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Solving Mixed Convection Near Stagnation-Point on a Vertical Surface using bvp4c Solver

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Abstract

This investigation focused on mixed convection near a stagnation point on a vertical surface. It validates the transformation of partial differential equations (PDEs) into ordinary differential equations (ODEs) using similarity transformations and solves them numerically with the shooting technique in MATLAB's bvp4c solver. The effects of parameters, such as velocity slip, thermal slip, mixed convection, and Prandtl number on velocity and temperature profiles are analysed. Results show excellent agreement with a reference study, confirming the approach's validity. Increasing slip parameters thickens boundary layers while reducing surface friction and heat transfer. The findings enhance understanding of slip effects in fluid flow and heat transfer near stagnation points.

1. Introduction

Studies by [1] and [2] analysed stagnation flows and viscoelastic fluids, showing how wall temperature and buoyancy affect flow patterns. [3] and [4] studied boundary layer flows near stagnation points, focusing on how temperature and velocity variations affect cooling systems and heat exchangers.

[5]and [6] explored flow behaviour under slip conditions and heat flux variations, offering valuable solutions for flow analysis. Similarly, [7] and [8] explored hybrid nanofluids, finding that stronger buoyancy forces enhance temperature fields, while melting effects reduce them. [9] emphasized the importance of mixed convection in permeable surfaces, relevant to engineering systems. Further research extends these findings to more complex scenarios. [10] highlighted the effects of velocity slip on three-dimensional flows, while [11] demonstrated dual solutions for hybrid nanofluid flows in porous media. These studies collectively emphasize the role of mixed convection in understanding complex fluid and thermal dynamics.

[12] focused on the effects of pressure and velocity on stagnation-point flow over stretching cylinders, while [13] analysed flame stagnation against flat walls, addressing combustion system challenges. Together, these studies deepen our understanding of stagnation-point flow and its wide-ranging industrial applications. This study highlights key insights into stagnation-point flow, which is crucial for understanding fluid behaviour near solid surfaces. [14] described how fluid slows down near blunt-nosed bodies and stops abruptly around objects like aircraft wings, providing a foundation for analysing pressure and heat transfer.

Besides that, the bvp4c solver is widely used in fluid dynamics for solving boundary value problems, especially in mixed convection flows.

Research, including works by [15] and [16], validated bvp4c's accuracy in solving nonlinear ODEs, calculating wall temperature gradients, skin friction coefficients, and heat transfer rates.

Therefore, the research on mixed convection near a stagnation point on a vertical surface involves the combined effects of buoyancy and forced convection in fluid flow is important, making it a key phenomenon in heat transfer studies. Understanding this behaviour is crucial for optimizing industrial and engineering applications, including cooling systems and energy devices. This current study focuses on mixed convection near the stagnation point on a vertical surface, using the bvp4c solver in MATLAB to solve the governing equations transformed via similarity transformation. Unlike [4], who used the shooting method in Maple, this research explores the shooting method in bvp4c to improve accuracy and computational efficiency. The study aims to validate the transformation process, solve the equations numerically, and compare the results with [4] to evaluate the advantages and limitations of the bvp4c solver.

1.1 Mathematical Formulation

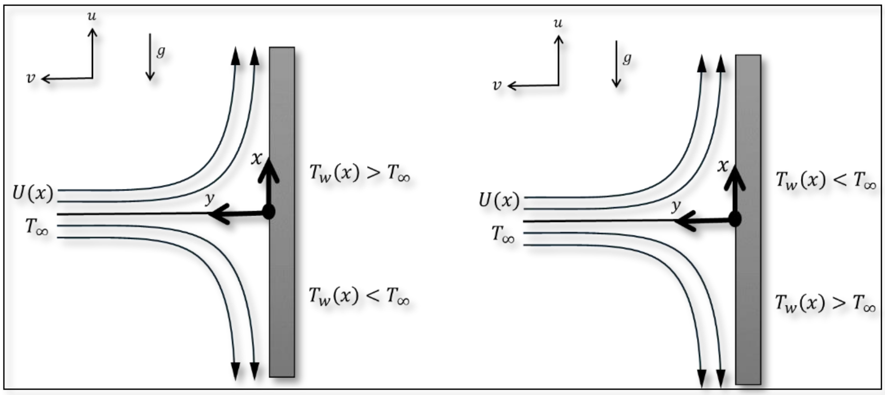

Fig. 1 Physical model and coordinate system by [4]

