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Solving Dengue Fever Model using Explicit Four-Step Multistep Method

Danish Daniel Sanusi[1], Syahirbanun Isa[1]*

[1] *Department of Mathematics and Statistics, Faculty of Applied Sciences and Technology, UTHM Kampus Cawangan Pagoh, Hub Pendidkan Tinggi Pagoh, KM 1, Jalan Panchor, 84600, Pagoh , Muar, Johor, MALAYSIA.*

*Corresponding Author: syahir@uthm.edu.my
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Abstract

This research investigates the modelling of dengue fever transmission using an explicit four-step multistep method. In tropical areas, dengue fever is a severe disease spread by mosquitoes that can cause significant public health problems. This study combines factors including infection, recovery, and natural mortality rates to model suspected, infected, and recovered populations. The Runge-Kutta approach was used to identify the starting values before applying Adam's Bashforth explicit four-step multistep method, which is renowned for its precision. Its accuracy and stability over the Bernoulli wavelet approach were shown via MATLAB (R2024B version) simulations. This strategy demonstrates the explicit multistep method's capacity to comprehend intricate disease dynamics and guide public health initiatives.

1. Introduction

Dengue fever (DF) is a serious worldwide health concern, especially in tropical and subtropical areas. Since the disease is mostly spread by Aedes mosquitoes and its incidence has significantly increased, it is essential to comprehend and anticipate how it will spread. In this endeavour, mathematical models play a vital role as they enable the researchers and public health officials to better allocate resources and predict outbreaks. Understanding the disease spreads requires a look at the history, the factors influencing the transmission, and methods used to model it [1], [2].

Foundational research on dengue origin established the groundwork for later studies that have looked at the virus's global distribution and the huge toll it takes with afflicted nations, highlighting the need for precise models that can forecast precisely when and where the next outbreak might happen [3].

As medical research developed, dengue transmission became more fully recognized and managed through the application of increasingly complex mathematical procedures. Several studies have explored the impact of various control strategies, offering valuable insights into how mathematical models can be used to curb the spread of the disease [4]. By introducing fractional calculus to their models, some have gone one step further and are able to anticipate the dynamics of the disease with more accuracy. This method opens new possibilities for improving the precision of these models, especially in regions with complex transmission patterns [5].

However, human behaviour and environment also have an impact on dengue outbreaks which are in addition to the virus itself. Climate change has been shown to influence the frequency and severity of the outbreaks, which are critical factors as global temperatures rise [6]. The migration of citizens in highly populated places can drastically change the way dengue spreads; Therefore, urban mobility also plays a big part in disease transmission [7].

These models require sophisticated mathematical models to function. Accurate methods for solving the complex equations involved in disease transmission models are essential for handling large amounts of data and calculations that come with mathematic modelling, something as dynamic as an epidemic. Numerous studies have advanced this field, improving techniques like wavelets and multistep schemes to increase the accuracy and robustness of the models [8], [9].

The explicit four-step multistep method is a robust mathematical tool to solve complex ordinary differential equations (ODEs) as this approach allows for the highly accurate modelling of complex operations in the real world. The computational efficiency and accuracy of this numerical method make it especially well-suited for capturing the dynamics of systems that change over time. In the context of disease modelling, such as DF transmissions, the explicit four-step multistep method facilitates the translation of real-world interactions between hosts and vectors into mathematical equations. This method promotes more successful public health interventions by predicting outcomes and providing insightful information about system behaviour through the analysis of these dynamics.

Thus, this study employs the explicit four-step multistep method to solve DF models, offering improved precision. Objectives include examining DF's mathematical modelling, solving DF problems using the method, and comparing results with existing studies for validation.

2. Methodology

This study develops and solves a mathematical model for DF transmission using an explicit four-step multistep method. The DF model is formulated as a system of ODEs representing interactions between human hosts and mosquito vectors. Important parameters, such as transmission rates and recovery rates, are derived from literature. The values of the initial conditions and parameters are determined by empirical data. The four-step explicit method is used to discretize and solve the ODEs because of its high accuracy and computational efficiency, particularly for stiff systems MATLAB is used to conduct simulations, and the results are validated against benchmark methods, that is the Bernoulli wavelet (BW) approach which the formula can be reviewed from [7], to ensure reliability. The BW method is a numerical technique that employs Bernoulli wavelets as basic functions to solve differential equations and other mathematical problems. This method leverages the orthogonality, compact support, and multi-resolution analysis of Bernoulli wavelets, making it highly efficient for solving ordinary differential equations (ODEs), partial differential equations (PDEs), and integral equations. **It** proves reliability by serving as continuous basic function with features such as orthogonality and compact support. These characteristics enhance the accuracy of wavelet-based numerical methods in solving mathematical problems which can be reviewed from [7]. The comparative analysis focuses on accuracy, stability, and computational performance. The model investigated was DF transmission model.

