

# A Study on Mixed Convection Heat Transfer in Lid-Driven L-Shaped Cavity Using COMSOL

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## Abstract

This study investigates the mixed convection heat transfer in a lid-driven L-shaped cavity using COMSOL Multiphysics. The dimensionless equations for mass, momentum, and energy were solved to produce streamlines and isotherms. The analysis was conducted using fixed values of Reynold number, Richardson number, and Prandtl number. The results showed that forced convection is dominant in the upper part of the cavity since it is where the lid motion creates a strong recirculating flow. Meanwhile, in the lower region, there seems to be a lack of streamlines because of the shape of the cavity and weak buoyancy. For isotherms, there is significant heat transfer at the bottom cavity shown by the closely spaced isotherm lines. The buoyancy forces generated by the heated wall caused warm air to rise, resulting in curving isotherms that show the interaction of natural and forced convection. This work highlights the potential of COMSOL Multiphysics to improve precision as well as the effectiveness of the study outcomes.

## 1. Introduction

Heat transfer occurs whenever there is a heat flow due to the temperature differences between states of matter, and it only happens from hot to cold [1]. Heat is transferred through three processes: conduction, convection, and radiation. In this study, we intend to examine mixed convection, which is a combination of natural and forced convection. Mixed convection flow occurs when there are combinations of natural and forced convection mechanisms simultaneously and significantly contribute to heat transfer [2]. Mixed convection occurs in a variety of technical and industrial applications. For example, mixed convection in cavities has many technological applications, including electronic component cooling, solar collectors, crystal formation, nuclear reactors, and gear wells [3]. An investigation of the effect of Reynolds and Prandtl numbers on mixed convective flow and heat transfer characteristics is conducted within a vented cavity with a heat-generating solid circular barrier in the centre. The factors indicated strongly affect the flow and thermal fields, heat transfer rate, drag force, and average fluid temperature in the cavity [4]. Next, an analysis used COMSOL-Multiphysics to study the effect of heat source location on several characteristics such as isotherms, velocity, pressure, temperature, Nusselt numbers, and air density was performed. The findings revealed that heat transmission increased when the heat source was close to the channel-cavity contact surface [5].

Mixed convection flow and heat transfer in cavities have always gained a lot of attention due to their potential applications in engineering. A cavity is a hollow or enclosed space within a solid boundary that allows flow to occur. Cavities come in a variety of shapes and configurations, including square, rectangular, and L-shaped. Cavities are classified as either open or closed. An open cavity refers to a type of flow configuration in which fluid flows over a hollow or recessed area. Several studies have looked at mixed convection in open cavities. Mixed convection from a bottom-heated open cavity exposed to an external flow for a wide range of governing

parameters over cavities was studied with different aspect ratios. It was discovered that the Reynolds number and Grashoff number influence the flow pattern and the presence of recirculating cells [6]. Another study researched mixed convection in an open hollow with a heated wall delimited by a horizontally unheated plate using experimental methods. The results revealed that the flow visualisation indicated that  $Re=1000$  has two nearly different fluid motions: a parallel forced flow in the channel and a recirculation flow within the cavity. For lower Reynolds numbers, the flow visualisation shows that forced motion penetrates the cavity and forms a vortex structure near the unheated vertical plate [7].

Meanwhile, a closed cavity is a confined space where fluid flows within solid boundaries, with no openings for fluid to enter or exit. An analysis of unsteady mixed convection heat transfer in a 3D closed cavity was analysed with constant heat flux applied on the center part of the bottom heated wall and isothermal sidewalls moving in the same vertical direction. The study used ANSYS FLUENT software to solve the governing equations of fluid flow and heat transfer using the finite volume method (FVM). The results revealed that increasing the Reynolds number leads to enhanced Nusselt number and turbulent kinetic energy of the fluid in the domain [8]. Another study researched the effects of a magnetic field on fluid flow and heat transfer in a 2D rectangular cavity. The findings indicated that the presence of a magnetic field retards mixed convection flow. However, the average Nusselt number, which represents the heat transfer rate, increased with the magnetic field angle [9]. Lastly, an investigation was conducted to study the effects of Reynold number, Grashoff number, and Darcy number on the velocity and temperature fields in a lid-driven L-shaped cavity filled with a porous medium. The research showed that the presence of a porous medium significantly affects the flow and heat transfer patterns within the cavity [10].