Fig. 1 illustrates the steady, two-dimensional laminar boundary layer flow of a viscous and incompressible fluid near the stagnation point on a vertical surface. The free stream velocity is represented by $u_e(x)$, the temperature of the ambient fluid by T_∞ , and the plate's temperature by $T_w(x)$. In compliance of the physical problem, the continuity equation, momentum equation and energy equation are as below [4]:

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=u_e\frac{du_e}{dx}+v\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

and subject to boundary conditions as follows:

$$u=L\frac{\partial u}{\partial y},\quad \upsilon=0,\quad T=T_w+S\frac{\partial T}{\partial y}\quad \text{at}\quad y=0,$$

$$u\to u_e,\quad T\to T_\infty\quad \text{as}\quad y\to\infty \tag{4}$$

where (u,v) are the components of the flow velocity in the (x,y).

The similarity variables below are used to fulfil transformation equation:

$$u=L\frac{\partial u}{\partial y},\quad \upsilon=0,\quad T=T_w+S\frac{\partial T}{\partial y}\quad \text{at}\quad y=0, \tag{5}$$

Eq. (2) and (3) are transformed by using eq. (5) respectively as follows:

$$f''' + ff'' - f'^2 + 1 + \lambda\theta = 0 \tag{6}$$

$$\frac{1}{\Pr}\theta'' + f\theta' - f'\theta = 0 \tag{7}$$

and the boundary conditions are transformed to the following form:

$$f(0) = 0, \quad f'(0) = \delta f''(0), \quad \theta(0) = 1 + \gamma\theta'(0)$$

$$f'(\eta) \to 1, \quad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty \tag{8}$$

where $\lambda = \frac{g\beta b}{a^2}$ is the mixed convection parameter, $\Pr = \frac{v}{\alpha}$ is the Prandlt number, $\delta = L\left(\frac{a}{v}\right)^{\frac{1}{2}}$ is the velocity slip parameter and $\gamma = S\left(\frac{a}{v}\right)^{\frac{1}{2}}$ is the thermal slip parameter.

The physical quantity of interest in this investigation is the coefficient for the local Nusselt number (Nu_x) and skin friction (C_f), which are elaborated as [4]:

$$Nu_x = \frac{xq_w}{k(T_w - T_\infty)} \quad \text{and} \quad C_f = \frac{\tau_w}{\rho u_e^2 / 2} \tag{9}$$

where τ_w is wall shear stress and q_w is surface heat flux are defined as below:

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \quad \text{and} \quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{10}$$

By using Eq. (9) and (10), the quantities can be described in equation (11):

$$\frac{1}{2}C_f \operatorname{Re}_x^{\frac{1}{2}} = f''(0), \qquad Nu_x \operatorname{Re}_x^{-\frac{1}{2}} = -\theta'(0) \tag{11}$$

Which $\operatorname{Re}_x^{\frac{1}{2}} = \left(\frac{a}{v}\right)^{\frac{1}{2}} x$ is the local Reynolds number and μ is a dimensionless parameter defined as $\mu = \rho v$.

2. Results and Discussion

The ordinary differential equations with the boundary conditions are solved numerically by using shooting method and the MATLAB software's bvp4c function by considering skin friction coefficient and local Nusselt number which are proportional to $f''(0)$ and $-\theta'(0)$, respectively, for various values of the governing parameters including velocity $f'(\eta)$ and temperature $\theta(\eta)$ profiles. To validate the findings, the current results are compared with previous work that has been obtained by [1]. Table 1 shows the comparison values of skin friction coefficient $f''(0)$ for different Prandtl numbers $\Pr$ with the study by [4].

