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Fehlberg Runge-Kutta Method for Solving System of Ordinary Differential Equations

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Abstract

Numerical methods are essential for solving linear and nonlinear ordinary differential equations (ODEs) encountered in various scientific and engineering applications. The objective of this study is to compare the fourth-order Runge-Kutta (RK4) and Fehlberg Runge-Kutta (RKF45) methods for solving such equations, focusing on their accuracy and efficiency under a fixed step size. The methods were implemented using MATLAB, enabling solution computations, graph plotting, and error analysis. The results show that RK4 achieves higher accuracy for smooth and predictable systems due to its simplicity and stability. In contrast, RKF45, while effective for complex systems, demonstrated reduced accuracy when constrained to a fixed step size. RKF45's potential is best realized when its adaptive step-sizing feature is employed. This study highlights the importance of selecting numerical methods based on the problem's characteristics and computational goals. Both RK4 and RKF45 are reliable tools, but their performance depends on proper implementation and problem dynamics.

1. Introduction

Systems of ordinary differential equations (ODEs) are a crucial mathematical tool for modelling a wide variety of phenomena in fields such as physics, biology, engineering, and economics. These systems consist of multiple interdependent ODEs, where each equation describes how a particular variable changes with respect to an independent variable, typically time. A system of ODEs can be linear or nonlinear, and it is often used to describe complex dynamic behaviours such as oscillations, growth processes, and interactions between multiple entities or species [1].

[2] had emphasized that the accuracy and efficiency of numerical methods for solving ODEs is evaluated based on the computation errors and computational time. Numerical solutions, often referred to as approximations, only approach the exact solution. The solution of an ODE is generally not unique unless an initial condition is specified, which determines the value of the dynamical system at its starting point. With an initial condition given, ODEs can be solved using numerical methods like the Euler method and the Runge-Kutta method [3].

[4] compared Taylor, Euler, and the classical fourth-order Runge-Kutta (RK4) methods for solving ODEs numerically. The study found that Taylor's method provides the highest accuracy but is computationally demanding, while Euler's method is computationally efficient but less accurate. RK4 offers a balance between accuracy and computational cost, making it a widely favoured choice. [5] focused on the development and evaluation of various orders of the Runge-Kutta method for solving ODEs. Their research involved comparing

the solutions obtained from different orders of the Runge-Kutta method to exact solutions. Results illustrated that solution curves from all categories closely matched, indicating the efficacy of the Runge-Kutta method across different orders in approximating exact solutions.

In this study, the RKF45 method is introduced to solve systems of ODEs. It is a numerical technique specifically designed for solving ODEs. As an embedded RKF45 method, it utilizes the same function evaluations to produce solutions of different orders while maintaining consistent error constants. RKF45 and the classical RK4 methods differ primarily in their step size control and error estimation approach [6]. RKF45 offers adaptive step size control, adjusting the step size dynamically based on error estimates to maintain accuracy efficiently, while RK4 uses a fixed step size throughout the integration process. RKF45 is an embedded method, providing both a fourth-order accurate solution and a fifth-order error estimate without additional function evaluations, whereas RK4 solely delivers a fourth-order accurate solution. This adaptive feature of RKF45 allows for more precise and computationally efficient solutions, especially in cases where accuracy requirements vary during integration [7].

The focus of this study is to apply the RKF45 method to solve systems of first-order ODEs, specifically modelling the behaviour of Resistor-Inductor (RL) circuits under constant and sinusoidal voltage inputs. This method, which offers adaptive step size control and higher-order accuracy compared to traditional Runge-Kutta methods, is implemented using MATLAB. The research aims to compare the results obtained using RKF45 with those from the widely used RK4 method, highlighting the potential advantages of RKF45 in terms of accuracy and computational efficiency for solving complex systems of ODEs.

2. Introduction

In this section, we overview the general formula of the system of first-order ODEs and the RKF45 method.