2.1 Dengue Fever Model

In the context of mathematical modelling of disease dynamics, population growth models can be applied to represent the spread diseases like DF. According to an exponential growth model, which is based on Malthusian law, the rate of disease transmission is directly proportional to the number of individuals infected. The spread of DF can be investigated from the model that was proposed. The mathematical modelling of DF is given by

$$\begin{aligned}\frac{dS(t)}{dt} &= \mu_h\big(1 - S(t)\big) - \alpha S(t)R(t)\\ \frac{dI(t)}{dt} &= \alpha S(t)R(t) - \beta I(t)\\ \frac{dR(t)}{dt} &= \gamma\big(1 - R(t)\big)I(t) - \delta_1 R(t)\end{aligned} \tag{1}$$

where S(t) is the potential victims of dengue virus, I(t) is the people who already been infected with the dengue virus , R(t) is the recovered patients from the dengue virus , μ_h the is natural death rate of human population , β is the rate of infection in the human population, α is the vaccine efficacy coefficient, γ is the recovery rate, and δ_1 is the number of deaths among the susceptible population.

2.2 Explicit Four-Step Multistep Method

The Adams-Bashforth (AB) method is a family of explicit multistep method used to solve ODEs numerically. It utilizes previously calculated values to forecast future points and is generated from the integration of the Taylor series expansion. The AB approach is particularly efficient for problem requiring multiple evaluations, as it reduces computational cost compared to single method like Runge-Kutta. The AB approach provides an equilibrium between computing economy and accuracy. In this study, the four-step AB methods is considered as an explicit multistep method for solving mathematical modelling of DF. In order to use the method, it requires minimum of three previous values that can be obtained using Runge-Kutta fourth order (RK4).

The mathematical modelling of DF will be solved numerically using four-step AB method. Starting values such as S, I, and Rat steps $S_1, S_2, S_3, I_1, I_2, I_3, R_1, R_2, R_3$ can be obtained through RK4 method. The RK4 method was shown as below:

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{2}$$

where;

$$\begin{aligned} k_1 &= hf(x_i, y_i) \\ k_2 &= hf\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1\right) \\ k_3 &= hf\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2\right) \\ k_4 &= hf(x_i + h, y_i + k_3) \end{aligned} \tag{3}$$

AB approach then calculates the solution commencing from $S_4, I_4, and\ R_4$ using the recurrence relations:

$$\begin{aligned} S_{(i+1)} &= S_i + \frac{h}{24}\left(55x_i - 59x_{(i-1)} + 37x_{(i-2)} - 9x_{(i-3)}\right) \\ I_{(i+1)} &= I_i + \frac{h}{24}\left(55y_i - 59y_{(i-1)} + 37y_{(i-2)} - 9y_{(i-3)}\right) \\ R_{(i+1)} &= R_i + \frac{h}{24}\left(55z_i - 59z_{(i-1)} + 37z_{(i-2)} - 9z_{(i-3)}\right) \end{aligned} \tag{4}$$

where *x, y,* and *z* denote the derivatives of *S, I,* and *R* respectively, and *h* is the step size. In order to evaluate accuracy and stability, the BW method is used to compare numerical solutions with the AB method. The precise solution is determined using the Ode45 solver, and the absolute error is calculated as :

$$\text{Absolute Error} = |\text{Numerical solution} - \text{Exact solution}|$$

3. Results and Discussions

The outcomes when utilizing the four-step AB method to solve the DF model are shown in this section. A test problem is examined, and the numerical results are compared with Ode45 solver for validation using MATLAB software. The precision of the model and the application of the numerical approach are highlighted when the discussion examines the relationships between the susceptible, infected, and recovered populations.

3.1 Test Problem

This section demonstrates the solution of mathematical models of DF solved using AB method. With initial values given $\boldsymbol{S(0) = 0.9999400530}$, $\boldsymbol{I(0) = 0.0000599472}$, and $\boldsymbol{R(0) = 0.1}$. From the research paper [7], the other parameters were also given such as $\boldsymbol{\mu_h = 0.0045}$, $\boldsymbol{\alpha = 0.006}$, $\boldsymbol{\beta = 0.333333}$, $\boldsymbol{\gamma = 0.375}$, $\boldsymbol{\delta_1 = 0.02941}$, and $\boldsymbol{h = 0.1}$.

3.2 Numerical Results

This section discusses the results of the mathematical model of DF. The four-step AB method is used to solve the system numerically. MATLAB software is utilized for all calculations. The Ode45 solver is also used to solve the problem, and the outcomes are contrasted with AB four-step and BW method's findings. The numerical solutions to test issue utilizing the Ode45 solver and the AB method, and the BW method approaches are shown in Table 1, Table 2, and Table 3 for *S(t), I(t)* and *R(t)* respectively. The absolute error between the two approaches is tabulated in Table 4, Table 5, and Table 6 respectively. The error is calculated by comparing the approach using the four-step AB method and BW method with Ode45 solver results.

