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Differential Transform Method for Solving Ordinary Differential Equations

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Abstract

This study compares the differential transform method (DTM) and Mohand transform (MT) for solving ordinary differential equations (ODEs) with constant coefficients. The research evaluates the performance of these methods in terms of accuracy and computational efficiency. Three examples of ODEs were solved using both methods, with MATLAB employed for computation and analysis. Numerical and graphical comparisons assessed the solutions' accuracy and computational feasibility. Results indicate that MT provides exact solutions, while DTM offers computational simplicity but with increased error in larger domains. MT demonstrates superior accuracy, while DTM remains computationally efficient for specific applications.

1. Introduction

Ordinary differential equations (ODEs) play a fundamental role in modelling a wide range of natural and engineered systems. From predicting population dynamics to simulating electrical circuits, ODEs provide a mathematical foundation for understanding and solving real-world problems. Historically, methods such as the Laplace transform have been instrumental in solving ODEs analytically. However, modern challenges often demand methods that balance accuracy with computational efficiency, paving the way for approaches like the differential transform method (DTM).

The DTM, introduced by [1], offers a semi-analytical solution by transforming differential equations into recurrence relations. It is particularly effective for linear and nonlinear initial value problems, with a focus on computational simplicity [2], [3]. Despite its advantages, DTM relies on truncated series solutions, limiting its accuracy in larger domains where approximation errors accumulate. To evaluate the accuracy of DTM, the Mohand transform (MT), proposed by [4], is used as a reference since it provides exact solutions by converting ODEs into algebraic equations. MT has been shown to solve a variety of equations with constant coefficients, fractional derivatives, and nonhomogeneous terms [5], [6].

This study evaluates DTM using three representative ODE examples and compares its numerical solutions with exact solutions obtained through MT. The performance of DTM is analysed in terms of convergence to the exact solution, accuracy, and computational efficiency. The findings aim to provide insights into the applicability of DTM for solving ODEs and to identify areas where improvements may be necessary to enhance its precision and usability.

2. Methodology

The one-dimensional differential transform function is formally defined as following:

$$Y(k) = \frac{1}{k!}\left[\frac{d^k y(x)}{dx^k}\right]_{x=0} \tag{1}$$

where $c(x)$ is the original function and $Y(k)$ is the transformed function.

Inverse differentials transform of $Y(k)$ is defines as follow:

$$y(x) = \sum_{k=0}^{\infty} Y(k)\, x^k \tag{2}$$

The fundamental differential transform theorem is shown in Table 1.

Table 1 The fundamental operations performed by differential transform

Original function	Transformed functions
$y(x)$	$Y(k)$
$y'(x)$	$(k+1)Y(k+1)$
$y''(x)$	$(k+1)(k+2)Y(k+2)$
x^m	$\delta\{k-m\} = \begin{cases} 1 \ if \ k = m \\ 0 \ if \ k \neq m \end{cases}$

2.1 Mohand Transform

The MT transform is defined for a function of exponential order. Consider following functions in the set A

$$A = \left\{ f(t) : \exists M, k_1, k_2 > 0. |f(t)| < Me^{\frac{|t|}{k_j}}, if\ t \in (-1)^j \times [0, \infty) \right\} \tag{3}$$

for the constant M must be finite number while k_1 and k_2 may be finite or infinite.

The MT denoted by the operator $M(.)$ defined by the integral equations

$$M[f(t)] = R(v) = v^2 \int_0^{\infty} f(t)e^{-vt} dt, t \geq 0, k_1 \leq v \leq k_2 \tag{4}$$

the variable v in this transform is used to factor the variable t in the function of the f.

The fundamental operations performed by MT is shown in Table 2.