2.1 System of First-order Differential Equations (ODEs)

A single first-order ODEs has the form as below [8].

$$\frac{dx}{dt} = f(t,x) \tag{1}$$

With x is the dependent variale, t is the independent variable and $f(t,x)$ is a function that defines how x changes with time. There is only one pair of ordinary differential equations in this problem, making it a system of ODEs. A system of first-order ODEs extends this idea to multiple dependent variables. The general form of an n-dimensional system is:

$$\begin{aligned} \frac{dx_1}{dt} &= f_1(t, x_1, x_2, \ldots, x_n) \\ \frac{dx_2}{dt} &= f_2(t, x_1, x_2, \ldots, x_n) \\ &\vdots \\ \frac{dx_n}{dt} &= f_n(t, x_1, x_2, \ldots, x_n) \end{aligned} \tag{2}$$

Where $x_1, x_2, \ldots, x_n$ are dependent variables that change over time and $f_1, f_2, \ldots, f_n$ are functions that define the rate of change of each variable. This system is coupled if at least one equation depends on multiple variables. The first-order system of ODEs has a wide range of applications across various fields but for this project, we will consider a simple RL circuit as the system of first-order ODEs. The series RL Circuit can be analysed using the first-order system of ODEs. RL circuit consists of electrical current I (in amperes), inductance L (in henrys), resistance R (in ohms), and electromotive force $E(t)$ (in volts), as shown in **Fig. 1**.

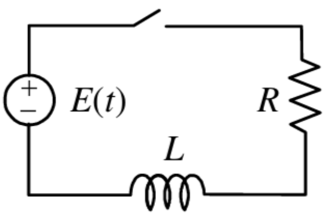

Fig. 1 *A simple RL-electrical circuit* [9]

It is assumed that R and L are constant. Furthermore, the fundamental relationship between the current and the charge q (measured in coulombs) is expressed as follows. By Kirchhoff's Voltage Law, the sum of voltages in a closed loop is equal to the applied voltage [10]:

$$L\frac{di}{dt}+Ri=E(t) \tag{3}$$

This equation describes how current changes due to inductance L and resistance R. The charge q equation is:

$$i=\frac{dq}{dt} \tag{4}$$

Substituting into Kirchhoff's equation:

$$L\frac{di}{dt}\left(\frac{dq}{dt}\right)+R\left(\frac{dq}{dt}\right)=E(t) \tag{5}$$

Rewriting as two first-order ODEs:
Charge equation:

$$\frac{dq}{dt}=i \tag{6}$$

Current equation:

$$\frac{di}{dt}=\frac{E(t)-Ri}{L} \tag{7}$$

Finally, this matches the general form:

$$\frac{dx_1}{dt}=f_1(t,x_1,x_2,...,x_n) \tag{8}$$

$$\frac{dx_2}{dt}=f_2(t,x_1,x_2,...,x_n) \tag{9}$$

With $x_1=q, x_2=i,\ f_1(t,q,i)=i$ and $f_2(t,q,i)=\dfrac{E(t)-R_i}{L}$. This confirms that the RL circuit is modelled as a system of first-order ODEs.

2.2 Fehlberg Runge-Kutta (RKF45)

RKF45 is a high-accuracy one-step numerical method that is part of the Runge-Kutta family, specifically of order four, but it offers accuracy up to order five [9]. The general formula of the RKF45 method is as follows:

$$\begin{aligned} q_{k+1}&=q_k+\frac{16}{135}k_{1q}+\frac{6656}{12825}k_{3q}+\frac{28561}{56430}k_{4q}+\frac{9}{50}k_{5q}+\frac{2}{55}k_{6q} \\ i_{k+1}&=q_k+\frac{16}{135}k_{1i}+\frac{6656}{12825}k_{3i}+\frac{28561}{56430}k_{4i}+\frac{9}{50}k_{5i}+\frac{2}{55}k_{6i} \end{aligned} \tag{10}$$

with $k=0,1,2,3,...$ where:

For charge:

$$
\begin{aligned}
k_{1q} &= h \cdot i_k \\
k_{2q} &= h \cdot \left(i_k + \frac{k_{1i}}{4} \right) \\
k_{3q} &= h \cdot \left(i_k + \frac{3k_{2i}}{8} \right) \\
k_{4q} &= h \cdot \left(i_k + \frac{12k_{3i}}{13} \right) \\
k_{5q} &= h \cdot \left(i_k + k_{4i} \right) \\
k_{6q} &= h \cdot \left(i_k + \frac{k_{5i}}{2} \right)
\end{aligned}
\tag{11}
$$