Table 1 Value of $f''(0)$ for different values of Pr when $\lambda = 1$, $\delta = 0$, and $\gamma = 0$

Pr	[1]	[17]	[18]	[19]	[4]	Current results
0.7	1.7063	1.7064	1.7064	1.70632	1.7063	1.7063
1	-	-	-	-	1.6754	1.6754
7	1.5179	1.5180	1.5180	-	1.5179	1.5179
10	-	-	-	1.49284	1.4928	1.4928
20	1.4485	1.4485	1.4486	-	1.4485	1.4485
40	1.4101	-	1.4102	-	1.4101	1.4101
50	-	-	-	1.40686	1.3989	1.3989
60	1.3903	1.3903	1.3903	-	1.3903	1.3903
80	1.3774	-	1.3773	-	1.3774	1.3774
100	1.3680	1.3680	1.3677	1.38471	1.3680	1.3680

Table 1 shows that at Pr = 0.7, this study and studies by [1], [19] and [4] show the same value of 1.7063, which means they match perfectly. For larger Pr values like Pr = 50, the results are also the same, with both studies showing $f''(0)$ = 1.3689. This consistency shows that the method used in this study, with the bvp4c solver, gives accurate results that align with the previous study.

Table 2 shows the comparison of Nusselt number $-\theta'(0)$ values for different Prandtl numbers Pr between the current outputs and the results obtained by previous researchers [4].

Table 2 Value of $-\theta'(0)$ for different values of Pr when $\lambda = 1$, $\delta = 0$ and $\gamma = 0$

Pr	[1]	[17]	[18]	[19]	[4]	Current results
0.7	0.7641	0.7641	0.7641	0.76406	0.7641	0.7641
1	-	-	-	-	0.8708	0.8708
7	1.7224	1.7223	1.7226	-	1.7224	1.7224
10	-	-	-	1.94461	1.9446	1.9446
20	2.4576	2.4574	2.4577	-	2.4576	2.4576
40	3.1011	-	3.1023	-	3.1011	3.1011
50	-	-	-	3.34882	3.3415	3.3415
60	3.5514	3.5517	3.5560	-	3.5514	3.5514
80	3.9095	-	3.9195	-	3.9095	3.9095
100	4.2116	4.2113	4.2289	4.23372	4.2116	4.2116

From Table 2, we can see that the current values match exactly with [4] values for every Prandtl number. For example, when $\mathrm{Pr} = 0.7$, both results give the same value of 0.7641. Similarly, for $\mathrm{Pr} = 1$, the value is 0.8708 for both cases. At $\mathrm{Pr} = 50$, the Nusselt number is 3.3415 from [4] and matches exactly with the current value of 3.3415. This agreement validates the accuracy of the current numerical method used, which is the bvp4c solver. Overall, the comparison confirms that the current work successfully reproduces the results of [4]. As Prandtl number increases, the Nusselt number also increases, and this trend can be observed consistently in both sets of results.

Fig. 1 to 5 illustrate the velocity profiles $f'(\eta)$ corresponding to variations in the velocity slip parameter δ, thermal slip parameter γ, buoyancy or mixed convection parameter λ and Prandtl numbers Pr respectively.

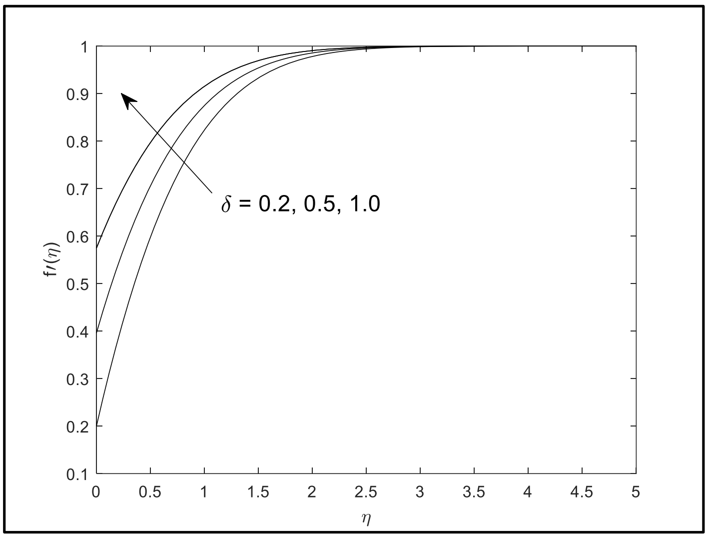

Fig. 1 Velocity profiles $f'(\eta)$ for some values of δ when $\Pr = 0.7$, $\gamma = 1$, and $\lambda = -0.2$

