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Mathematical Modelling of Gross Domestic Product Prediction in United States Based on Logistic Map

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Abstract

Gross domestic product (GDP) measures a country's economic development. However, uncertain factors affecting the growth of a GDP cause the GDP prediction to be difficult and inaccurate. This study aims to propose a logistic map model for predicting the GDP in the United States. The GDP historical data from 1960 to 2023 are collected and visualized for prediction. A least square optimization problem is introduced, where the objective function is to minimize the sum of square errors for the differences between the model and the actual GDP data. The gradient method is applied to solve the least squares optimization problem and, in turn, to estimate the model parameters optimally and to update the model solution iteratively until convergence. The predictive solution of the GDP is obtained with these optimal parameters. For illustration, we examine some trial models for an appropriate predictive model for the GDP. From the simulation results, the parameters in the logistic map model are successfully estimated, and the predicted GDP results in the United States give a small mean square error value. Also, we find that a logistic map model with chaotic behaviour provides a more accurate prediction solution closely aligned with the actual GDP data. In conclusion, the logistic map model efficiently predicts the United States' GDP.

1. Introduction

A gross domestic product (GDP) measures the total value of all final goods and services produced within a country's border at a specific time. In many countries, the GDP is the primary way to measure the health of a country's economy, and it is used to measure whether the economy is growing or shrinking. A growing GDP indicates that more goods and services are produced, whereas a shrinking GDP suggests that fewer goods and services are produced [1]. There are many factors affecting the growth of GDP, such as the pandemic [2], decreased fertility rates [3], gold, crude oil and the stock market [4]. The United States is one of the largest economic entities in the world, and it impacts the global market and the world's economy.

In fact, the GDP model can date back to the 1600s suggested by [5]. The current GDP system was invented during the Great Depression in the twentieth century by [6]. This system is designed to assess the state of the national economy and its performance over time. Many mathematical models can be used to predict the GDP trend [7], such as time series [8], [9], neural networks [10], [11] and differential equations models [12], [13]. Economists apply these mathematical models to describe the GDP phenomenon and draw inferences from basic assumptions.

However, the GDP is a highly complex real-world system with multivariate interactions involving many variables. These interactions are often nonlinear relationships, and a slight change in variables can affect the GDP trend. Traditional linear models are unsuitable for multi-variables and nonlinear systems with chaotic behaviour.

This is because the linear model assumes constant relationships and oversimplifies the interactions of the GDP system. Thus, it is necessary to use a nonlinear model to capture nonlinear and chaotic dynamics in the system.

A logistic map is the simplest dynamical system with complex behaviour described by a nonlinear model. Parameters in the logistic map with increasing periodicity led to robust chaotic behaviour. Discrete-time chaotic maps are widely used in creating cryptographic keys due to the confusion and diffusion properties. It involves using cryptographic algorithms and chaotic maps to encrypt digital images [14], [15]. Nevertheless, a logistic map is one of the simplest nonlinear models in difference equations. Its solution sequences are in time series form, which is appropriately employed for GDP prediction as GDP is in a time series form. GDP exhibiting fluctuation reveals that the logistic map is suitable for GDP prediction. Therefore, the prediction process will be precise and efficient, resulting in a more accurate prediction solution.

The study in this paper mainly focuses on the mathematical modelling of the GDP prediction in the United States using a logistic map model. The actual data on GDP in the United States are obtained from the World Bank Group website. Three objectives of the study are established. First, to predict the GDP of the United States by using a logistic map simulation. Second, to estimate the parameters in the logistic map model by solving a least squares optimization problem. Third, to verify the performance of the computational algorithm for the GDP prediction. Throughout the study, the mathematical model of the logistic map is studied. The optimal parameters are estimated when the least squares optimization problem is solved. The simulation results of the GDP prediction of the United States using the logistic map are discussed and interpreted. Finally, a conclusion is made.

1.1 Predictive Modelling

Consider a least squares optimization problem [16],

$$\text{Minimize } J = \frac{1}{2}\sum_{k=0}^{N-1}(y_k - x_k)^2, \tag{1}$$

where J is the objective function, which represents the sum of squared error, y_k is the actual data of the GDP at time step k in year, and N is the number of GDP data points. Here, x_k is the solution sequence to a logistic map model [17],

$$x_{k+1} = a x_k (b - x_k) \tag{2}$$

with a and b are the model parameters. Thus, this is the prediction problem of the GDP using a logistic map model.

