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Solving Facility Location-Allocation Problem with Zone-Dependent Fixed Cost using Modified Cooper's Method

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Abstract

Location analysis is a vital area within Operations Research that focuses on optimizing the placement of facilities and the allocation of customers to achieve a balance between service efficiency and cost reduction. This research examines the incapacitated multisource Weber problem, incorporating zone-dependent fixed costs, which adds complexity by considering regional cost differences. Since the multisource Weber Problem is classified as an NP-hard problem due to its complexity, therefore a heuristic technique named Modified Cooper's method was proposed. Algorithm for the solution technique was coded using Python programming. Previously, research had been done to solve this problem using Simulated Annealing (SA), which is a meta-heuristic approach known for its global search capabilities. Modified Cooper's method resulted in a total cost of 171.02, significantly lower than the 196.73 achieved through SA. This finding highlights the effectiveness and practicality of the Modified Cooper's method in addressing location-allocation challenges with zone-dependent costs, providing valuable insights for sectors such as logistics and urban planning.

1. Introduction

Location analysis plays a vital role in operations research, concentrating on determining the most advantageous sites for facilities within a network and assigning demand points to these locations. The main goal is to enhance efficiency and customer satisfaction while reducing overall expenses, which generally encompasses fixed and transportation costs. The mathematical models employed in location analysis are typically divided into three categories: network models (where the solution space, x, consists of a finite union of linear, continuous sets), discrete models (suitable for finite solution spaces, x), and continuous models (where $x \subseteq \Re^q$)[1]. Facility location decisions typically involve two main types of costs: fixed costs, which are tied to the establishment of the facility at a specific site, and transportation costs, which pertain to the distribution of goods or services from the facilities to the points of demand. A key element of these analyses is the need to balance service efficiency with the economical use of resources. The goal of optimizing facility locations is to minimize overall operational expenses while effectively meeting customer demand.

Table 1 *Nomenclature*

Symbol	Description
x	Location of the facility
m	Number of Facility
n	Number of customer or demand points
$f(x)$	Location fixed cost
$c(x,y)$	Transportation cost from location x to demand y

This study explores facility location issues that involve the interplay between fixed and transportation costs, emphasizing their practical implications. Typically, facility location-allocation challenges are analysed in static environments, allowing for a clear connection between theoretical models and real-world scenarios. Nevertheless, actual applications frequently encounter dynamic elements, including fluctuations in construction expenses, workforce availability, and regional economic conditions, all of which affect fixed costs. It is essential to incorporate these dynamic cost factors into the analytical framework to ensure that decision-making is both relevant and resilient.

The study explores the multi-facility location challenge, often known as the multisource Weber problem. This challenge focuses on identifying the best locations for facilities within Euclidean space to reduce the total transportation and fixed costs. The presence of zone-dependent fixed costs adds further complexity, enhancing the problem's significance for real-world applications. This issue is classified as NP-hard, indicating that there is no polynomial-time algorithm capable of ensuring an optimal solution across all scenarios [2]. Consequently, heuristic approaches are frequently utilized to approximate solutions effectively.

Among the heuristic methods, SA has gained prominence due to its simplicity and effectiveness in solving complex optimization problems. SA is a probabilistic technique inspired by the annealing process in metallurgy, where controlled cooling allows atoms to settle into a low-energy state [3]. In the context of facility location problems, SA explores the solution space by iteratively moving to neighbouring solutions, accepting worse solutions with a certain probability of escaping local optima. Over time, the probability of accepting worse solutions decreases, converging to a near-optimal solution. Its adaptability to various cost functions and ability to handle non-convex solution spaces make it a valuable tool in addressing the complexities of location analysis [4]. The study builds upon Cooper's method, a well-established approach in facility location analysis, and extends it to incorporate zone-dependent fixed costs. This method was refined by incorporating Algorithm 1 [5] to improve the solution obtained at each iteration. Algorithm 1 was introduced by [5] to improve the solution obtained at each iteration when dealing with zone-dependent fixed costs in facility location analysis. Algorithm 1 was created to refine the method by incorporating a more accurate way to address the problem of changing costs based on geographical zones. The proposed heuristic method is further refined using algorithms developed in previous studies [6]. Its performance is evaluated using a set of customer locations to determine the optimal positions of two facilities. The heuristic approach of Modified Cooper's method with the aid of Algorithm 1 has been implemented and the algorithm of the solution technique was coded using Python programming.