Table 2 The fundamental operations performed by Mohand transform

Original function	Transformed functions
t	1
$y(x)$	$R(v)$
$y'(x)$	$vR(v) - v^2 f(0)$
$y''(x)$	$v^2 R(v) - v^3 f(0) - v^2 f'(0)$

The inversed MT for $\boldsymbol{R(v)}$ is denoted by $\boldsymbol{F(t)}$

$$\boldsymbol{F(t) = M^{-1}\{R(v)\}} \tag{5}$$

where $\boldsymbol{M^{-1}}$ is an operator and it is called as inverse MT operator.

The inverse MT functions is shown in Table 3.

Table 3 Inversed Mohand Transform

Original function	Transformed functions
1	t
$\frac{v^2}{v-a}$	e^{at}
$\frac{v^2}{v^2+a^t}$	$\frac{\sin at}{a}$
$\frac{v^3}{v^2+a^2}$	$\cos at$

3. Result and Discussion

Examples for ODEs are solved using DTM. The results obtained illustrate the effectiveness of the DTM in solving linear ODEs. Further, the results are compared with MT.

Example 1: Consider the first order differential equation

$$\frac{dy}{dt}+y=0 \tag{6}$$

with the initial condition

$$y(0)=1 \tag{7}$$

the theorem of differential transform from Table 1 are applied and transformed the equation (6) into

$$(k+1)Y(k+1)+Y(k)=0 \tag{8}$$

the following recurrence relation is

$$Y(k+1)=-\frac{Y(k)}{(k+1)} \tag{9}$$

and initial conditions are transformed into

$$Y(0)=1 \tag{10}$$

at $k=0,1,2,3,4$ the following terms are obtained

$$Y(1)=-1, Y(2)=\frac{1}{2}, Y(3)=-\frac{1}{6}, Y(4)=\frac{1}{24}, Y(5)=-\frac{1}{120} \tag{11}$$

then all the terms are combined and expressed in Taylor series such as

$$y(x)=1-t+\frac{1}{2}t^2-\frac{1}{6}t^3+\frac{1}{24}t^4-\frac{1}{120}t^5 \tag{12}$$

where the MT solution of this example is

$$y(x)=e^{-t} \tag{13}$$

The results obtained highlight the efficiency of DTM in solving ODEs.

Table 4 shows the numerical solutions from the DTM for Example 1, a first-order ODE, comparing exact and DTM values with absolute errors. The error increases as t approaches 1, reflecting DTM's series truncation limitations.

Table 4 The numerical solution of DTM for Example 1

t	MT	DTM 5th terms	Absolute error	DTM 6th terms	Absolute error
0.0	1.0000000	1.0000000	0.0000000e+00	1.0000000	0.0000000e+00
0.1	0.9048374	0.9048375	8.1964040e-08	0.9048374	1.3692929e-09
0.2	0.8187308	0.8187333	2.5802554e-06	0.8187307	8.6411315e-08
0.3	0.7408182	0.7408375	1.9279318e-05	0.7408172	9.7068172e-07
0.4	0.6703200	0.6704000	7.9953964e-05	0.6703147	5.3793690e-06
0.5	0.6065307	0.6067708	2.4017362e-04	0.6065104	2.0243046e-05
0.6	0.5488116	0.5494000	5.8836391e-04	0.5487520	5.9636094e-05
0.7	0.4965853	0.4978375	1.2521962e-03	0.4964369	1.4838712e-04
0.8	0.4493290	0.4517333	2.4043692e-03	0.4490027	3.2629745e-04
0.9	0.4065697	0.4108375	4.2678403e-03	0.4059167	6.5290974e-04
1.0	0.3678794	0.3750000	7.1205588e-03	0.3666667	1.2127745e-03