In this study, the input is expressed by the solution sequences of a logistic map, and the output is the GDP historical data set. Notice that only solving the logistic map model (2) will not predict the GDP values, which are in a fluctuating trend. However, estimating the model parameters a and b by observing the GDP trend and applying these parameter estimates in the logistic map to update the solution sequences can predict the GDP data points. Hence, an appropriate logistic map model will be suggested for the GDP prediction in the study.

The gradients of the objective function in (1) are calculated from

$$\frac{\partial J}{\partial a} = \frac{1}{2}\sum_{k=0}^{N-1}(-1)(2)(y_k - x_k)(x_k(b - x_k)), \tag{3}$$

$$\frac{\partial J}{\partial b} = \frac{1}{2}\sum_{k=0}^{N-1}(-1)(2)(y_k - x_k)(a x_k). \tag{4}$$

The model parameters a and b are estimated through the following recursion equations [18],

$$a^{(i+1)} = a^{(i)} + \alpha_1 \cdot \left(\frac{\partial J}{\partial a}\right)^{(i)}, \tag{5}$$

$$b^{(i+1)} = b^{(i)} + \alpha_2 \cdot \left(\frac{\partial J}{\partial b}\right)^{(i)}, \tag{6}$$

where α_1 and α_2 are the step sizes, ranging from 0 to 1, and i is the iteration number. The initial values $a^{(0)}$ and $b^{(0)}$ are provided to begin the iteration. Here, (5) and (6) are known as the gradient method.

When the convergence is achieved, the optimal parameter estimates

$$a^{(i+1)} \approx a^{(i)} \text{ and } b^{(i+1)} \approx b^{(i)} \tag{7}$$

will minimize the objective function (1), where the first order necessary conditions

$$\frac{\partial J}{\partial a} = 0 \text{ and } \frac{\partial J}{\partial b} = 0 \tag{8}$$

are satisfied. Denote the optimal parameter estimates be $\hat{a}$ and $\hat{b}$, generated from (7), and substitute them into the logistic map model (2) to give

$$\hat{x}_{k+1} = \hat{a}x_k(\hat{b} - x_k) \tag{9}$$

with

$$x_k = \hat{x}_k + \alpha_3(y_k - \hat{x}_k) \tag{10}$$

is a line search equation that satisfies the first-order necessary condition ∂J/∂x = 0, where a_3 is the step size, ranging from 0 to 1. Thus, the solution sequences of the logistic map model shall approximate the actual values of the GDP with a small mean square error (MSE) value, given by [19, 20],

$$\text{MSE} = \frac{1}{N}\sum_{k=0}^{N-1}(y_k - \hat{y}_k)^2, \tag{11}$$

where $\hat{x}_k$ is the predictive solution sequence from (9), and the predicted solution to the GDP values is given by

$$\hat{y}_k = \hat{x}_k. \tag{12}$$

Hence, the solution sequences of (9) measured by (12) approximate the actual values of the GDP in (1). These solution sequences are referred to as the predictive solution of the actual values of the GDP.

Therefore, we summarize the calculation procedure as an iterative algorithm given below.

Data Given the GDP in the normalized value form, the initial values $a^{(0)}$ and $b^{(0)}$.
Step 1 Solve the logistic map model (2) for the initial solution curve. Set $i = 0$.
Step 2 Evaluate the objective function from (1).
Step 3 Compute the gradients of the objective function from (3) and (4).
Step 4 Estimate the model parameters a and b from (5) and (6).
Step 5 Update the model sequences from (9) and (10).
Step 6 Examine the MSE value (11) by using the predicted solution (12).
Step 7 Check the convergence. If the model parameters in (7) are satisfied, stop the iteration. Otherwise, set $i = i + 1$, repeat from Step 2.

There are various types of simulation data techniques, each with its own advantages and limitations. Their effectiveness often depends on the specific characteristics of the data. Numerous previous studies have applied simulation methods combined with mathematical techniques across a wide range of disciplines worldwide [21–23].