Thus, the goal of this research is to use COMSOL software to explore the mixed convection in the bottom heated wall of L-shaped lid-driven cavity. Compared to the [3] this research used COMSOL Multiphysics to improve the precision as well as the effectiveness of the study outcomes. The fluid that will be used in this research is air. This research involves solving unsteady equations to analyse heat transfer. The governing conservation equations for mass, momentum, and energy will be converted into dimensionless equations. It is important to analyse the streamlines and isotherm to understand the effect of Reynold number, Richardson number, and Prandtl number on the mixed convection.

#### ***Nomenclature***

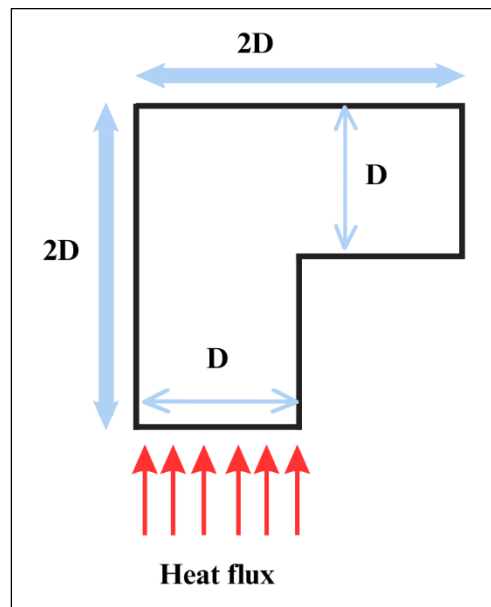
|       |                                                 |
|-------|-------------------------------------------------|
| $D$   | Height of the heated wall                       |
| $g$   | Gravitational acceleration                      |
| $Gr$  | Grashoff number                                 |
| $k$   | Thermal conductivity                            |
| $p$   | Pressure                                        |
| $Pr$  | Prandtl number                                  |
| $q$   | Heat flux                                       |
| $Re$  | Reynold number                                  |
| $Ri$  | Richardson number                               |
| $t$   | Time                                            |
| $T$   | Temperature                                     |
| $T_i$ | Ambient Temperature                             |
| $u$   | Velocity component in x-direction               |
| $U$   | Dimensionless velocity component in X-direction |
| $u_i$ | Inlet velocity                                  |
| $v$   | Velocity component in Y-direction               |
| $V$   | Horizontal coordinate distance                  |
| $x$   | Horizontal coordinate distance                  |
| $X$   | Dimensionless horizontal coordinate distance    |
| $y$   | Vertical coordinate distance                    |
| $Y$   | Dimensionless vertical coordinate distance      |

**Greek Symbols**

|           |                                  |
|-----------|----------------------------------|
| $\alpha$  | Thermal diffusivity              |
| $\beta_T$ | Volumetric expansion coefficient |
| $\theta$  | Dimensionless temperature        |
| $\mu$     | Dynamic viscosity                |
| $\nu$     | Kinematic viscosity              |
| $\rho$    | Density                          |
| $\tau$    | Dimensionless time               |
| $\beta$   | Liquid fraction                  |

**2. Research Method**

The rectangular 2-D L-shaped cavity at the heated wall used in this work is shown in Fig. 1. Only one heated wall location, the wall below—will be used in this study. The heating wall is assessed for Reynolds numbers,  $Re = 200$  and Richardson numbers,  $Ri = 20$ . The streamlines and isotherms will be compared. The following equations describe the fluid and heat flow in a two-dimensional channel in a closed cavity. The inlet air's thermophysical characteristics remained constant, except for the air's temperature-dependent density, which will be estimated using the Boussinesq approximation and resulted in an increase in buoyancy forces. The conservation of mass, momentum, and energy represented the governing equations for steady, incompressible, laminar flow with minimum viscous dissipation [3].



