ENHANCED KNOWLEDGE IN SCIENCES AND TECHNOLOGY
e-ISSN: 2773-6385

EKST

Vol. 5 No. 1 (2025) 1-10
https://publisher.uthm.edu.my/periodicals/index.php/ekst

Effects of Predator's Consumption on Crop-Pest-Predator Models Subject to Pest Harvesting with Bifurcation Analysis

Liu Qiu Yan[1], Hamizah Mohd Safuan[1]*

[1] *Department of Mathematics and Statistics, Faculty of Applied Sciences and Technology, UTHM Kampus Cawangan Pagoh, Hab Pendidikan Tinggi Pagoh, KM 1, Jalan Panchor, 86400 Pagoh, Muar, Johor, MALAYSIA.*

*Corresponding Author: hamizahs@uthm.edu.my
DOI: https://doi.org/10.30880/ekst.2025.05.01.001

Article Info

Received: 30 December 2024
Accepted: 16 January 2025
Available online: 30 July 2025

Keywords

Crop-Pest-Predator System, Consumption Rate of Pests by Predators, Pest Harvesting, Bifurcation Analysis

Abstract

Mathematical modelling translates real-world scenarios into mathematical expressions, such as equations, to represent the dynamics of a system. This approach enhances the accuracy of predictions regarding system behaviour. In this study, ecological model consists of crop, pest, and predator populations is utilised to examine how dynamic interactions impact ecosystem stability. The model considers pests consuming crops and predators preying on pests. This study focuses on the dynamics of a crop-pest-predator system which utilises two ecological models, one with and one without pest harvesting control. The consumption rate of pests by predators serves as the main bifurcation parameter, and both models are analysed to determine their respective dynamics. The stability of steady states and equilibrium points is evaluated in each model through numerical and bifurcation analysis. Numerical and graphical analyses are applied to explore the effects of varying consumption rates of pests on the dynamics and stability of the system. Excessive rate of consumption rates of pests by predators may lead to oscillatory dynamics. The model with the pest harvesting control demonstrates a broader range of stability, highlighting its potential to enhance the stability of the ecological system. However, the findings also highlight the importance of maintaining balance, as excessive control measures can disrupt the ecosystem's equilibrium. These insights contribute to the development of more effective and resilient agricultural systems that optimise the pest management strategies.

1. Introduction

Food crop production is a critical component of global food security and human health, but it continues to face challenges from crop pests, diseases, and many environmental conditions [1]. Agricultural pests, including insects and mites, are significant contributors to considerable declines in crop productivity, resulting in extensive economic harm [2]. Mathematical models represent real-world ecological systems through equations that characterise their behaviour. These models are extremely helpful for understanding complex dynamics, such as prey-predator relationships. The Lotka-Volterra model is widely used to demonstrate systems where predators control the prey population, such as agricultural pests [3]. The impact of crop pest populations on the yield of valued crops has been emphasised in earlier studies [4]. In agriculture, pest control is crucial, with the prey-predator system being a common method used to manage pest populations.

[5] studied how changes in parameter value influence the dynamic behaviour of the crop, pest, and predator populations. Similarly, [6] developed a prey-predator model that incorporates susceptible prey, infected prey, and predator populations. The primary objective of their investigation was to examine the control strategies for the management of the prey-predator system, which is primarily concerned with the reduction of the pest population. According to previous research, [7] focused on the ecological model to investigate the interaction between pest and predator populations.

The dynamic behaviour of ecological model is influenced by various factors. [8] found that the dynamics of the prey-predator model are largely determined by the rate at which predators consume prey. The consumption rate of pests by predators is the rate of the predator consuming the pests. It acts as a key variable in the stability of the system. Pest harvesting, which involves the removal of pest from the population, is another factor that can significantly affect the dynamics and stability of the ecosystem. Furthermore, [9] proposed a prey-predator model with disease in pests, including optimal pesticide implementation and pest-harvesting. The impact of prey harvesting on the Leslie-Gower pest-predator model was studied by [10].

In this study, the model proposed in previous research [8] was modified by incorporating the effects of pest harvesting. A bifurcation analysis was conducted on two models to explore the dynamic behaviour of crop, pest and predator populations. Bifurcation theory is the mathematical study of qualitative changes in system behaviour as key parameters are changed, such as shifts from a stable state to an unstable state or the formation of oscillations. This study uses mathematical software such as MATLAB and Maple to perform bifurcation analysis to determine specific threshold values for predator consumption rate and effects of pest harvesting that influence system stability.

2. Research Methodology

2.1 Mathematical Models

This study mainly focused on the mathematical model that consists of the interaction of crops, pests, and predators to investigate the dynamics of a system. The model developed by [8] is considered the primary reference for conducting the stability analysis in this study. In this study, the model in [8] is extended to consider pest harvesting that could affect the consumption rate of pests by predators. The crop-pest-predator model without pest-harvesting is based on the system proposed by [8]:

$$\begin{aligned}
\frac{dx}{dt} &= rx\left(1-\frac{x}{k}\right)-\frac{Rxy}{a+x}-qEx,\\
\frac{dy}{dt} &= \frac{LRxy}{a+x}-\mu y-\frac{gyz}{b+y},\\
\frac{dz}{dt} &= \frac{mgyz}{b+y}-ez
\end{aligned} \tag{1}$$

where x, y, and z represent the populations of crop, pest, and predator, respectively.