Table 1 Numerical solutions of four-step AB method, BW method [7] and Ode45 Solver for S(t)

t	Four-step AB method	BW method	Ode45 Solver
0	0.9999400530	0.9999400530	0.9999400530
0.1	0.9998801862	0.9998801861	0.9998801862
0.2	0.9998205234	0.9998205233	0.9998205234
0.3	0.9997610629	0.9997610628	0.9997610629
0.4	0.9997018030	0.9997018029	0.9997018030
0.5	0.9996427420	0.9996427419	0.9996427420
0.6	0.9995838783	0.9995838783	0.9995838783
0.7	0.9995252104	0.9995252103	0.9995252104
0.8	0.9994667367	0.9994667366	0.9994667367
0.9	0.9994084557	0.9994084556	0.9994084557
1	0.9993503659	0.9993503659	0.9993503659

Table 1 Numerical solutions of four-step AB method, BW method [7] and Ode45 Solver for I(t)

t	Four-step AB method	BW method	Ode45 Solver
0	0.0000599472	0.0000599472	0.0000599472
0.1	0.0001169012	0.0001169012	0.0001169012
0.2	0.0001718139	0.0001718139	0.0001718139
0.3	0.0002247537	0.0002247537	0.0002247537
0.4	0.0002757869	0.0201957869	0.0002757869
0.5	0.0003249776	0.0003249776	0.0003249776
0.6	0.0003723876	0.0003723877	0.0003723877
0.7	0.0004180769	0.0004180770	0.0004180770
0.8	0.0004621032	0.0004621033	0.0004621033
0.9	0.0005045225	0.0005045226	0.0005045226
1	0.0005453888	0.0005453889	0.0005453889

Table 2 Numerical solutions of four-step AB method, BW method [7] and Ode45 Solver for R(t)

t	Four-step AB method	BW method	Ode45 Solver
0	0.1000000000	0.1000000000	0.1000000000
0.1	0.0997093188	0.0997093188	0.0997093188
0.2	0.0994213779	0.0994213779	0.0994213779
0.3	0.0991361027	0.0991361027	0.0991361027
0.4	0.0988534212	0.0988534211	0.0988534211
0.5	0.0985732634	0.0985732633	0.0985732634
0.6	0.0982955619	0.0982955618	0.0982955618
0.7	0.0980202513	0.0980202512	0.0980202512
0.8	0.0977472684	0.0977472682	0.0977472682
0.9	0.0974765519	0.0974765517	0.0974765518
1	0.0972080427	0.0972080425	0.0972080425

Table 3 Absolute error of four-step AB method and BW method for R(t)

t	Four-step AB method	BW method
0	0.00	0.00
0.1	1.3322676296E-14	8.2265749768E-11
0.2	2.5868196474E-14	9.6155527984E-11
0.3	3.7414515930E-14	9.6063157429E-11
0.4	5.5422333389E-13	8.1640472160E-11
0.5	1.0157430452E-12	9.6338825806E-11
0.6	1.4677148386E-12	2.5996538255E-11
0.7	1.8810508706E-12	9.7473140670E-11
0.8	2.2677415501E-12	7.7328476955E-11
0.9	2.6267876763E-12	7.0544348141E-11
1	2.9606317398E-12	1.9288459718E-11

Table 4 Absolute error of four-step AB method and BW method for I(t)

t	Four-step AB method	BW method
0	0.00	0.00
0.1	6.8950753310E-13	3.2880762918E-11
0.2	1.3329349831E-12	2.0884305732E-11
0.3	1.9325913482E-12	9.9490520802E-12
0.4	2.9039565837E-11	1.9919999974E-02
0.5	5.3155242360E-11	3.2312711155E-11
0.6	7.6756651599E-11	1.4264921980E-11
0.7	9.8275354272E-11	7.6440502691E-12
0.8	1.1835299760E-10	4.8322886382E-11
0.9	1.3694394371E-10	3.8975379456E-11
1	1.5417255265E-10	1.3653383003E-11

Table 5 Absolute error of four-step AB method and BW method for R(t)

t	Four-step AB method	BW method
0	0.00	0.00
0.1	7.6039174957E-13	2.0175278115E-11
0.2	1.4703793738E-12	1.0599618405E-11
0.3	2.1324886301E-12	2.7914656697E-11
0.4	2.9920649292E-11	4.1273665041E-11
0.5	5.4557081075E-11	5.3814980250E-11
0.6	7.8721598684E-11	2.6650431861E-11
0.7	1.0070781709E-10	5.2908372128E-12
0.8	1.2120701665E-10	4.8433243527E-11
0.9	1.4016436622E-10	5.9036386890E-11
1	1.5771046968E-10	1.7613452363E-11

Fig. 1 the comparison of absolute errors as in Table 4, Table 5, and Table 6 respectively.