2. Result and Discussion

Consider the historical data of GDP of United States (https://data.worldbank.org/indicator/NY.GDP.MKTP.CD), which are collected from 1960 to 2023, and the GDP values are in trillions. These data are used to estimate the model parameters, namely the growth rate and carrying capacity, in the logistic map and the solution sequence is employed to predict the GDP trend. Before using the algorithm proposed, the data is normalized into the range of 0 to 1, as follows,

$$\text{Normalized value }(y_{\text{normalized}}) = \frac{\text{Original GDP value }(y_{\text{real}})}{\text{Maximum GDP value }(y_{\max})}. \tag{9 }$$

2.1 Solution Curve of Logistic Map Model

Consider the logistic map models that are used as an initial model as follows,

$$x_{k+1} = 0.79x_k(1.00 - x_k), \tag{10}$$

$$x_{k+1} = 1.48x_k(1.00 - x_k), \tag{11}$$

$$x_{k+1} = 2.17x_k(1.00 - x_k), \tag{12}$$

with the initial condition x_0 = 0.0198. We only consider the growth rates 0.79, 1.48 and 2.17 because, after the numerical testing in the study, the growth rates greater than 2.17 caused the divergence during the calculation. Also, we fix the value of the carrying capacity b = 1 to use the standard logistic map as the initial model and to identify the appropriate carrying capacity value for the GDP data points after convergence.

Fig. 1 shows the decay solution curve for the model in (14), which decreases from the initial value in 1960 to about zero after 20 years. Fig. 2 shows the solution curve for the model in (15), where the curve increases from the initial value in 1960 to about 0.32 after 1975 and stays along this value for the following years. Fig. 3 shows the solution curve for the model in (16), and the solution curve increases rapidly from the initial value in 1960 to the value of 0.54 after 6 years and remains at this value till 2023. These are the initial solution curves for implementing the computational algorithm to predict the GDP. Nevertheless, these solution curves are impossible to happen in an actual situation.