For current:

$$
\begin{aligned}
k_{1i} &= h \cdot \frac{E(t) - Ri_k}{L} \\
k_{2i} &= h \cdot \frac{E\left(t + \frac{h}{4} \right) - R\left(i_k + \frac{k_{1i}}{4} \right)}{L} \\
k_{3i} &= h \cdot \frac{E\left(t + \frac{3h}{8} \right) - R\left(i_k + \frac{3k_{2i}}{8} \right)}{L} \\
k_{4i} &= h \cdot \frac{E\left(t + \frac{12h}{13} \right) - R\left(i_k + \frac{12k_{3i}}{13} \right)}{L} \\
k_{5i} &= h \cdot \frac{E\left(t + h \right) - R\left(i_k + k_{4i} \right)}{L} \\
k_{6i} &= h \cdot \frac{E\left(t + h \right) - R\left(i_k + \frac{k_{5i}}{2} \right)}{L}
\end{aligned}
\tag{12}
$$

3. Results and Finding

In this section, the results obtained for the system of ODEs using RKF45 were discussed. The results are obtained using RKF45 are compared with the RK4 method from the [8]. Applying and using the RKF45 method, are two examples of solving the system of ODEs, where we take a constant voltage in the first example and a sinusoidal voltage in the second example.

3.1 RL Circuit with a Constant Voltage

[9] solved problems related to the system of first-order ODEs using the RK4 method. In this project, we want to solve the same example which was to involve the constant voltage using the RKF45 method and compare the results. Firstly, we considered that $R = 15$, $E(t) = 120\,volts$, $L = 3\,henrys$, and the initial conditions $q = 0$ and $i = 0$ when $t = 0$. By using the RKF45 method, we want to find the value of i and q for $0 \le t \le 5$ with $\Delta t = 0.25$.

Parameters [9]:

$$R = 15,$$
$$E(t) = 120\, volts,$$
$$L = 3\, henrys,$$
$$h = 0.25 \quad (13)$$

Consider the following system of first-order ODEs for RL circuit [9]:

$$\frac{dq}{dt} = i = f(t, q, i), \quad (14)$$

$$\frac{di}{dt} = \frac{E(t) - Ri}{L} = \frac{120 - 15i}{3} = 40 - 5i = g(t, q, i) \quad (15)$$

The exact solution for the charge q, and current i are given as follows.

$$q(t) = \frac{8}{5}(e^{-5t} - 1) + 8t,$$
$$i(t) = 8(1 - e^{-5t}) \quad (16)$$

Table 1 The numerical solution and absolute error for the charge in RL Circuit with a Constant Voltage

No.	t	Exact Solution	RKF45	|Error|, |Exact–RKF45|	RK4	|Error|, |Exact – RK4|
0	0	0	0	0	0	0
1	0.25	0.8584	0.9949	0.1365	0.8919	0.0335
2	0.50	2.5313	2.4517	0.0796	2.5512	0.0199
3	0.75	4.4376	4.1221	0.3155	4.4465	0.0089
4	1.00	6.4108	5.8915	0.5193	6.4143	0.0035
5	1.25	8.4031	7.7067	0.6964	8.4044	0.0013
6	1.50	10.4009	9.5430	0.8579	10.4014	0.0004
7	1.75	12.4003	11.3891	1.0112	12.4004	0.0001
8	2.00	14.4001	13.2397	1.1604	14.4001	0.0000
9	2.25	16.4000	15.0925	1.3075	16.4000	0.0000
10	2.50	18.4000	16.9462	1.4538	18.4000	0.0000
11	2.75	20.4000	18.8003	1.5997	20.4000	0.0000
12	3.00	22.4000	20.6547	1.7453	22.4000	0.0000
13	3.25	24.4000	22.5092	1.8908	24.4000	0.0000
14	3.50	26.4000	24.3637	2.0363	26.4000	0.0000
15	3.75	28.4000	26.2182	2.1818	28.4000	0.0000
16	4.00	30.4000	28.0727	2.3273	30.4000	0.0000
17	4.25	32.4000	29.9273	2.4727	32.4000	0.0000
18	4.50	34.4000	31.7818	2.6182	34.4000	0.0000
19	4.75	36.4000	33.6364	2.7636	36.4000	0.0000
20	5.00	38.4000	35.4909	2.9091	38.4000	0.0000