2. Research Method

This part is an overview of constructive heuristics. It starts with a basic problem, that is to find the best location for a single facility, such as a warehouse or a factory. Setup cost varies from zone to zone. After setting up this basic problem, proceeding further will entail tackling the more complex problem of locating multiple facilities at the same time.

2.1 Incapacitated Multisource Weber Problem with Zone-Dependent Fixed Cost

Incapacitated multi-source Weber problem with zone-dependent fixed costs can be formulated mathematically as follows [7]:

Given:
Parameters:

n : the number of fixed points (or customer points);

w_j : demand or weight of customer $j (j = 1,\dots,n)$;

a_j : (a_j^1, a_j^2) : location of customer *j* where $a_j \in \Re^2$, $(j = 1,\dots,n)$;

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M : the number of fixed points (or customer points).

Decision Variables:
x_f, y_f : coordinates of the facility, where $x_f, y_f \in \Re$.

Derived Variables:
q_j : quantity assigned from the facility to the customer j .
Objective:
Minimize the total cost, given by:

$$\text{Total cost} = \sum_{i=1}^{M}\sum_{j=1}^{n} x_{ij} d(x_i, a_j) + \sum_{i=1}^{M} f(x_i), \tag{1}$$

where:
$c(a_j, x_f, y_f)$ represents the distance between the facility f and customer j .
$f(x_f, y_f)$ represents the fixed cost associated with the facility at the location f .

Subject to

$$\sum_{i=1}^{M} x_{ij} = w_j, \forall_j = 1, \dots, n \tag{2}$$

$$x_{ij} \geq 0, \forall_i = 1, \dots, M; j = 1, \dots, n \tag{3}$$

(2) ensures that each customer's demand is met.
(3) prevents negative quantities or distances.

The setup makes no restriction whatsoever on the amount of shipment that can be handled at all, which means the customers are assigned to the nearest facilities, with arbitrarily broken ties. However, with the opening of each facility, a fixed cost is incurred, depending on the location.

2.2 Measurement of Distance

In location-allocation problems, the accurate measurement of distances is critical. Two common distance metrics are used [8]:

Rectilinear Distance: (also known as Manhattan or city-block distance) is useful in grid-like urban environments, where travel is constrained by streets.
Euclidean Distance: the straight-line distance, commonly used in open, less structured areas

This study largely focuses on Manhattan distance due to its practical application in grid-based environments, though in some circumstances, the Euclidean distance will be considered, such as in the application of Weiszfeld's equation in a way proposed by [9].