**Fig. 1** Geometric configurations of heated wall position at the bottom

The governing equations (1) to (4) by [3] are shown below:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\mu}{\rho} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g\beta_T(T - T_i) \quad (3)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (4)$$

The boundary conditions are as follows:

The initial boundary conditions:

$$U = 0, \quad V = 0$$

The temperature for the bottom heated wall:

$$\theta = \text{Constant}$$

The adiabatic boundary condition is applied for all walls except the bottom heated wall:

$$\frac{\partial \theta}{\partial X} = 0 \quad \text{or} \quad \frac{\partial \theta}{\partial Y} = 0$$

No-slip conditions are applied at solid surfaces:

$$u = v = 0$$

The following variables are substituted into the equations (1) to (4) to be converted into their dimensionless forms as shown in equation (5),

$$\begin{aligned} X &= \frac{x}{D}, \quad Y = \frac{y}{D}, \quad U = \frac{u}{u_i}, \quad V = \frac{v}{u_i}, \\ P &= \frac{pD}{\mu u_i}, \quad Re = \frac{u_i D}{\nu}, \quad \tau = \frac{u_i t}{D}, \quad Pr = \frac{\nu}{\alpha}, \\ \theta &= \frac{(T - T_i)}{\left(\frac{qD}{k}\right)}, \quad Gr = \frac{g\beta_T q D^4}{\nu^2 k}, \quad Ri = \frac{Gr}{Re^2} \end{aligned} \quad (5)$$

where there are components of velocity in  $x$  and  $y$  directions, fluid pressure, density, gravitational acceleration, volumetric expansion coefficient, fluid temperature, ambient temperature, thermal diffusivity, the height of the vertical heated wall, heat flux, thermal conductivity and inlet velocity.

By using the variables in equations (5), equations (1)-(4) are transformed into dimensionless equations. To ensure they were accurate and consistent, the obtained equations were compared to dimensionless equations by [3].

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (6)$$

$$\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (7)$$

$$\frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = - \frac{\partial P}{\partial X} + \frac{1}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) + Ri\theta \quad (8)$$

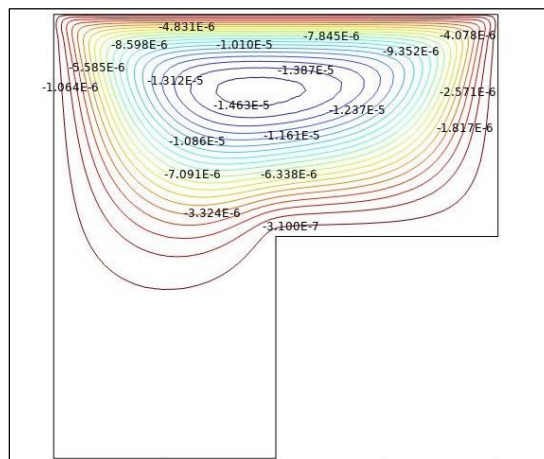
$$\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{RePr} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (9)$$

### 3. Results and Discussion

The Prandtl number is set to 0.71, which represents the thermal and momentum diffusivities of air at the specified temperature. The mixed convection behaviour in an L-shaped lid-driven cavity is examined for this study using the parameters  $Re = 200$  and  $Ri = 20$ . The reason these values are selected is that  $Ri = 20$  creates a significant buoyancy force, allowing the interaction of both forced and natural convection to be observed, while  $Re = 200$  indicates a reasonably strong lid-driven forced convection effect.