Table 2 The numerical solution and absolute error for the current in RL Circuit with a Constant Voltage

No.	t	Exact Solution	RKF45	|Error|, |Exact–RKF45|	RK4	|Error|, |Exact – RK4|
0	0	0	0	0	0	0
1	0.25	5.7080	4.2981	1.4099	5.5404	0.1676
2	0.50	7.3433	6.2870	1.0563	7.2438	0.0995
3	0.75	7.8119	7.2073	0.6046	7.7675	0.0444
4	1.00	7.9461	7.6332	0.3129	7.9285	0.0176
5	1.25	7.9846	7.8303	0.1543	7.9780	0.0066
6	1.50	7.9956	7.9215	0.0741	7.9932	0.0024
7	1.75	7.9987	7.9637	0.0350	7.9979	0.0008
8	2.00	7.9996	7.9832	0.0164	7.9994	0.0002
9	2.25	7.9999	7.9922	0.0077	7.9998	0.0001
10	2.50	8.0000	7.9964	0.0036	7.9999	0.0000
11	2.75	8.0000	7.9983	0.0017	8.0000	0.0000
12	3.00	8.0000	7.9999	0.0001	8.0000	0.0000
13	3.25	8.0000	8.0000	0.0000	8.0000	0.0000
14	3.50	8.0000	8.0000	0.0000	8.0000	0.0000
15	3.75	8.0000	8.0000	0.0000	8.0000	0.0000
16	4.00	8.0000	8.0000	0.0000	8.0000	0.0000
17	4.25	8.0000	8.0000	0.0000	8.0000	0.0000
18	4.50	8.0000	8.0000	0.0000	8.0000	0.0000
19	4.75	8.0000	8.0000	0.0000	8.0000	0.0000
20	5.00	8.0000	8.0000	0.0000	8.0000	0.0000

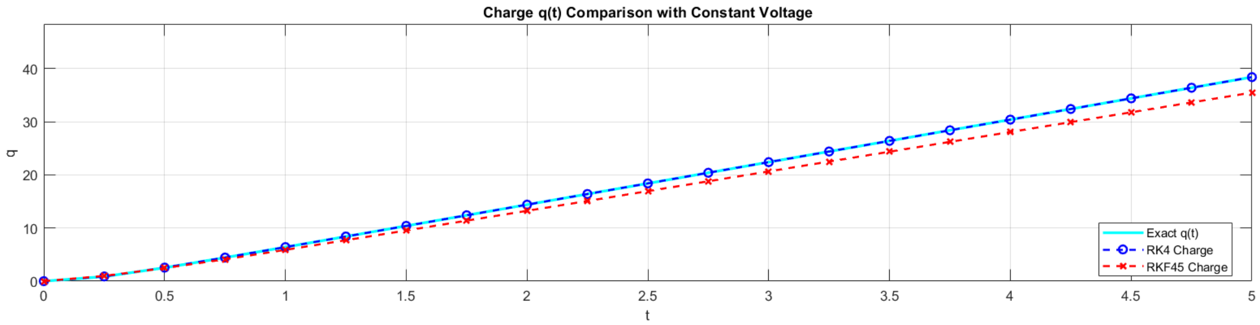

Fig. 2 Graph of the exact solution, RKF45, and RK4 for the charge in RL Circuit with a Constant Voltage