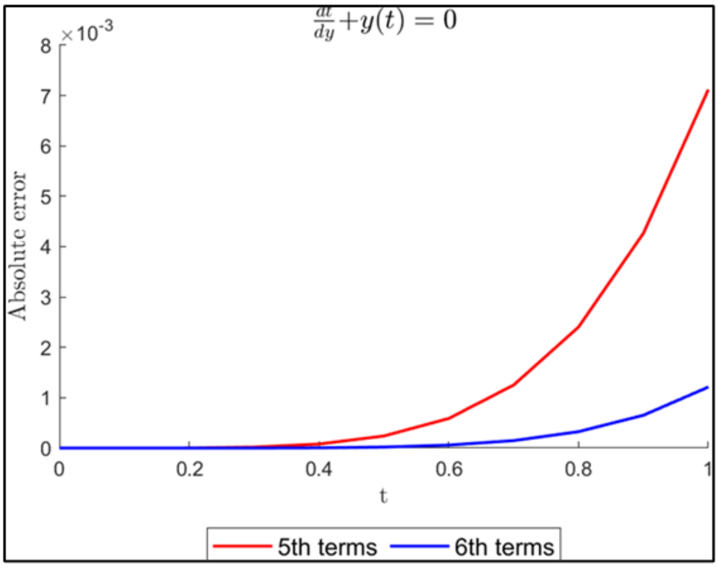

Fig. 1 5th terms and 6th terms solutions for Example 1

Example 2: Consider the following second order differential equation

$$y'' + y = 0 \tag{11}$$

with the initial condition

$$y(0) = y'(0) = 1 \tag{12}$$

after DTM theorems are applied, the following recurrence relation is obtained

$$Y(k+2) = -\frac{Y(k)}{(k+2)(k+2)} \tag{13}$$

then, the initial conditions are transformed as initial terms

$$Y(0) = 1, Y(1) = 1 \tag{14}$$

at $k = 0,1,2,3,4$ the following recurrence is expressed as

$$y(t) \approx 1 + t - \frac{t^2}{2} - \frac{t^3}{6} + \frac{t^4}{24} - \frac{t^5}{120} \tag{15}$$

where the exact solution of this example is

$$y(t) = \sin t + \cos t \tag{16}$$

Table 5 presents the numerical results for Example 2, a second-order ODE. It shows a greater deviation between the DTM and exact solutions as t increases, highlighting the increased sensitivity of second-order equations to truncation errors.

Table 5 Numerical solution of DTM for Example 2

t	MT	DTM 5th terms	Absolute error	DTM 6th terms	Absolute error
0.0	0.0000000	0.0000000	0.0000000e+00	1.0000000	0.0000000e+00
0.1	0.6973387	0.6973333	8.1924854e-08	1.0948376	1.4084793e-09
0.2	1.3788367	1.3786667	2.5753030e-06	1.1787360	9.1363697e-08
0.3	2.0292849	2.0280000	1.9195787e-05	1.2508577	1.0542131e-06
0.4	2.6347122	2.6293333	7.9336312e-05	1.3104853	5.9970218e-06
0.5	3.1829420	3.1666667	2.3726716e-04	1.3570312	2.3149505e-05
0.6	3.6640782	3.6240000	5.7808830e-04	1.3900480	6.9911695e-05
0.7	4.0708995	3.9853333	1.2223745e-03	1.4092381	1.7820881e-04
0.8	4.3991472	4.2346667	2.3294669e-03	1.4144640	4.0119975e-04
0.9	4.6476953	4.3560000	4.0993779e-03	1.4057582	8.2137210e-04
1.0	4.8185949	4.3333333	6.7732907e-03	1.3833333	1.5600427e-03