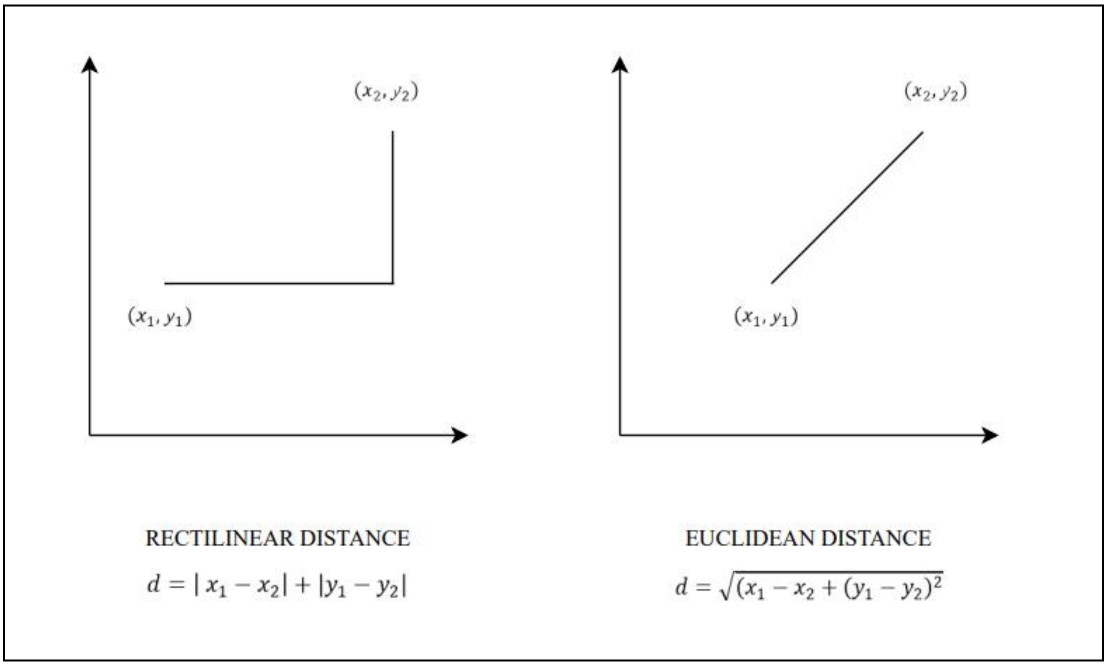

Fig. 1 Rectilinear and Euclidean and Distance [8]

2.3 Solving Single-Source Weber Problem with Zone-Dependent Fixed Cost

To understand the multi-facility problem, we first address the simpler single-facility problem with zone-dependent fixed costs.

2.3.1 Weiszfeld Equation

The Weiszfeld equation proposed by Endre Vaszonyi Weiszfeld in 1936 is a well-known method to find the best location of a single facility minimizing the total weighted Euclidean distance between the facility and existing infrastructure. The Weiszfeld method is iterative, improving the location estimate through successive computations.

To find the optimal location of a single facility, the Weiszfeld equation is applied iteratively:

$$x_f(k+1) = \frac{\sum_{j=1}^{n} d_j . x_j}{\sum_{j=1}^{n} d_j} \quad \text{and} \quad y_f(k+1) = \frac{\sum_{j=1}^{n} d_j . y_j}{\sum_{j=1}^{n} d_j} \tag{4}$$

Where k represents the iteration number. The process continues until the location converges.

2.1.1 Algorithm 1: Solving the Single-Facility Location Problem

Algorithm 1[5] is developed to find the optimal location for a single facility in the non-existence of zone-dependent fixed costs. The goal is to minimize both transport and fixed costs:

$$\text{minimize} \sum_{j=1}^{n} c(a_j, x_f, y_f).q_j + \sum_{f=1}^{m} f(x_f, y_f) \tag{5}$$

The algorithm proceeds with the following steps:

Step 1: Solve the single-facility minimum problem and check if the solution lies in the zone with the smallest fixed cost.
Step 2: For each candidate polygon, determine its visibility and boundary.
Step 3: Use a one-dimensional interval bisection search to find the optimal facility location.

This method involves solving the location problem iteratively while adjusting for zone-dependent fixed costs.

2.3.2 Numerical Example of Algorithm 1

Algorithm 1 is illustrated using a numerical example [5] as shown in Fig. 2. The square is divided into several rectangular zones, with customer locations specified. The algorithm begins with solving the single facility minimum problem using the Weiszfeld equation and subsequently selects the optimum facility location according to the zones fixed costs.