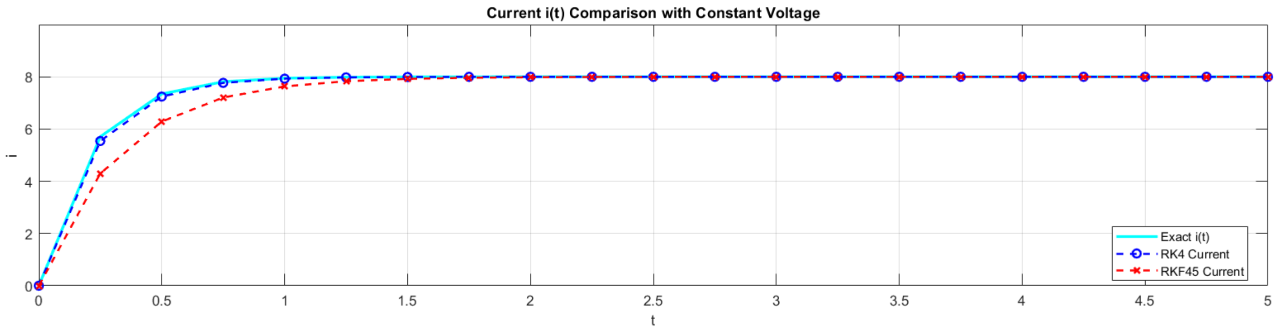

Fig. 3 Graph of the exact solution, RKF45, and RK4 for the current in RL Circuit with a Constant Voltage

Fig. 2 presents a comparison of the exact solution for charge with numerical approximations using the RK4 and RKF45 methods. Observing the results, the RK4 method closely follows the exact solution, indicating a high level of accuracy. The RKF45 method, on the other hand, shows a slight deviation in the early stages but gradually converges with the exact solution as time progresses. Fig. 3 displays the behaviour of the current under a sinusoidal voltage of 120 volts for an RL circuit. The graph shows that the current reaches the steady state solution at amperes as time t goes to infinity. From the figure, it can be observed that RKF45 produces slight deviations from the exact solution, which are further confirmed by the absolute error values in **Table 2.** In comparison, RK4 delivers results that are more accurate and closely match the exact solution, highlighting its effectiveness in numerical computations.

3.2 RL Circuit with a Sinusoidal Voltage

[9] also solved problems related to the system of first-order ODEs using the RK4 method. Now, we will proceed to solve the same examples but with additional sinusoidal voltage using the RKF45 method and compare the results. Firstly, we considered that $R = 15,\ E(t) = 120\sin 2t,\ L = 3\,henrys$, and the initial conditions $q = 0$ and $i = 0$ when $t = 0$. By using the RKF45 method, we want to find the value of i and q for $0 \le t \le 5$ with $\Delta t = 0.25$.

Parameters:

$$\begin{aligned} R &= 15, \\ E(t) &= 120\sin 2t, \\ L &= 3\,henrys, \\ h &= 0.25 \end{aligned} \tag{17}$$

Consider the following system of first-order ODEs [9]:

$$\begin{aligned} \frac{dq}{dt} &= i \\ &= f(t,q,i), \end{aligned} \tag{18}$$

$$\begin{aligned} \frac{di}{dt} &= \frac{E(t) - Ri}{L} \\ &= \frac{120\sin 2t - 15i}{3} \\ &= 40\sin 2t - 5i \\ &= g(t,q,i) \end{aligned} \tag{19}$$

The exact solution for the charge *q*, and current *i* are given as follows.

$$\begin{aligned} q(t) &= 4 - \frac{16}{29}e^{-5t} - \frac{40}{29}\sin 2t - \frac{100}{29}\cos 2t, \\ i(t) &= \frac{80}{29}e^{-5t} + \frac{200}{29}\sin 2t - \frac{80}{29}\cos 2t \end{aligned} \tag{20}$$

Table 3 The numerical solution and absolute error for the charge in RL Circuit with a Sinusoidal Voltage

No.	t	Exact Solution	RKF45	|Error|, |Exact–RKF45|	RK4	|Error|, |Exact – RK4|
0	0	0	0	0	0	0
1	0.25	0.1545	0.0810	0.0735	0.1417	0.0128
2	0.50	0.9310	0.9198	0.0112	0.9262	0.0048
3	0.75	2.3672	2.4288	0.0616	2.3704	0.0032
4	1.00	4.1771	4.2870	0.1099	4.1850	0.0079
5	1.25	5.9360	6.0597	0.1237	5.9456	0.0096
6	1.50	7.2188	7.3212	0.1024	7.2272	0.0084

