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Numerical Solution of Casson Fluid Flow Under Viscous Dissipation

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Abstract

The Casson fluid model plays a critical role in describing non-Newtonian fluid behaviour, particularly in systems where Newtonian assumptions are inadequate. This study focuses on investigating the influence of viscous dissipation on the flow characteristics of Casson fluid under boundary layer conditions. The governing nonlinear partial differential equations are transformed into ordinary differential equations using the Similarity Transformation. These equations are then solved numerically using the Fourth Order Runge-Kutta (RK4) method in conjunction with the Shooting Method. The results reveal how viscous dissipation significantly alters velocity and temperature profiles within the fluid flow, influencing thermal boundary thickness and energy transfer rates. Graphical representations illustrate these effects, offering deeper insights into the dynamics of non-Newtonian fluids under dissipative conditions. The study establishes the importance of considering viscous dissipation for accurate modelling of Casson fluid flow, enhancing its applicability to industrial, medical, and engineering systems.

1. Introduction

Casson fluid flow has garnered significant attention in fluid mechanics due to its distinctive non-Newtonian properties, including yield stress and a nonlinear relationship between shear stress and shear rate. These characteristics make Casson fluids vital in understanding energy dissipation under shear forces and have practical applications in various fields. For instance, the modelling of blood as a Casson fluid is critical in biomedical engineering, enabling the design and optimization of medical devices such as artificial hearts, blood pumps, and dialysis systems [1, 2].

In the food industry, many products like chocolate, ketchup, and mayonnaise exhibit Casson-like behaviour. Understanding their flow dynamics facilitates improvements in manufacturing processes, enhances product consistency, optimizes efficiency, and ensures superior quality [5]. Furthermore, in geophysics, materials such as mudflows, lava flows, and debris flows are studied using Casson fluid models to predict and mitigate the impact of natural disasters like landslides, volcanic eruptions, and floods [15].

Key factors influencing viscosity dissipation in Casson fluids, such as shear rate, temperature, pressure, and boundary conditions, are analysed through advanced mathematical models, computational simulations, and experimental methods. These approaches contribute to improved process design, efficient energy utilization, and safer handling of non-Newtonian fluids across various industries [17]. By enhancing the understanding of Casson fluid dynamics, these studies drive innovation in fluid mechanics, foster advancements in technology, and support practical applications in biomedical engineering, geophysics, and industrial processing [20].

Nomenclature

P_y	yield stress of fluid
π_a	critical value based on non-Newtonian model
μ, ν	velocity components
ρ	density
C_p	specific heat
k^*	fluid medium permeability
C_w	concentration of fluid at surface
C_∞	concentration in the free stream
T_w, T_∞	temperature at the sheet and free stream
q_r	Rossland approximation
P_r	Prandtl number
Q	heat source
R_d	radiation parameter
M	magnetic number
E_c	Eckert number
L_e	Lewis number
c_f	coefficient of skin friction
$N\mu_x$	local Nusselt number
q_w	wall heat flux from plate
Re_x	Reynolds number

Greek Symbols

μ_B	plastic dynamic viscosity of non-Newtonian model
κ	thermal diffusivity
β	Casson fluid parameter
σ^*	Stefan-Boltzman constant
τ_w	skin friction from the plate

2. Research Method

An extensive review of the use and utilization of the fourth order Runge-Kutta with shooting method for the duration of the project is given in this section. The section is divided into three main sections, each of which focuses on a different topic. (1) Deal with similarity transformations for partial differential equations, (2) Deals with physical quantities, (3) Provides an explanation of the fourth order Runge-Kutta with the shooting method.