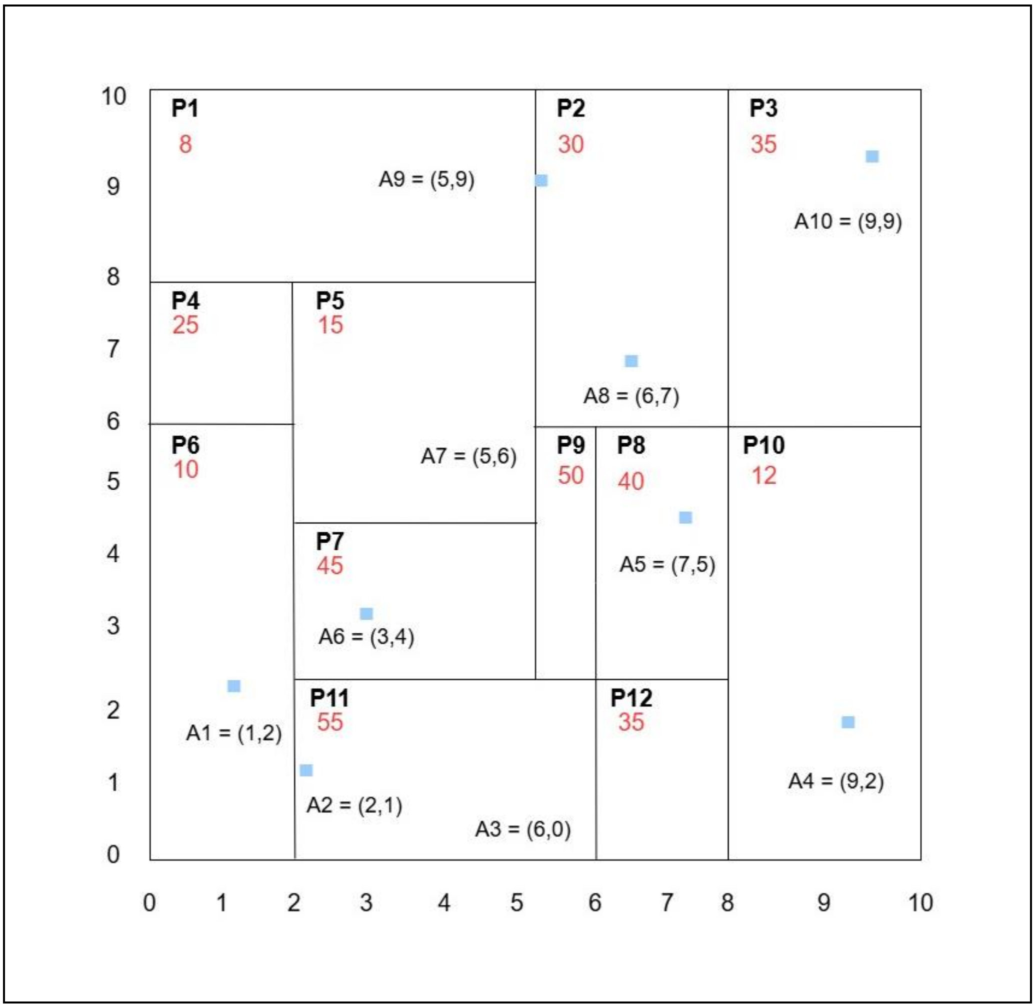

Fig. 2 Numerical Example of Algorithm 1[5]

2.4 Solving the Multisource Weber Problem with Zone-Dependent Fixed Cost

When more than one facility is to be located, the problem becomes the multisource Weber problem. This section introduces a modified version of Cooper's method [10] for solving the multisource problem with zone-dependent fixed cost.

2.4.1 Modified Cooper's Method

Cooper's method involves alternating between location and allocation steps until a local minimum is reached. In this study, the method is modified by incorporating Algorithm 1 to improve the location of facilities. The modified steps are:

Step 1: Choose m initial facility locations randomly.
Step 2: Allocate each customer to its nearest facility.
Step 3: Relocate the facilities by solving the m single-facility location problem using Algorithm 1.
Step 4: Repeat **Step 2** and **Step 3** until no further improvement is achieved over three successive iterations.

3. Results and Discussion

This study uses the well-documented 50-point dataset from [11] as a test case, focusing on a scenario with two facilities $(m = 2)$. The chapter presents the implementation process of the Modified Cooper's Method, discusses the results obtained, and provides insights into the relative strengths and weaknesses of both methods in solving the Incapacitated Multisource Weber Problem efficiently.