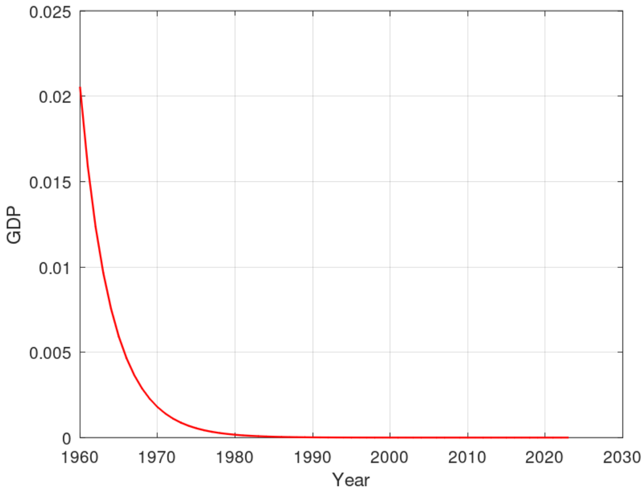

Fig. 1 Solution curve for logistic map model with a = 0.79

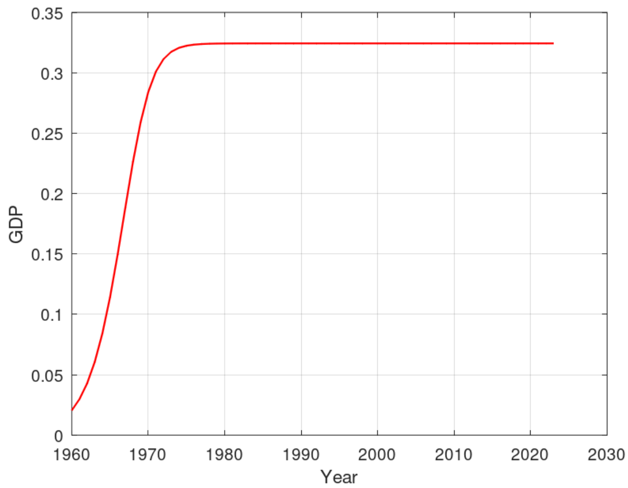

Fig. 2 Solution curve for logistic map model with a = 1.48

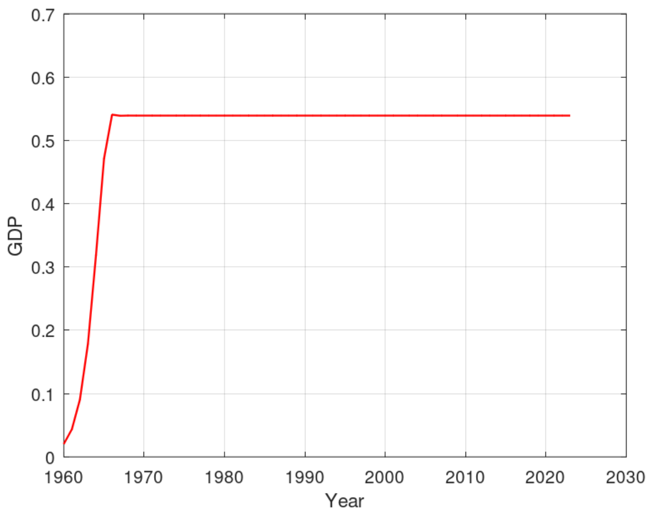

Fig. 3 Solution curve for logistic map model with a = 2.17

2.2 Prediction Solution of Gross Domestic Product

To apply the computational algorithm discussed previously, we set the tolerance of 1.0×10^{-6} and the maximum iteration number 50,000. We also assign the step sizes $\alpha_1 = 0.0010, \alpha_2 = 0.0010$ and $\alpha_3 = 0.8000$. Table 1 shows the simulation results using the initial models given by (14), (15) and (16). The mean square errors presented by using these initial models have a small value of 8.7534×10^{-4}, which verify that the computational algorithm accurately predicts the GDP. The iteration numbers are 29,594, 32,145 and 38,152 for growth rates of 0.79, 1.48 and 2.17, respectively.

Table 1 Simulation Results Using Different Logistic Map Models

Model	Mean Square Error	Iteration Number	Elapsed Time (second)
Model (14)	8.7534×10-4	29,594	28.4942
Model (15)	8.7534×10-4	32,145	37.0920
Model (16)	8.7534×10-4	38,152	35.5770

At the end of convergence, the model parameters *a* and *b* are optimally estimated, and the prediction model of the GDP is resulted by

$$x_{k+1} = 0.4219x_k(3.0671 - x_k) \quad (13)$$

It is observed that the carrying capacity is *b* = 3.0671. Consequently, the solution sequences in (17) are scaled by maximum value of the GDP (y_{max}) to give the prediction solution to the GDP. Fig. 4 shows the prediction solution of the GDP when the logistic map model in (16) is an initial model. The solution curve follows the trend of the GDP accordingly. The prediction error, which presents a small error ranging from -0.10 to 0.20, is shown in Fig. 5. The optimal model parameters shown in Fig. 6 reached their equilibrium points at the end of the convergence.