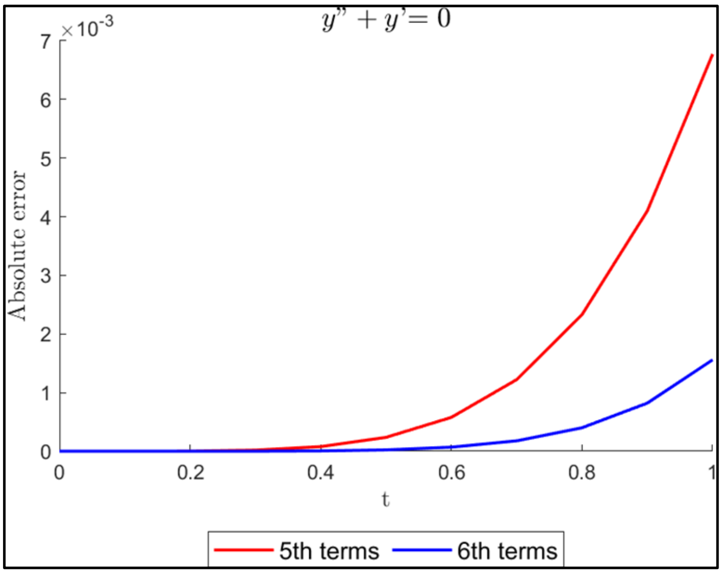

Fig. 2 5th and 6th terms solutions for Example 2

Example 3: Consider the following initial value problem

$$y'' + 4y = 12t \tag{17}$$

with the initial conditions

$$y(0) = 0, y'(0) = 7 \tag{18}$$

the following recurrence relations is obtained with DTM theorems

$$Y(k+2) = \frac{12\delta(k) - 4Y(k)}{(k+1)(k+2)} \tag{19}$$

initial conditions transformed into

$$Y(0) = 0, Y(1) = 7 \tag{20}$$

at $k = 0,1,2,3$ we obtained

$$Y(2) = 0, Y(3) = -\frac{8}{3}, Y(4) = 0, Y(5) = \frac{8}{15} \tag{21}$$

the Taylor series as follows

$$y(t) \approx 7t - \frac{8}{3}t^3 + \frac{8}{15}t^5 \tag{22}$$

The MT solution given of this example is

$$y(t) = 3t + 2\sin 2t \tag{23}$$

Table 5 presents the results for a nonhomogeneous linear ODE.

Table 6 Numerical solution of DTM for Example 3

t	Exact	DTM 5th terms	Absolute error	DTM 6th terms	Absolute error
0.0	0.0000000	0.0000000	0.0000000e+00	0.0000000	0.0000000e+00
0.1	0.6973387	0.6973333	5.3282568e-06	0.6973387	5.0765442e-09
0.2	1.3788367	1.3786667	1.7001795e-04	1.3788373	6.4871603e-07
0.3	2.0292849	2.0280000	1.2849468e-03	2.0292960	1.1053210e-05
0.4	2.6347122	2.6293333	5.3788485e-03	2.6347947	8.2484868e-05
0.5	3.1829420	3.1666667	1.6275303e-02	3.1833333	3.9136372e-04
0.6	3.6650782	3.6240000	4.0078172e-02	3.6654720	1.3938281e-03
0.7	4.0708995	3.9853333	8.5566127e-02	4.0749707	4.0712067e-03
0.8	4.3991472	4.2346667	1.6448054e-01	4.4094293	1.0282127e-02
0.9	4.6476953	4.2346667	2.9169526e-01	4.6709280	1.0282127e-02
1.0	4.8185949	4.3333333	4.8526152e-01	4.8666667	4.8071813e-02