3.1 Computational Result

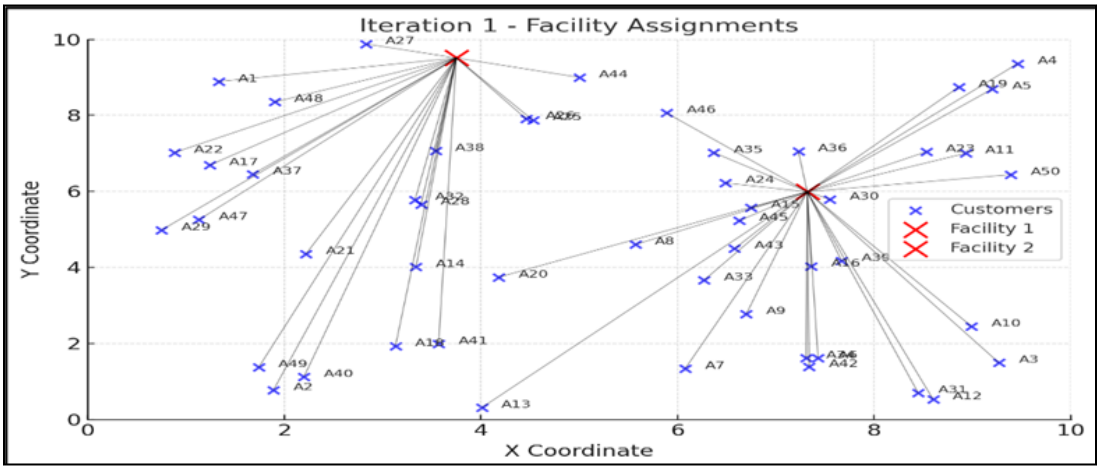

Fig. 3 Iteration 1

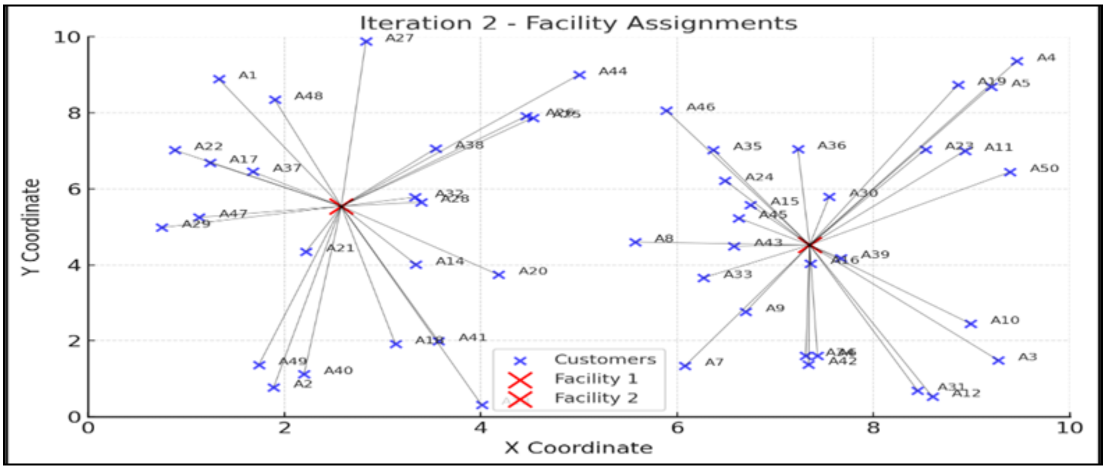

Fig. 4 Iteration 2

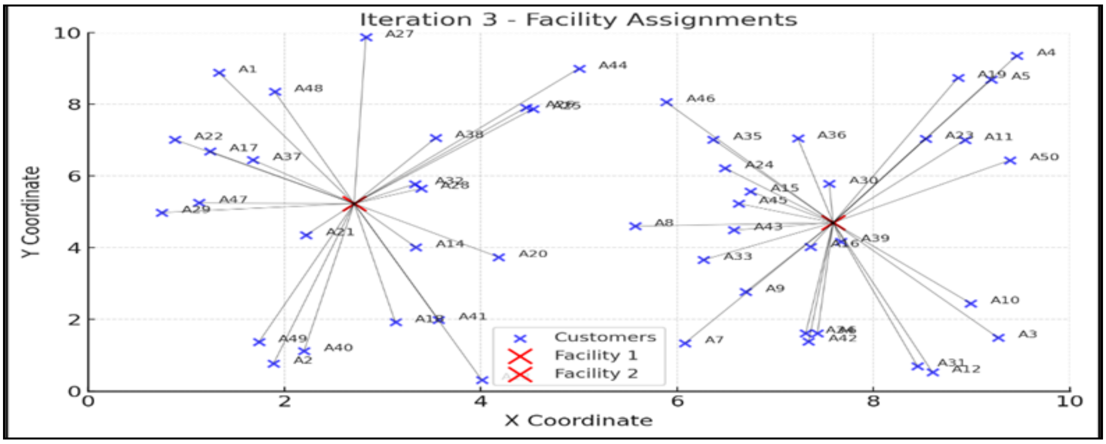

Fig. 5 Iteration 3

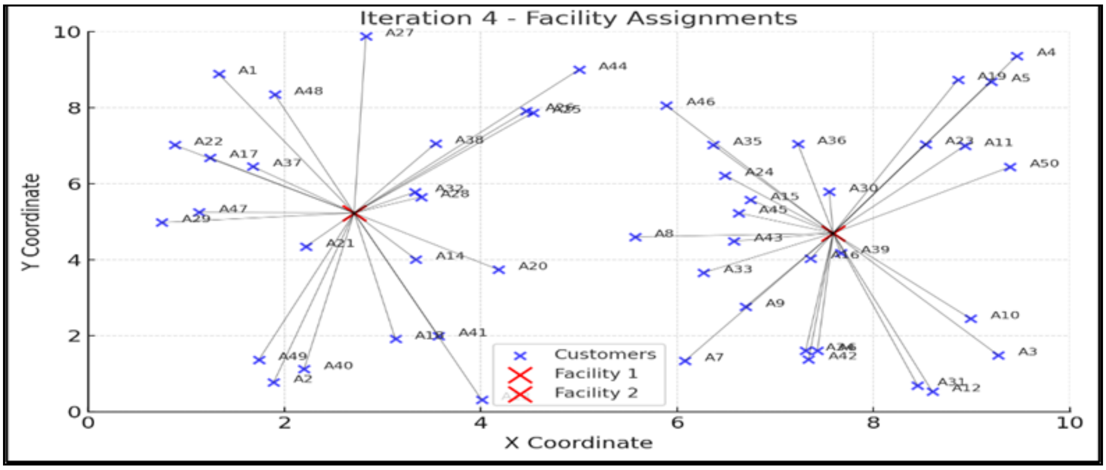

Fig. 6 Iteration 4

Fig. 4 to Fig. 6 display a detailed description of the iterative optimization process. The blue points represent customer locations with each label to identify individual customers. The red points represent facility locations at each iteration, which indicates the process by which the facilities adjust their positions throughout optimization. Additionally, the black lines connect the customer locations identified by the letters with their assigned facilities showing visually where customers are assigned facilities for each iteration. These connections show the reallocation of customers to facilities as the facility locations change to lower total costs.

Table 2 Modified Cooper's Method Iteration

Iteration	Facility location	Total Cost
1	(3.745, 9.507) and (7.320, 5.987)	221.86
2	(2.575, 5.539) and (7.348, 4.520)	172.18
3	(2.707, 5.233) and (7.589, 4.705)	171.02
4	(2.707, 5.233) and (7.589, 4.705)	171.02

The Modified Cooper's method has been able to optimize facility locations in four iterations by minimizing the total cost, including the transport and zone-dependent fixed costs. The algorithm began with randomly assigned facility locations, which yielded a total cost of 221.86. To cover the costs after this, customers were allocated to their nearest facilities based on rectilinear distance in the first iteration.