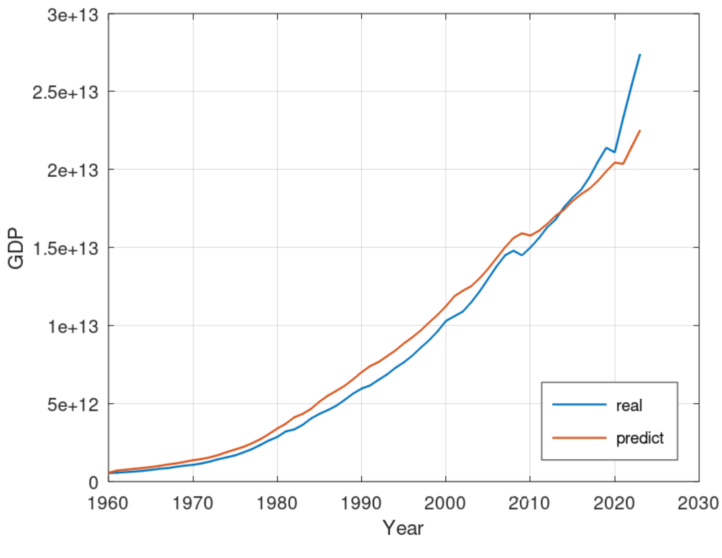

Fig. 4 Prediction solution curve for GDP prediction

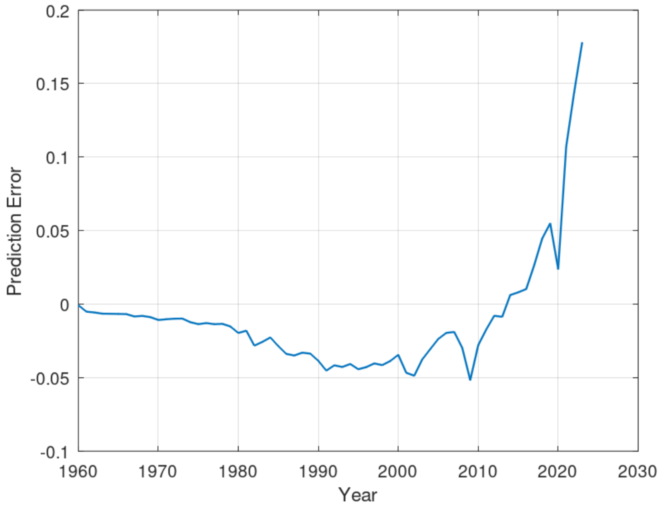

Fig. 5 Prediction error for GDP prediction

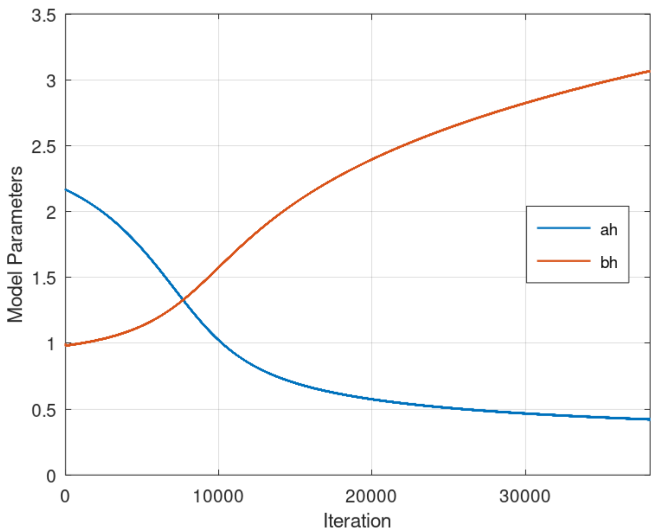

Fig. 6 Parameter estimates for GDP prediction

2.3 Simulation Results for Trial Models

In this section, we examine the prediction solution with different values of model parameters a and b to understand the predictive modelling of the GDP better. To avoid divergence, we choose the following values for testing purposes,

(a) a = 0.79 and b = 1.00, 2.00, 3.00, 4.00, 5.00.

(b) a = 1.48 and b = 1.00, 2.00.

Table 2 shows the simulation results for the parameters in (a). In these trial models, the first three trial models have the initial solution in decay (D), the fourth trial model presents the initial solution in periodic (P), as shown in Fig. 7, while the fifth trial model has the initial solution in chaotic (C), as shown in Fig. 8. The mean square error values are reduced due to the increasing carrying capacity. However, the mean square error is the largest value at the carrying capacity of 3.00. The iteration number reduces from 29,594 iterations to 38 iterations when the carrying capacity increases. Hence, the fifth trial model gives the best prediction model that is given by

$$x_{k+1} = 0.2655 x_k (4.5428 - x_k) \tag{14}$$

with the estimated carrying capacity of 4.5428. The prediction error presents a small error ranging from -0.04 to 0.12. The optimal model parameters reached their equilibrium points at the end of the convergence of 38 iterations.

Table 2 Simulation Results with a = 0.79 and b Different Values

Carrying Capacity *b*	Mean Square Error	Iteration Number	Elapsed Time (second)	Parameter Estimates (*a*, *b*)
1.00 (D)	8.7534×10^{-4}	29,594	28.4942	(0.4219, 3.0671)
2.00 (D)	8.7534×10^{-4}	24,267	37.9656	(0.4219, 3.0671)
3.00 (D)	1.0180×10^{-3}	70	0.10052	(0.4618,2.8536)
4.00 (P)	6.0388×10^{-4}	56	0.08021	(0.3369, 3.6997)
5.00 (C)	4.1786×10^{-4}	38	0.05442	(0.2655, 4.5428)