2.1 Similarity Transformations for Partial Differential Equations

Assume that 2D Casson fluid flow that is stable and incompressible as it passes through a porous stretching surface at y = 0. While the x and y-axes are taken along and normal to the surface, respectively, the flow is restricted to $y > 0$. The geometrical picture of fluid flow with the boundary layer conditions and coordinate system under consideration is shown in Fig. 1. For anisotropic and incompressible Casson fluid flow, the rheological equation of state is

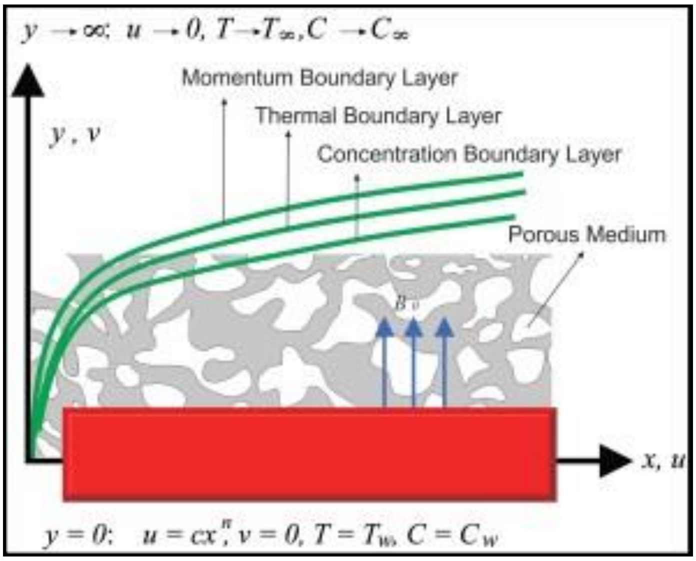

Fig. 1 Geometrical view of the physical model

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + p_y/\sqrt{2\pi}\right)e_{ij} & \text{for } \pi > \pi_a \\ 2\left(\mu_B + p_y/\sqrt{2\pi}\right)e_{ij} & \text{for } \pi < \pi_a \end{cases} \tag{1}$$

Similarity transformations are those that enable a partial differential system with n independent variables to be transformed into one with $n - 1$ independent variables. When $n = 2$, the situation is ideal since one is working with an ordinary differential equation rather than a partial differential equation. These are some of the greatest tools available for creating similarity transformations. The Prandtl equations are asymptotically approximated in the surface region. The equations make it simple to solve these. Consequently, the equations are

$$\frac{\partial \mu}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$\mu\frac{\partial \mu}{\partial x} + v\frac{\partial u}{\partial y} = v\left(1+\frac{1}{\beta}\right)\frac{\partial^2 \mu}{\partial y^2} - \frac{\sigma}{p}\beta_0^2\mu - \frac{v}{k^*}\mu \tag{3}$$

$$\mu\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \kappa\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_0}\frac{\partial q_r}{\partial y} + \frac{Q_0}{\rho c_0}(T - T_0) + \frac{v}{c_0}\left(1+\frac{1}{\beta}\right)\frac{\partial^2 \mu}{\partial y^2} \tag{4}$$

$$\mu\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_\beta\frac{\partial^2 C}{\partial y^2} \tag{5}$$

The boundary conditions determined by:

$$\mu = cx^n, v = 0, T_w = T, C = C_w \text{ at } y = 0 \tag{6}$$

$$\mu \to 0, T \to T_\infty, C \to C_\infty \text{ as } y \to \infty$$

Additionally, the similarity variables used in this formula can be expressed as

$$\left.\begin{aligned} &\mu = cx^{n} f^{'}(\eta) \\ &\eta = y\sqrt{\frac{a(n+1)}{2v}}\, x^{\frac{n-1}{2}} \\ &T = (T_w - T_\infty)\theta(\eta) + T_\infty \\ &C = (C_w - C_\infty)\phi(\eta) + C_\infty \\ &v = -\sqrt{cv\left(\frac{n+1}{2}\right)}\, x^{\frac{n-1}{2}}\left[f(\eta) + \frac{n-1}{n+1}\eta f^{'}(\eta)\right] \end{aligned}\right\} \tag{7}$$