#### 3.1 Streamlines

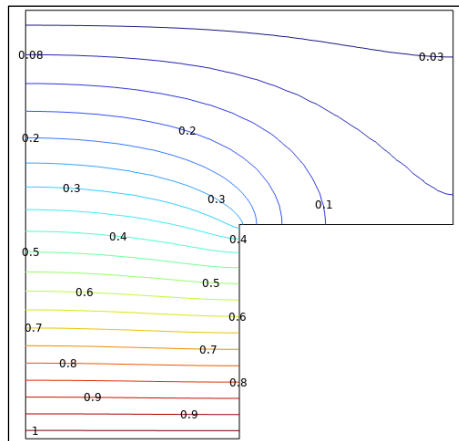
Fig. 2 illustrates the streamlines for  $Re = 200$  and  $Ri = 20$ . At  $Re = 200$ , the flow is driven by the lid's motion and buoyancy effects. A significant circulation zone may be seen in the cavity's upper clockwise eddy. The circulation is caused by the lid sliding in one direction. Additionally, there is contour density close to the upper wall due to the shear effect brought on by lid-driven motion and boundary conditions that have no-slip boundary conditions. The streamline plot reveals that the bottom part of the L-shaped cavity lacks streamlines. This indicates that this area has very little fluid movement. A significant recirculation zone is generated close to the center of the cavity by the sliding lid, which primarily drives the flow in the upper portion. Since less momentum was transferred to the fluid, the conditions in the bottom section became nearly stationary as the distance from the lid increased. The minimal buoyancy forces and geometric constraints also resulting in low fluid motion in this region.



**Fig. 2** Streamlines for  $Re = 200$ ,  $Ri = 20$  and  $Pr = 0.71$

#### 3.2 Isotherms

Fig. 3 displays the isotherm for  $Re = 200$  and  $Ri = 20$ . Higher temperatures are represented by red, while lower temperatures are represented by blue on the colour scale. The isotherms are uniformly distributed and correspond to the L-shaped cavity's curved shape. Compared to the top wall, the isotherm in the bottom heated wall is more closely spaced, implying an increased rate of heat transfer there. The effect of convection, where the fluid moves and transfers heat, is shown by the isotherm becoming more curved and spaced out as the isotherm moves away from the heated wall. The temperature distribution illustrates the flow of the mixed convection nature, with forced convection dominating the upper half of the cavity and natural convection almost entirely influencing the lower region near the heated wall. Heat is efficiently distributed by the moving lid's forced convection, creating consistent isotherms in the upper cavity. However, buoyant forces from the heated wall drive vertical heat transfer towards the bottom, resulting in densely packed isotherms in this region. These two mechanisms cooperate to produce the observed curved isotherms.



**Fig. 3** Isotherm for  $Re = 200$ ,  $Ri = 20$  and  $Pr = 0.71$

#### 4. Conclusion

The mixed convection heat transfer in a lid-driven L-shaped cavity has been analysed in this study. Based on the analysis of the results, forced convection is dominant in the upper part of the cavity since it is where the lid motion creates a strong recirculating flow. Meanwhile, in the lower region, there seems to be a lack of streamlines there because of the shape of the cavity and weak buoyancy. For isotherms, there is significant heat transfer at the bottom cavity shown by the closed-spaced isotherm. The buoyancy forces generated by the heated wall caused warm air to rise, resulting in curving isotherms that show the interaction of natural and forced convection.

It is suggested to explore the effect of variable parameters on mixed convection heat transfer in complex geometrical systems. Changing the parameters such as the value of the Reynolds number, and Richardson number, or adding other new parameters will affect heat transfer within an irregular or intricate shape.

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#### Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

#### Author Contribution

The authors confirm contribution to the paper as follows: **study conception and design, solve the governing equation, analysis, and interpretation of results:** Nurul Hamidah Sudirman, Muhamad Ghazali Kamardan; **draft manuscript preparation:** Nurul Hamidah Sudirman, Muhamad Ghazali Kamardan. All authors reviewed the results and approved the final version of the manuscript.

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