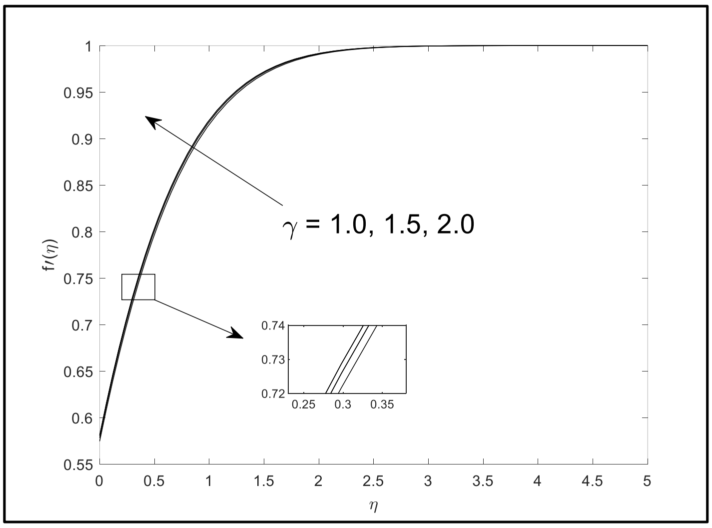

Fig. 2 Velocity profiles $f'(\eta)$ for some values of γ when $\Pr = 0.7$, $\delta = 1$, and $\lambda = -0.2$

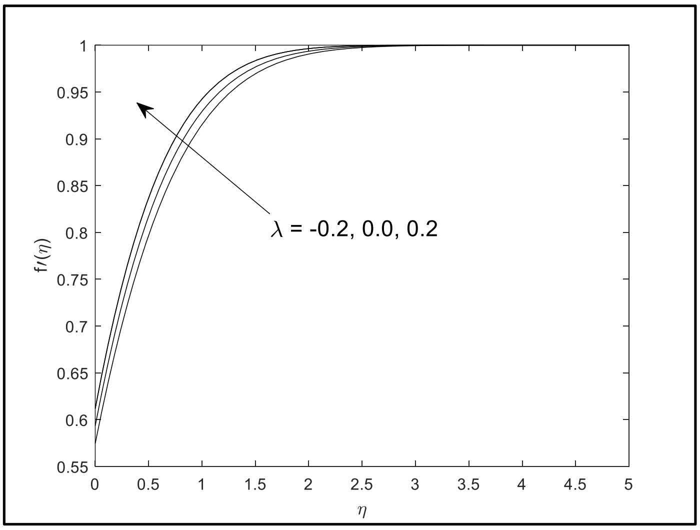

Fig. 3 Velocity profiles $f'(\eta)$ for some values of λ when $\Pr = 0.7$, $\delta = 1$, and $\gamma = -0.2$

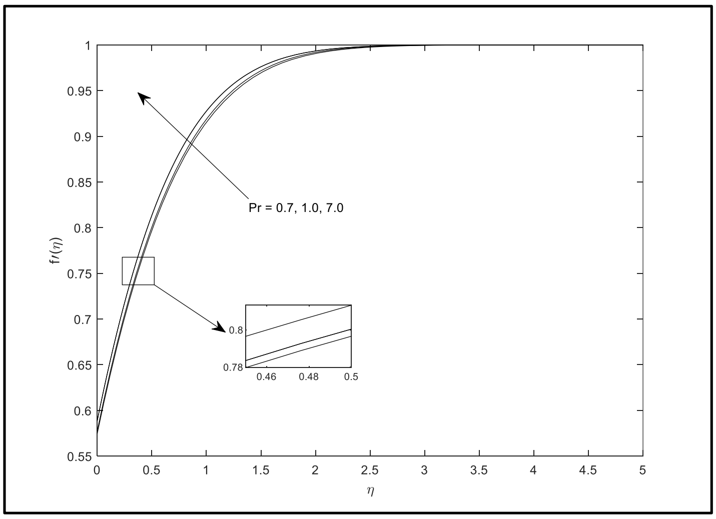

Fig. 4 Velocity profiles $f^{'}(\eta)$ for some values of $\Pr$ when $\lambda = -0.2$, $\delta = 1$ and $\gamma = 1$