During the second iteration, significant adjustments were observed to the locations of the facilities, with Facility 1 relocating to (2.58, 5.54), and Facility 2 moving to (7.35, 4.52). This led to a large cost reduction of 170.18, demonstrating the algorithm's ability to make impactful adjustments early on. By the beginning of the third iteration, the locations of the facilities were slightly modified to (2.71, 5.23) for Facility 1 and (7.59, 4.70) for Facility 2. The total cost was slightly up to 171.02; however, this small adjustment indicated that the algorithm had converged toward optimal transportation and fixed costs balance.

During the final iteration, no further changes to the facility placement had been made, indicating that the algorithm had stabilized since there was no further significant lowering of costs. The total cost of 171.02 has been achieved, which marked the optimal solution upon which the process ended. The iterative correction demonstrates the efficiency of this algorithm through high-cost reductions in earlier runs and only slight adjustments in later iterations. The location of the final facilities reflects a strategic placement that minimizes overall costs, thus making this method well-suited for solving complex location-allocation problems.

3.2 Comparison of Modified Cooper's Method and Simulated Annealing (SA)

The comparison between the Modified Cooper's method and the SA approach, as applied by [11], reveals distinct trade-offs in facility placement and associated costs.

Table 3 Comparison of each method

Method	Facility location	Cost
Modified Cooper's method	(2.71, 5.23) and (7.59, 4.70)	171.02
Simulated Annealing (SA)	(2.00, 5.65) and (8.00, 4.60)	196.73

The Modified Cooper's method obtained optimal facility locations, considering (2.71, 5.23) and (7.59, 4.70), which resulted in the overall expense amounts to. 171.02, conversely, SA gave only slightly different facility placements at (2.00, 5.65) and (8.00, 4.60) with a higher total cost of 196.73. This shows how, with fewer iterations, the Modified Cooper's method yields a better and more cost-effective solution that is easy to replicate.