The radiative heat flux (qr)described by Rossland approximation takes the form

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^{*}T_\infty^{3}}{3k^{*}}\frac{\partial^2 T}{\partial y^2} \tag{8}$$

with

$$q_r = -\frac{4\sigma^{*}}{3k^{*}}\frac{\partial T^4}{\partial y} \tag{9}$$

where k^{*} denotes the Rossland mean absorption coefficient and σ^{*} denotes the Stefan Boltzmann constant. Eq. (2) is similarly satisfied after using the similarity variables. Eq. (3) tp Eq. (5) are thus reduced to linked nonlinear ordinary differential equations of the following form

$$\left(1+\frac{1}{\beta}\right) f^{'''} + ff'' - 2\left(\frac{nf'^{2}}{n+1}\right) - \frac{2}{n+1}(K+M) f^{'} = 0 \tag{10}$$

$$\left(1+\frac{4}{3}Rd\right)\theta^{''} + \Pr\left(f\theta^{'} + \frac{2}{n+1}Q\theta\right) + \Pr\left(1+\frac{1}{\beta}\right)E_c f^{''2} = 0 \tag{11}$$

and

$$\phi^{''} + L_e f \phi^{'} = 0 \tag{12}$$

Similarly, the boundary conditions may take the form

$$\left.\begin{aligned} &f(0)=0, \quad f^{'}(0)=1, \quad \theta(0)=1, \quad \phi(0)=1 \quad \text{at } y \to 0 \\ &f^{'}(\infty)=0, \quad \theta^{'}(\infty)=0, \quad \phi(\infty)\to 0, \quad \text{as } y\to\infty \end{aligned}\right\} \tag{13}$$

where Pr symbolize the Prandtl Number, $Q > 0$ and $Q < 0$ represent the heat source and the heat sink, Rd is the radiation parameter, M is the magnetic number, β is the Casson fluid parameter, K is the permeability parameter, E_c is the Eckert number and L_e is the Lewis number. These parameters may be expressed as

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$$\left.\begin{aligned} P_r &= \frac{\mu c_p}{\kappa} \\ Q &= \frac{Q_0 x}{\mu_w(x)\rho c_p} \\ R_d &= \frac{4\sigma^* T_\infty^3}{kk^*} \\ M &= \frac{\sigma \beta_0^2}{c\rho} \\ \beta &= \frac{\mu_B \sqrt{2\pi_c}}{p_y} \\ K &= \frac{\nu}{ck^*} \\ Ec &= \frac{\mu_w^2(x)}{c_p(T_w - T_\infty)} \\ Le &= \frac{\nu}{D_R} \end{aligned}\right\} \tag{14}$$

when the surface is stretched linearly (n=1.0) then by setting K=M=0, the exact solution of Eq. (14) may take the form

$$f(\eta) = \sqrt{1+\frac{1}{\beta}}\left(1-\exp\left(-\frac{n}{\sqrt{1+\frac{1}{\beta}}}\right)\right) \tag{15}$$

2.2 Fourth Order Runge-Kutta with Shooting Method

The set of Eq. (10) and Eq. (11) in this study have been computed using the fourth-order Runge-Kutta technique with shooting approach, pursuant to the conditions specified in Eq. (13), using MATLAB computing software. We begin with an arbitrary value for $f''(0)$ and work our way up until the boundary requirements for high values of η are fully satisfied, even if the values of $f(0)$ and $f'(0)$ are known. The iterative process used to find a suitable value of $f''(0)$ is known as the shooting technique.

3. Result and Discussion

This chapter explained the result for numerical solution of equation from nonlinear ordinary differential equations using fourth-order Runge-Kutta method with shooting technique.