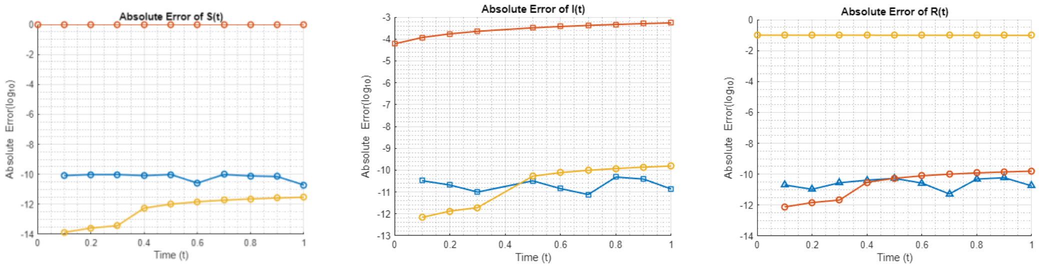

Fig. 1 The graphical comparison of Absolute Error of Bernoulli Wavelet and Adams-Bashforth with Ode45 solver solution for S(t), I(t), and R(t) where the blue line indicates the "BW method vs Ode45 solver" , red line indicates the "AB vs Ode45 solver", and yellow line indicates the "Ode45 solver".

The interactions between the susceptible (S), infected (I), and recovered (R) populations over time show clear tendencies, according to the numerical findings of solving the DF transmission model using the Explicit four-step multistep method. Beginning with the initial values of $S_0 = 0.9999400530$, $I_0 = 0.0000599472$, and $R_0 = 0.1$, the dynamics of these populations exhibit clear and interpretable patterns.

Based on the graph of *S(t)*, it shows that the four-step AB approach performs greater than the BW method. The four-step AB method consistently shows smaller and more stable errors, particularly during the early stages of the simulations. In contrast, while the BW method is initially accurate, it begins to show bigger differences from the analytical answer as the simulations develop. The four-step AB method is more stable and produces fewer errors over time than the BW method, showcasing that the four-step AB method is more effective in simulating the suspected population in this circumstance.

Next, for the infected population *I(t)*, the graph shows that the BW method produces smaller and more stable errors than the four-step AB method. The error noticeably around $t = 0.4$, where a sharp transition in the infected population occurs. The BW method closely approximates the analytical solution, which makes it very useful for modelling the infected population. Although the four-step AB method provides a good approximation, the increase in error shows as the infection level rises making the four-step AB method is not well-suited for capturing the dynamics of the infected population over time as the BW method.

Similar tendencies were observed for the recovered population, *R(t)* where the BW method consistently exhibits the least error across the time points, closely approximating the analytical solution. The four-step AB method introduces slightly larger errors as the simulation progresses, indicating that it struggles to accurately model the recovery phase.

In conclusion, the four-step AB method is more accurate in modelling the susceptible population, *S(t)* whereas the BW method provides higher accuracy for the infected population, *I(t)* and recovered population, *R(t).* The four-step AB method higher accuracy for *S(t)* is attributed to the utilization of prior data points, which stabilizes the estimation. The BW method excels in calculating *I(t)* and *R(t)* with minimal mistakes throughout the simulation.

4. Conclusion

Based on the results and discussion, the four-step AB method effectively solved the DF transmission model, providing a precise depiction of the interactions between susceptible, infected, and recovered populations. The numerical results, which accurately depicted the dynamics of disease transmission and recovery processes, showed a steady decline in susceptible people, a brief increase in infected cases, and the ultimate equilibrium of the recovered group.

The comparison with Ode45 solver by using MATLAB software validated the accuracy and reliability of four-step AB method, showcasing its capability to model such dynamic epidemiological systems with minimal errors. This study emphasizes how effective and reliable four-step AB method is in capturing the complex behaviour of disease transmission models, demonstrating its use as a tool for understanding DF dynamics and assisting in development of disease control and preventive methods.

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Conflict of Interest

Authors declare that there is no conflict of interest regarding the publication of the paper.

Author contribution

The authors confirm contribution to the paper as follows: ***study conception and design:*** *Danish Daniel Sanusi, Syahirbanun Isa;* ***data collection:*** *Danish Daniel Sanusi;* ***analysis and interpretation of results:*** *Danish Daniel Sanusi, Syahirbanun Isa;* ***draft manuscript preparation:*** *Danish Daniel Sanusi, Syahirbanun Isa. All authors reviewed the results and approved the final version of the manuscript*

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