7	1.75	7.7129	7.7660	0.0531	7.7178	0.0049
8	2.00	7.2978	7.2866	0.0112	7.2980	0.0002
9	2.25	6.0752	6.0010	0.0742	6.0706	0.0046
10	2.50	4.3445	4.2242	0.1203	4.3363	0.0082
11	2.75	2.5295	2.3914	0.1381	2.5196	0.0099
12	3.00	1.0745	0.9512	0.1233	1.0655	0.0090
13	3.25	0.3357	0.2564	0.0793	0.3298	0.0059
14	3.50	0.4941	0.4770	0.0171	0.4927	0.0014
15	3.75	1.5109	1.5590	0.0481	1.5144	0.0035
16	4.00	3.1371	3.2376	0.1005	3.1446	0.0075
17	4.25	4.9745	5.1017	0.1272	4.9843	0.0098
18	4.50	6.5734	6.6950	0.1216	6.5831	0.0103
19	4.75	7.5422	7.6273	0.0851	7.5494	0.0072
20	5.00	7.6437	7.6704	0.0267	7.6467	0.0030

Table 4 The numerical solution and absolute error for the current in RL Circuit with a Sinusoidal Voltage

No.	t	Exact Solution	RKF45	|Error|, |Exact – RKF45|	RK4	|Error|, |Exact – RK4|
0	0	0	0	0	0	0
1	0.25	1.6758	1.9867	0.3109	1.7397	0.0639
2	0.50	4.5392	4.6113	0.0721	4.5633	0.0241
3	0.75	6.7490	6.5508	0.1982	6.7339	0.0151
4	1.00	7.4376	7.0617	0.3759	7.3985	0.0391
5	1.25	6.3428	5.9129	0.4299	6.2956	0.0472
6	1.50	3.7058	3.3437	0.3621	3.6648	0.0410
7	1.75	0.1646	-0.0334	0.1980	0.1408	0.0237
8	2.00	-3.4160	-3.3980	0.0180	-3.4163	0.0002
9	2.25	-6.1600	-5.9290	0.2310	-6.1366	0.0235
10	2.50	-7.3958	-7.0077	0.3881	-7.3543	0.0414
11	2.75	-6.8207	-6.3704	0.4503	-6.7714	0.0493
12	3.00	-4.5757	-4.1733	0.4024	-4.5306	0.0451
13	3.25	-1.2104	-0.9544	0.2560	-1.1806	0.0299
14	3.50	2.4512	2.4982	0.0470	2.4585	0.0073
15	3.75	5.5127	5.3391	0.1736	5.4957	0.0170
16	4.00	7.2245	6.8729	0.3516	7.1873	0.0372
17	4.25	7.1675	6.7240	0.4435	7.1193	0.0483
18	4.50	5.3557	4.9287	0.4270	5.3081	0.0475
19	4.75	2.2325	1.9268	0.3057	2.1974	0.0351
20	5.00	-1.4372	-1.5469	0.1097	-1.4513	0.0141

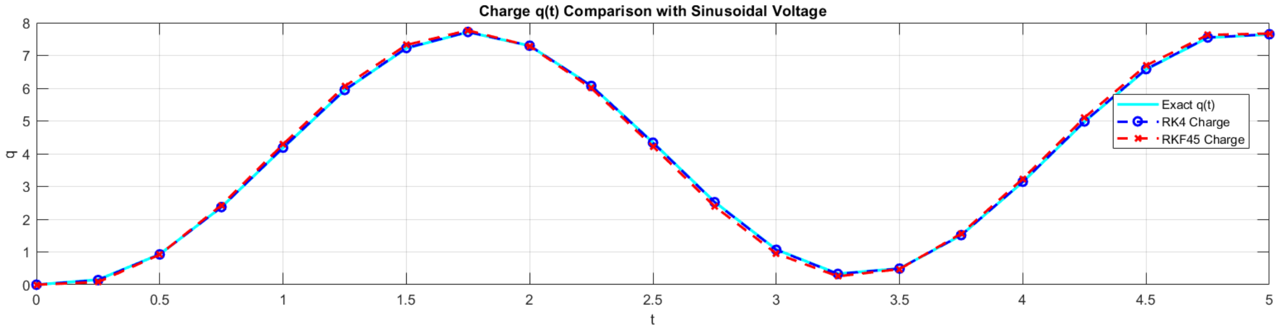

Fig. 4 Graph of the exact solution, RKF45, and RK4 for the charge in RL Circuit with a Sinusoidal Voltage