The wider exploration of the solution space afforded by simulated annealing sometimes results in better facility placements, though in this case, Modified Cooper's method had a more competitive edge concerning cost. The SA method incurred an extra cost, indicating the given problem's sensitivity to the iterative refinement technique applied by Modified Cooper's method.

An analysis was made between the Modified Cooper's method and SA. The strengths and weaknesses are apparent from the analyses thus made. The Modified Cooper's method had a total cost of 171.02 and showed a fast convergence within 4 iterations, thus proving highly efficient in cases where computational time is of the essence.

4. Conclusion

The Modified Cooper's Method successfully demonstrated its efficiency for solving Incapacitated Multisource Weber Problem with zone-dependent fixed costs, incurring a lower total cost of 171.02 as compared to 196.73 obtained on Simulated Annealing [8]. Its computational simplicity, involving merely a few iterations, is clear evidence that the method can be a tool for practical applications in optimizing the location of facilities and minimizing the corresponding costs in industries like logistics and supply chain management. However, being deterministic, the method is vulnerable to being trapped in local minima, besides which, they possess limited scalability and ability to cope with changing circumstances, which is another limitation for the applicability of this method on larger or more complicated issues. Moreover, the dataset, while controlled, does not represent the complexities of the real world, such as dynamic demand and regulatory constraints. Future research can explore hybrid approaches by integrating this approach into probabilistic algorithms, working with bigger datasets, and including machine learning for the prediction of contextual factors in a realistic manner to improve its robustness, scalability, and applicability in diverse real-world scenarios.

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Conflict of Interest

The authors state that they have no conflicts of interest related to the publication of this paper.

Author Contribution

The authors acknowledge their contributions to the paper as detailed below: ***study conception and design:*** *Siti Nur Hurul Aini Mohamad Zarazilah;* ***solve the equations:*** *Siti Nur Hurul Aini Mohamad Zarazilah;* ***analysis and interpretation of results:*** *Siti Nur Hurul Aini Mohamad Zarazilah, Azila Md Sudin;* ***draft manuscript preparation:*** *Siti Nur Hurul Aini Mohamad Zarazilah, Azila Md Sudin.*

References

[1] Mladenović, N., Brimberg, J., Hansen, P., & MorenoPérez, J. A. (2007). European Journal of Operational Research, 179(3), 927-939.
[2] Drezner, Z., & Hamacher, H. W. (2001). Facility Location: Applications and Theory. Springer.
[3] Kirkpatrick, S., Gelatt, C. D., & Vecchi, M. P. (1983). Optimization by Simulated Annealing. *Science, 220*(4598), 671-680. https://doi.org/10.1126/science.220.4598.671
[4] Cerny, V. (1985). A Thermodynamical Approach to the Traveling Salesman Problem: An Efficient Simulation Algorithm. *Journal of Optimization Theory and Applications, 45*(1), 41-51. https://doi.org/10.1007/BF00940812
[5] Salhi, S., Gamal, M. (2003) A Genetic Algorithm Based Approach for the Uncapacitated Continuous Location–Allocation Problem. *Annals of Operations Research* **123**, 203–222. https://doi.org/10.1023/A:1026131531250
[6] Brimberg, J., Mladenovic, N., & Salhi, S. (2004). The multi-source Weber problem with constant opening cost. Journal of the Operational Research Society, 55, 640-646.
[7] Abdullah, T. (2007). Application of Simulated Annealing in Solving Uncapacitated Continuous Locatio-Allocation Problem with Zone-Dependent Fixed Cost. Degree of Master dissertation, Universiti Teknologi Malaysia, Fakulti Sains
[8] Love, R. F., & Morris, J. G. (1979). Mathematical models of location and relocation. Operations

Research, 27(6), 947–965.
[9] Weiszfeld, E. (1936). "Sur le Point Pour Lequal la Somme des Distances the n Points Donnes est Minimum". The Tohoku Mathematical Journal, 43:355-386.
[10] Cooper, L. (1963). Location-allocation problems. Operations research, 11(3), 331- 343.
[11] Eilon, S., C.D.T. Watson-Gandy, and N.Christofields. (1971). Distribution Management. Mathematical Modelling and Practical Analysis. Hafiner, New York: Charles Griffin & Company Limited.