3.1 Numerical Solution of RK4 with Shooting Technique using MATLAB

This MATLAB code uses the shooting method to solve a system of ordinary differential equations (ODEs) that model fluid flow and heat transfer. It starts by defining various problem parameters such as the Prandtl number (Pr), Eckert number (Ec), and radiation parameter (Rd), which are essential for the system's behaviour. The boundary conditions at $\eta = 0$ are set (e.g., $f(0) = 0$ and $f'(0) = 1$. The ODE system is then formulated to describe the velocity and temperature profiles of the fluid, incorporating the effects of viscous dissipation, heat transfer, and radiation. The shooting method is used to iteratively adjust the initial guess for the second derivative of f at $\eta = 0$ and solve the system of ODEs using $ode15s$. The solution is checked for convergence, ensuring that the computed values at the end of the domain $(at\ \eta \rightarrow \infty)$ match the target conditions. Finally, the script plots the velocity and temperature 34 profiles along with their gradients, providing insight into the fluid's behaviour under the given conditions.

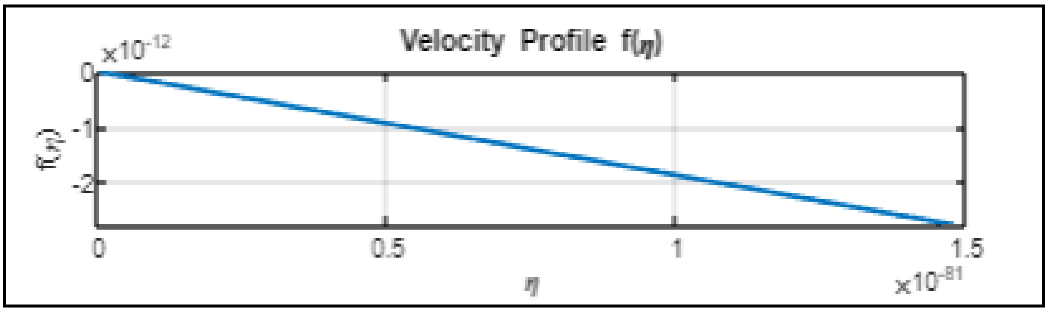

Fig. 2 Velocity Profile

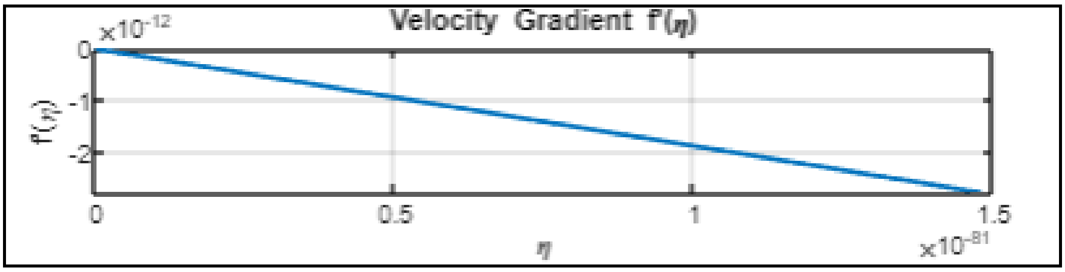

Fig. 3 Velocity Gradient

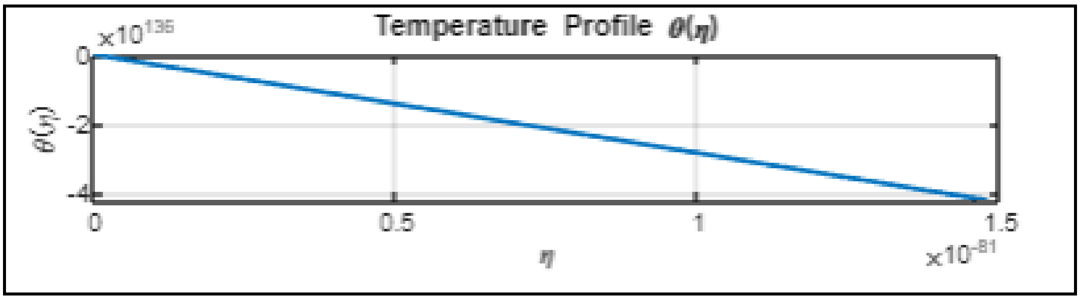

Fig. 4 Temperature Profile

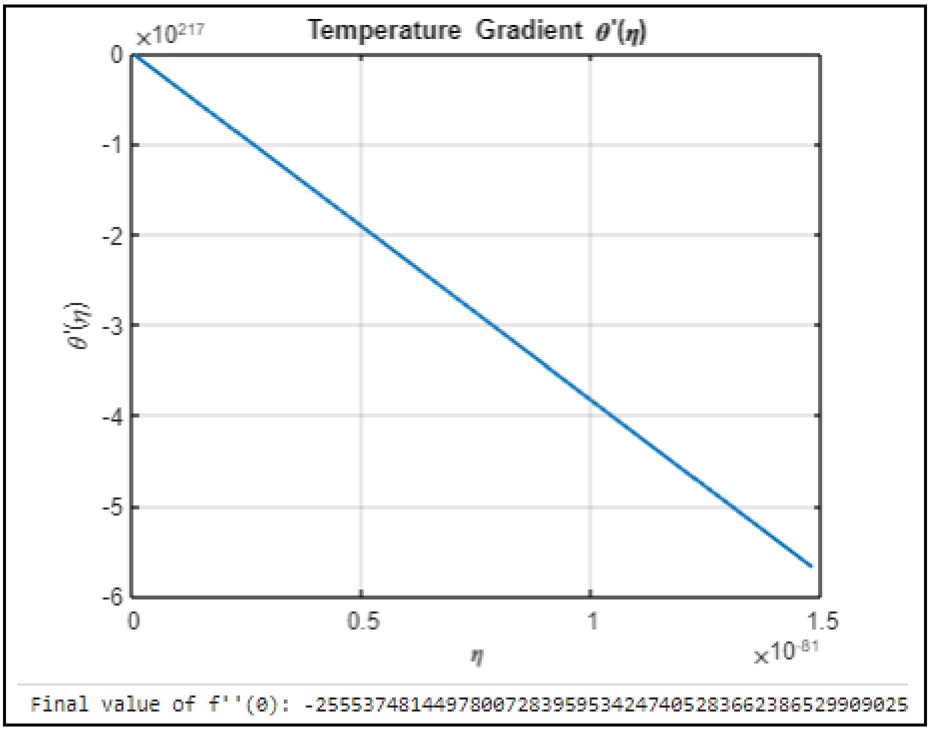

Fig. 5 Temperature Gradient

4. Conclusion

