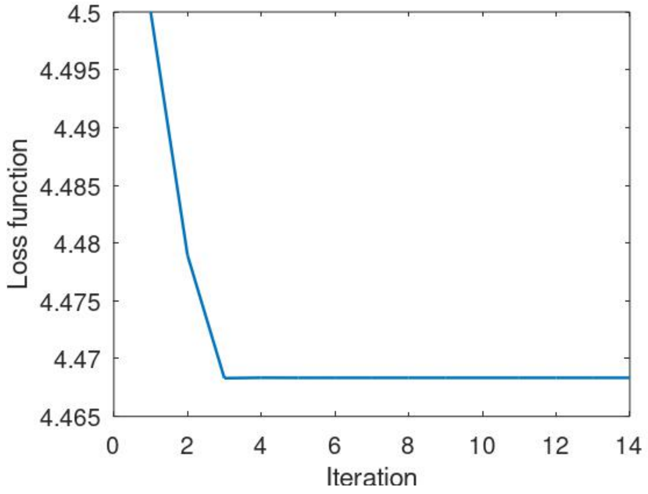

Fig. 5 *Loss function using final model*

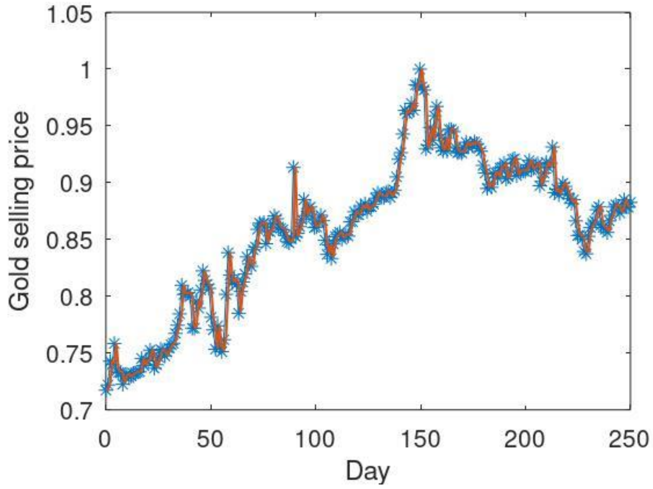

Fig. 6 *Prediction solution for improved final model and gold selling price*

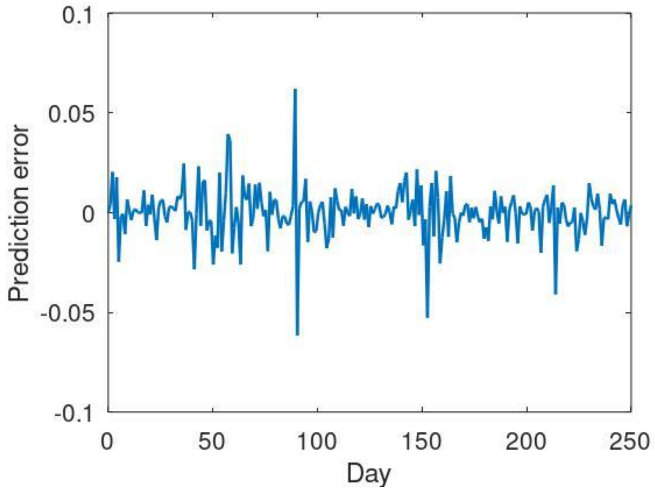

Fig. 7 *Prediction error for improved final model*

3.3 Performance Measure Comparison

In this section, we use the simple moving average and nonlinear regression to predict gold selling prices, and their MSE values are compared with our proposed method. Table 2 shows the comparison result of the performance measures. The proposed method has the smallest MSE value, which expresses an accurate prediction result.

Table 2 *Comparison of performance measures*

Method	MSE
Growth exponential model	3.6601×10^{-8}
Simple moving average (5-day)	7.3011×10^{-5}
Nonlinear regression	9.8315×10^{-4}

Figure 8 shows the prediction solution after using the simple moving average method over five days. This prediction solution matches the actual gold selling prices with the MSE of 7.3011×10^{-5}. This is a satisfactory prediction solution, but the accuracy of this method is not as high as the exponential growth model. The prediction error of using the simple moving average is shown in Figure 9, where the random characteristic ranging from -0.06 to 0.06 is presented.

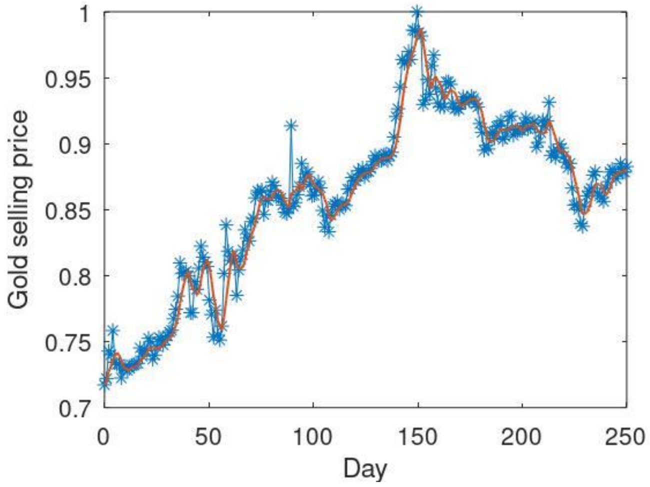

Fig. 8 *Prediction solution for simple moving average model and gold selling prices*