The velocity profiles illustrate that as the velocity slip parameter δ or thermal slip parameter γ increases, the boundary layer thickness increases, causing the velocity to take longer to reach the free stream value. This increase reduces the gradient at the surface, resulting in a decrease in the skin friction coefficient. Similarly, as the mixed convection parameter λ increases, the velocity near the surface becomes steeper, increasing fluid motion within the boundary layer. In contrast, a higher Prandtl number causes thinner boundary layers with steeper velocity gradients, leading to increased skin friction. These trends highlight the influence of slip, convection, and thermal properties on boundary layer behaviour.

Fig. 5 to 9 illustrate the temperature profiles $\theta(\eta)$ corresponding to variations in the velocity slip parameter δ, thermal slip parameter γ, buoyancy or mixed convection parameter λ and Prandtl numbers $\Pr$ respectively.

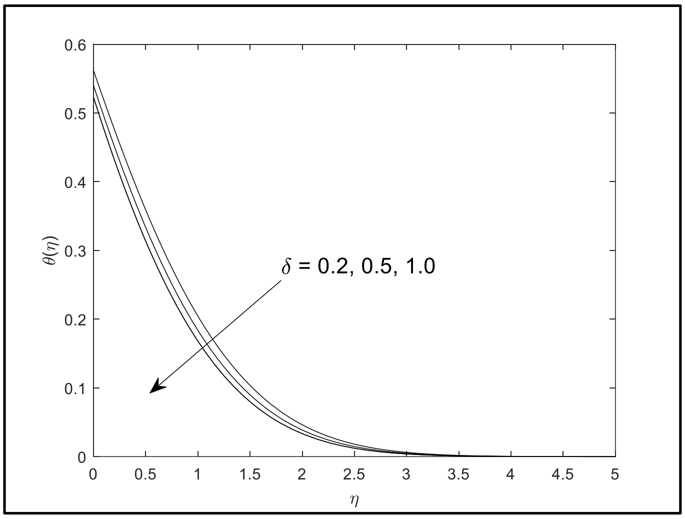

Fig. 5 Temperature profiles $\theta(\eta)$ for some values of δ when $\Pr = 0.7$, $\gamma = 1$, and $\lambda = -0.2$

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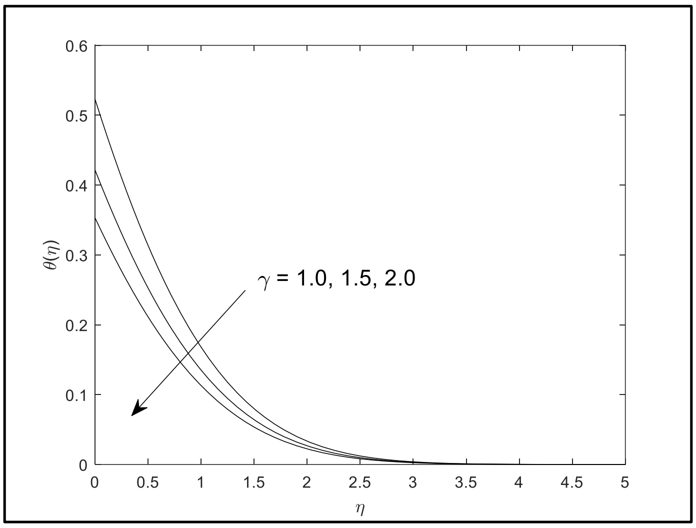

Fig. 6 Temperature profiles $\theta(\eta)$ for some values of γ when $\Pr = 0.7$, $\delta = 1$, and $\lambda = -0.2$