This study successfully investigates the Casson fluid flow under the influence of viscous dissipation using the Similarity Transformation and RK4 Shooting Method implemented in MATLAB software. By reducing complex partial differential equations to solvable ordinary differential equations, the approach provides efficient and accurate numerical solutions while adhering to boundary conditions. The results reveal that viscous dissipation significantly impacts both the velocity and temperature profiles, leading to thicker thermal boundary layers and higher energy transfer rates. Graphical representations highlight the sensitivity of system behaviour to parameter variations, including Casson fluid properties and dissipation effects, offering valuable insights into the interplay between fluid dynamics and heat transfer in non-Newtonian systems. The findings demonstrate that the combined use of similarity transformations and advanced numerical techniques provides a reliable framework for modelling Casson fluid flow, enhancing its applicability to industrial, medical, and engineering systems.

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Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design:*** *Muhammad Fakhrul Syazwan Zainuddin;* ***solve the governing equation:*** *Muhammad Fakhrul Syazwan Zainuddin;* ***analysis and interpretation of results:*** *Muhammad Fakhrul Syazwan Zainuddin, Mahathir Mohamad;* ***draft manuscript preparation:*** *Muhammad Fakhrul Syazwan Zainuddin, Mahathir Mohamad. All authors reviewed the results and approved the final version of the manuscript.*

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