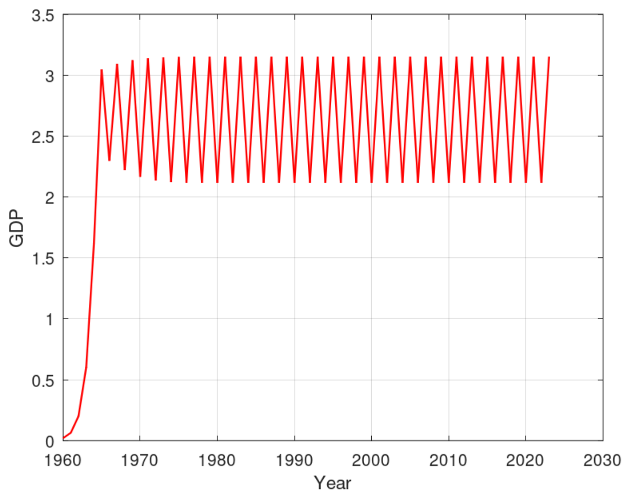

Fig. 7 Solution curve for logistic map model with a = 0.79 and b = 4.00

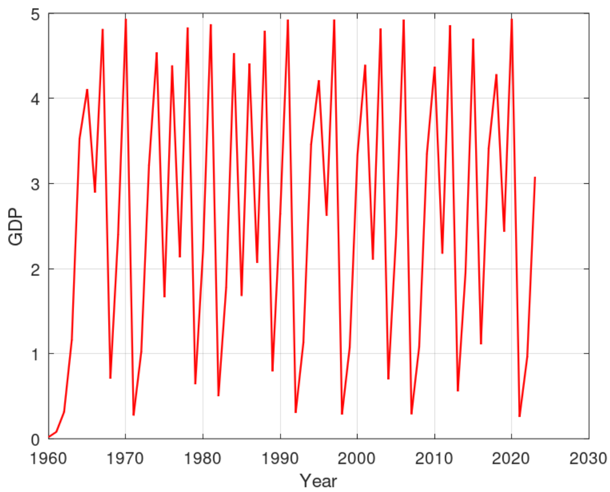

Fig. 8 Solution curve for logistic map model with a = 0.79 and b = 5.00

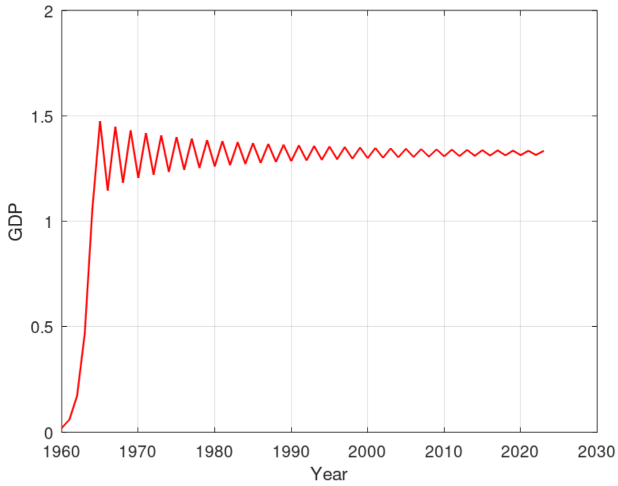

Fig. 9 Solution curve for logistic map model with a = 1.48 and b = 2.00

3. Conclusion

This paper used logistic map model simulation to discuss the mathematical modelling of GDP prediction in the United States. The actual GDP data was used to estimate the model parameters optimally and to update the solution sequences of the model by solving the least squares optimization problem. At the end of the calculation procedure, the iterative solution approximated the GDP closely. The final prediction model was validated through a small mean square error value. In addition, trial models with different parameter values were studied to examine the proposed computational algorithm's performance to the model parameter changes. The testing results showed that the logistic map model can be the best predictive model for predicting the United States GDP.

In conclusion, the logistic map model is efficient for GDP prediction. However, the study was limited to using the optimal parameter values that are constant to fit the GDP, for which the high volatility of the GDP cannot be well-captured. For further studies, we shall investigate a model with time-based parameters for GDP prediction. By allowing the parameters to change over time, the model can be better adapted to disruptive events and improve the predictive performance. Furthermore, a detailed convergence analysis of the logistic map is suggested to improve the accuracy and efficiency of the parameter estimation.

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Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design:*** *Feng Yann Lim;* ***data collection:*** *Feng Yann Lim;* ***analysis and interpretation of results:*** *Feng Yann Lim, Kek Sie Long;* ***draft manuscript preparation:*** *Feng Yann Lim, Kek Sie Long. All authors reviewed the results and approved the final version of the manuscript.*

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