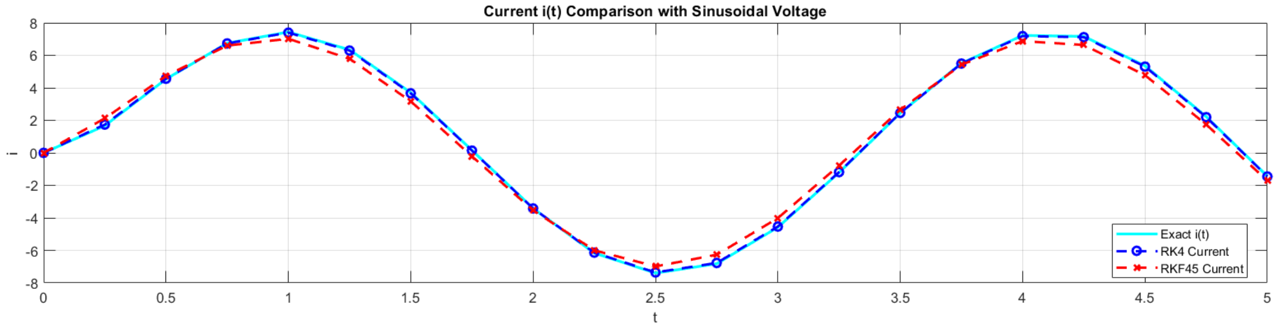

Fig. 5 Graph of the exact solution, RKF45, and RK4 for the current in RL Circuit with a Sinusoidal Voltage

Fig. 4 illustrates the charge at a sinusoidal voltage for a RL circuit. It can be seen that RKF45 showed slightly different from the exact solution as the absolute error calculated in Table 3 but RK4 provided more accurate numerical solutions compared to RKF45. Fig. 4 compares the current obtained from the exact solution with the RK4 and RKF45 numerical methods for the sinusoidal voltage. Similar to the current graph, the RK4 solution aligns closely with the exact charge curve, while the RKF45 method exhibits minor discrepancies.

4. Discussion

From the results presented in the graphs, we can analyse the accuracy and behaviour of the RK4 and RKF45 methods in solving the differential equations for charge and current under constant and sinusoidal voltage. The comparison between RK4 and RKF45 with a fixed step size shows that both methods can approximate the exact solutions for current and charge effectively. However, the RK4 method demonstrates slightly better accuracy in this scenario, particularly in capturing the transient response of the system. The RKF45 method, which is inherently designed for adaptive step-size control, does not show its full potential when restricted to a fixed step size, leading to small deviations from the exact solution. This suggests that if accuracy is the main priority and a fixed step size must be used, RK4 is the more suitable choice. On the other hand, RKF45 would be more beneficial in cases where an adaptive step-size approach is allowed, as it would improve computational efficiency while maintaining accuracy. Therefore, the selection of the numerical method should depend on whether flexibility in step size is permitted and on the accuracy requirements of the system being modelled.

5. Conclusion

The system of ODEs was solved using the RKF45 method for two examples which is the RL circuit with a constant voltage and one with a sinusoidal voltage. The numerical results were compared to the RK4 solutions from [9], with MATLAB used to compute the results, visualize the exact solution graphs, and calculate the absolute errors between RKF45 and RK4. For the RL circuit with a constant voltage, RK4 showed minimal error compared to the exact solution, while RKF45 exhibited a more noticeable deviation. Similarly, for the RL circuit with a sinusoidal voltage, RK4 was slightly more accurate than RKF45. Overall, RK4 was found to provide more precise results, requiring less computational work. However, RKF45's adaptive step size feature offers improved accuracy for problems with rapid changes, despite slightly larger errors in this study.

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Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design:*** *Nur Afiqah Azarani, Azila Md Sudin;* ***data collection:*** *Nur Afiqah Azarani;* ***analysis and interpretation of results:*** *Nur Afiqah Azarani;* ***draft manuscript preparation:*** *Nur Afiqah Azarani, Azila Md Sudin. All authors reviewed the results and approved the final version of the manuscript.*

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