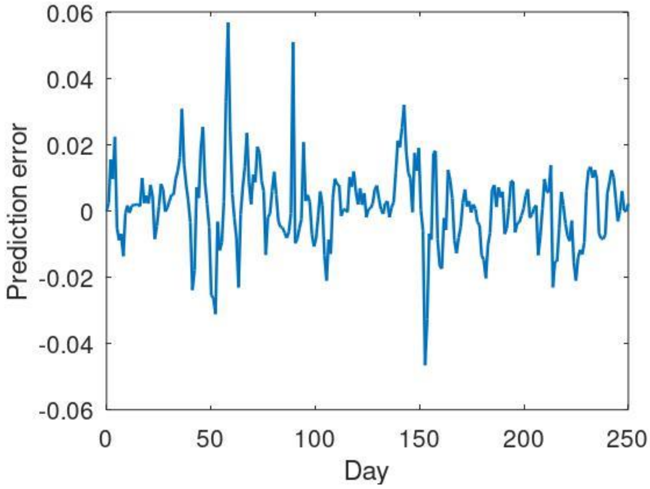

Fig. 9 *Prediction error for simple moving average model*

Figure 10 shows the prediction solution when we use the nonlinear regression model

$$h(t) = b_1 \exp(-b_2 t) \tag{22}$$

to predict the actual gold selling prices. The parameter estimates are

$$\hat{b}_1 = 0.77953 \text{ and } \hat{b}_2 = -7.6735 \times 10^{-4}, \tag{23}$$

and the prediction solution shown in Figure 10 is similar to the prediction solution given by the final model (20). The prediction error for using the nonlinear regression model (22) is shown in Figure 11, where this error expresses the fluctuation behaviour and is non-stationary.

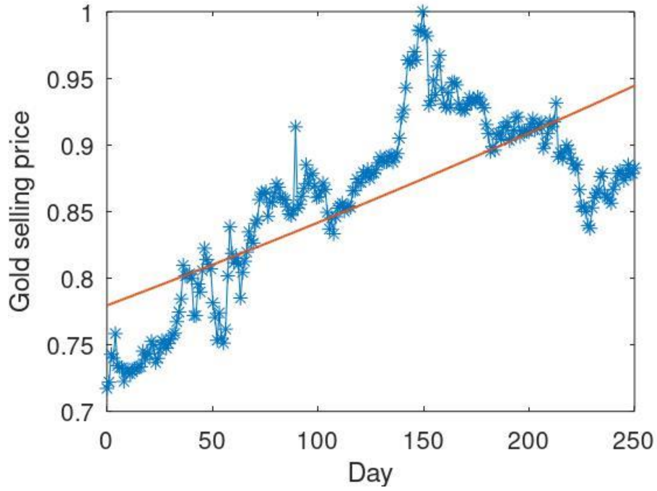

Fig. 10 *Prediction solution for nonlinear regression model and gold selling prices*

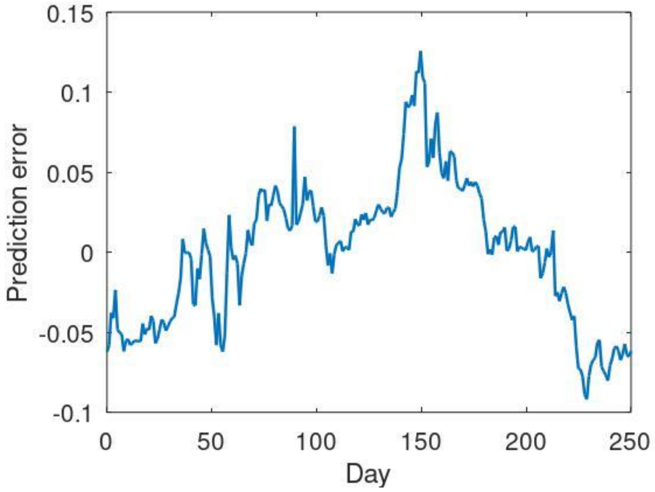

Fig. 11 *Prediction error for nonlinear regression model*

4. Conclusion

In this paper, using an exponential growth model to predict gold selling prices was discussed, where the methodology used gives a new insight into the prediction of gold selling prices based on the first-order ODE model. The simulation results verified that the model's prediction accuracy is higher than simple moving averages and nonlinear methods. With the lowest MSE value, the exponential growth model reveals the applicability of predicting the gold selling prices, and the satisfactory prediction result is well presented. For further study, it is recommended to use a higher-order ODE for predicting gold selling prices for which the accuracy of prediction might be well-expressed. Using more data points of gold selling prices for two years and above should be selected, which might make the estimated parameter more efficient in predicting gold selling.

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Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The authors conform contribution to the paper as follows: ***study conception and design:*** *Teo Hui Ting, Kek Sie Long;* ***data collection:*** *Teo Hui Ting;* ***analysis and interpretation of results:*** *Teo Hui Ting, Kek Sie Long;* ***draft manuscript preparation:*** *Teo Hui Ting, Kek Sie Long. All authors reviewed the results and approved the final version of the manuscript.*

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