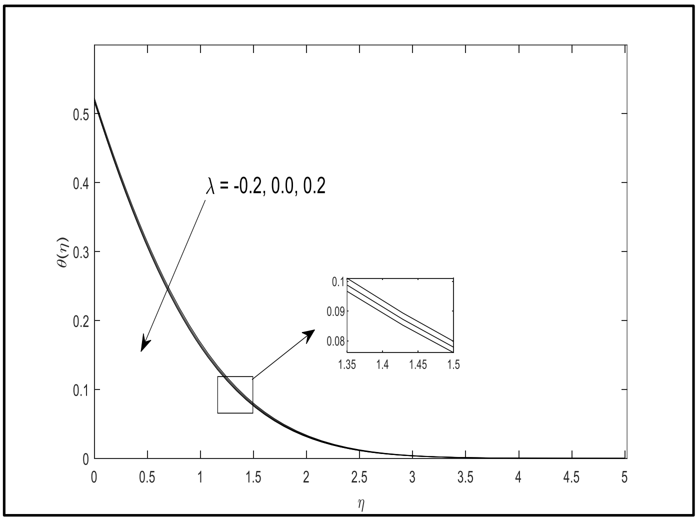

Fig. 7 Temperature profiles $\theta(\eta)$ for some values of λ when $\Pr = 0.7$, $\delta = 1$, and $\lambda = 1$

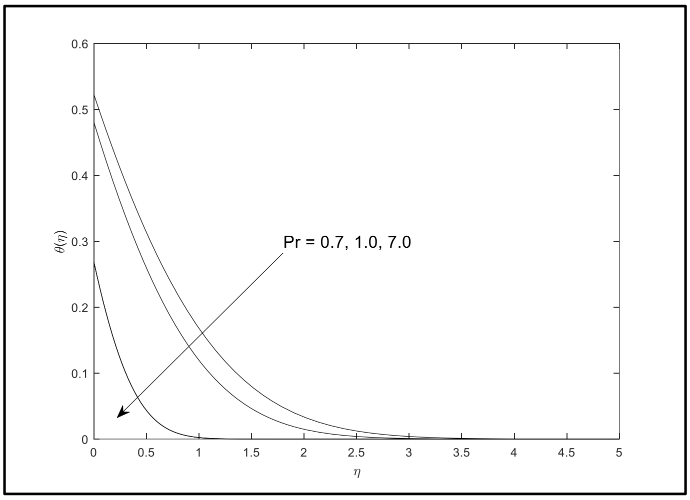

Fig. 8 Temperature profiles $\theta(\eta)$ for some values of $\Pr$ when $\lambda = -0.2$, $\delta = 1$, and $\gamma = 1$

From Figs. 6 and 7, an increase in velocity and thermal slip parameters causes the thermal boundary layer to thicken, reducing surface temperature, velocity, and heat transfer efficiency. In Fig. 8, there are only slight changes in the temperature profiles, where the mixed convection parameter slows the heat transfer process. Additionally, increasing the velocity slip, thermal slip, and mixed convection parameters thickens the boundary layers, reduces surface gradients, and decreases both skin friction and heat transfer efficiency. In contrast, as shown in Fig. 9, a higher Prandtl number leads to steeper gradients, enhancing both skin friction and heat transfer near the surface.

3. Conclusion

This study successfully solved the problem of mixed convection near a stagnation point on a vertical surface using the bvp4c solver in MATLAB. The governing equations have been transformed into ordinary differential equations using similarity transformations and solved numerically with the shooting method. The results for the skin friction coefficient and the Nusselt number have been validated against the findings of [4] demonstrating excellent agreement and confirming the accuracy of the present numerical approach. Furthermore, the effects of governing parameters such as the velocity slip parameter, thermal slip parameter, buoyancy or mixed convection parameter, and Prandtl number on velocity and temperature profiles have been examined. It is observed that an increase in slip parameters and mixed convection parameters led to thicker boundary layers and a reduction in surface friction and heat transfer, while a higher Prandtl number enhances both skin friction and heat transfer near the surface. These findings highlight the critical role of the governing parameters in influencing fluid flow and temperature behaviour near a stagnation point.

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Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design:*** *Anis Sulwana Azizi;* ***solve the governing equation:*** *Anis Sulwana Azizi, Fazlina Aman;* ***data collection:*** *Anis Sulwana Azizi;* ***analysis and interpretation of results:*** *Anis Sulwana Azizi, Fazlina Aman;* ***draft manuscript preparation:*** *Anis Sulwana Azizi, Fazlina Aman. All authors reviewed the results and approved the final version of the manuscript.*

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