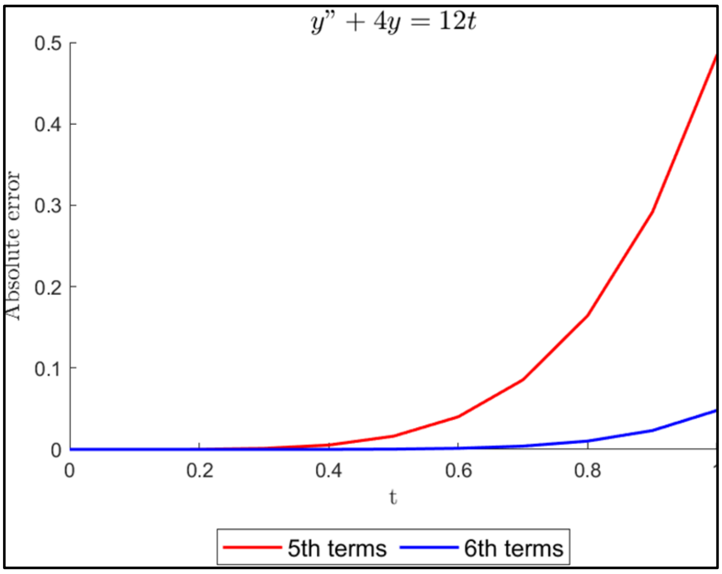

Fig. 3 5th and 6th terms solutions for Example 3

The first-order ODE was solved using DTM, and the numerical solutions are presented in Table 2. The results demonstrate that the absolute error increases as t approaches 1, indicating the impact of series truncation in DTM. The 6th-term approximation reduces the error compared to the 5th-term approximation, suggesting that increasing the number of terms improves accuracy. Fig. 1 illustrates the numerical solutions obtained for both approximations. It can be observed that DTM provides an accurate approximation for small values of t, but the deviation from the exact solution increases over a larger domain. This highlights the method's limitation in handling extended solution ranges due to truncation errors.

For the second-order ODE, Table 4 presents the numerical solutions obtained using DTM. The results indicate that the error accumulates as t increases, confirming the greater sensitivity of second-order equations to truncation effects in DTM. Fig. 2 shows the 5th-term and 6th-term solutions, demonstrating that while increasing the number of terms improves accuracy, truncation errors still persist. This suggests that alternative refinement techniques or extended series terms may be necessary for higher accuracy when using DTM.

The third example involves solving a nonhomogeneous linear ODE. The numerical solutions presented in Table 6 indicate that errors grow significantly for larger values of t. This confirms that DTM's accuracy diminishes more rapidly in nonhomogeneous equations compared to homogeneous cases. Fig. 3 illustrates the divergence of DTM solutions as t increases. The results highlight the challenge of approximating nonhomogeneous equations using a truncated series, further emphasizing the need for refinement techniques to enhance DTM's precision.

Overall, the findings suggest that DTM effectively approximates solutions for linear ODEs, with higher accuracy achieved by increasing the number of terms. However, truncation errors become a limiting factor, particularly in higher-order and nonhomogeneous equations. The accuracy of DTM solutions is influenced by the complexity of the ODE and the number of terms retained in the transformation process.

4. Conclusion

The results from Example 1 demonstrate that DTM provides accurate solutions for first-order ODEs, particularly for small values of t, but its accuracy declines as increases due to series truncation errors. Example 2 highlights the sensitivity of second-order equations to truncation effects, with the 6th-term approximation providing improved accuracy but still showing deviation from the exact solution. Example 3 further illustrates that DTM struggles with nonhomogeneous equations, as errors grow significantly over larger domains. These findings confirm that while DTM is computationally efficient and effective for linear ODEs, it is limited in handling extended solution ranges and complex equations.

Future research could focus on three key areas. First, extending the series expansion or integrating hybrid approaches, such as the Padé approximation, can help reduce truncation errors and improve solution accuracy. Second, DTM should be tested on nonlinear differential equations to assess its applicability beyond linear cases, expanding its usability in complex mathematical modelling. Finally, a comparative analysis between DTM and other numerical methods, such as finite difference and Runge-Kutta techniques, would provide valuable insights into its strengths and limitations, helping to identify the most appropriate contexts for its application

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Conflict of Interest

Authors declare that there is no conflict of interest regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design****,* ***solve the equations, analysis, and interpretation of results:*** *Mohd Shamezan Sham, Noor Azliza Abd Latif;* ***draft manuscript preparation****: Mohd Shamezan Sham, Noor Azliza Abd Latif; All authors reviewed the results and approved the final version of the manuscript.*

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