In the absence of pests and harvesting, crops are presumed to grow at a logistic rate, defined by a growth rate r and a carrying capacity k. The consumption of crops by pests is represented by a Holling Type II functional response, which measures the rate of pest consumption of crops. This is characterised by the conservation rate L, the half-saturation constant a, and the consumption rate of crops by pests R, which measures the efficacy of converting consumed crops into pest biomass. Moreover, μ represents the natural death rate of pests. Crop harvesting is also included in the model, denoted by a catchability coefficient q, which indicates the effect of harvesting operations and a harvesting rate, E.

The model implied that pest populations diminish as a result of predation by their predators. The predation of pests by predators is regulated by parameters including the consumption rate of pests by predators denoted by g; b represents the half-saturation constant, m represents the conversion efficiency of consumed pests into predator biomass and e represents the natural mortality rate of predators. This relationship emphasises the importance of predation in the management of pest populations, as the dynamics of pest management and the overall stability of the system can be significantly impacted by increased predation rates, which can considerably reduce pest density.

Previous research has demonstrated that the presence of control variables in dynamic systems has an essential effect on the overall stability and behaviour of the ecosystem [9] . Consequently, the pest population equation has been modified by the incorporation of pest harvesting, which is denoted by pE into the equation (1).

The proposed model is:

$$\begin{aligned}
\frac{dx}{dt} &= rx\left(1-\frac{x}{k}\right)-\frac{Rxy}{a+x}-qEx, \\
\frac{dy}{dt} &= \frac{LRxy}{a+x}-\mu y-\frac{gyz}{b+y}-pEy, \\
\frac{dz}{dt} &= \frac{mgyz}{b+y}-ez
\end{aligned} \tag{2}$$

2.2 Stability Analysis of Crop-Pest-Predator Model Without Pest Harvesting

The equilibrium points of the model are determined by using mathematical software such as MATLAB and Maple. The equilibrium points are obtained in each system, which include trivial equilibrium points, semi trivial equilibrium points and interior equilibrium points. Therefore, the Jacobian matrix has been obtained to conduct the stability analysis of the model. The Jacobian matrix for the equation (1) is:

$$J(x,y,z)=\begin{bmatrix} r-qE-\frac{2rx}{K}-\frac{aRy}{(a+x)^2} & -\frac{Rx}{a+x} & 0 \\ \frac{aLRy}{(a+x)^2} & \frac{LRx}{a+x}-\mu-\frac{bgz}{(b+y)^2} & -\frac{gy}{b+y} \\ 0 & \frac{bmgz}{(b+y)^2} & \frac{mgy}{b+y}-e \end{bmatrix} \tag{3}$$

The eigenvalues of the Jacobian matrix are determined by using the determinant of $(J-\lambda I)$, where λ is the eigenvalue and I is the identity matrix. These eigenvalues represent the stability characteristics of the equilibrium point and can be determined through numerical analysis.

2.3 Stability Analysis of Crop-Pest-Predator Model with Pest Harvesting

The model investigated by [8] has been modified to include pest harvesting in the pest population. The Jacobian matrix is evaluated as in equation (4) after the equilibrium points are obtained:

$$J(x,y,z)=\begin{bmatrix} r-qE-\frac{2rx}{K}-\frac{aRy}{(a+x)^2} & -\frac{Rx}{a+x} & 0 \\ \frac{aLRy}{(a+x)^2} & \frac{LRx}{a+x}-\mu-\frac{bgz}{(b+y)^2}-pE & -\frac{gy}{b+y} \\ 0 & \frac{bmgz}{(b+y)^2} & \frac{mgy}{b+y}-e \end{bmatrix} \tag{4}$$

Hence, the characteristic equation of Jacobian matrix is used to examine the stability characteristics of each equilibrium points. An equilibrium is unstable if there is at least one positive real part in the eigenvalues of the Jacobian matrix, while it is asymptotically stable if all eigenvalues have negative real parts [11].

2.4 Bifurcation Analysis

Bifurcation analysis is a mathematical technique that examines how a dynamical behaviour of a system changes when the key parameters are varied [11]. The consumption rate of pests by predators has been set as the bifurcation parameter in the bifurcation analysis. This method helps identify the critical points where the system's stability shifts. The goal of this study is to investigate the stability of both ecological models when there is a variation of the system's parameter value. The consumption rate of pests by predators was chosen as the bifurcation parameter in this analysis.

3. Results and Discussion

3.1 Numerical Analysis

This study focused on the dynamical behaviour of both models to investigate the effects of consumption rate of pests by predators and the pest harvesting on the models. For this analysis, we assume that the initial populations of crops, pests, and predators are 1.2, 11, and 1 respectively. The parameter values proposed by [8] are utilised to conduct numerical and bifurcation analysis. The set of parameter values is shown in Table 1.

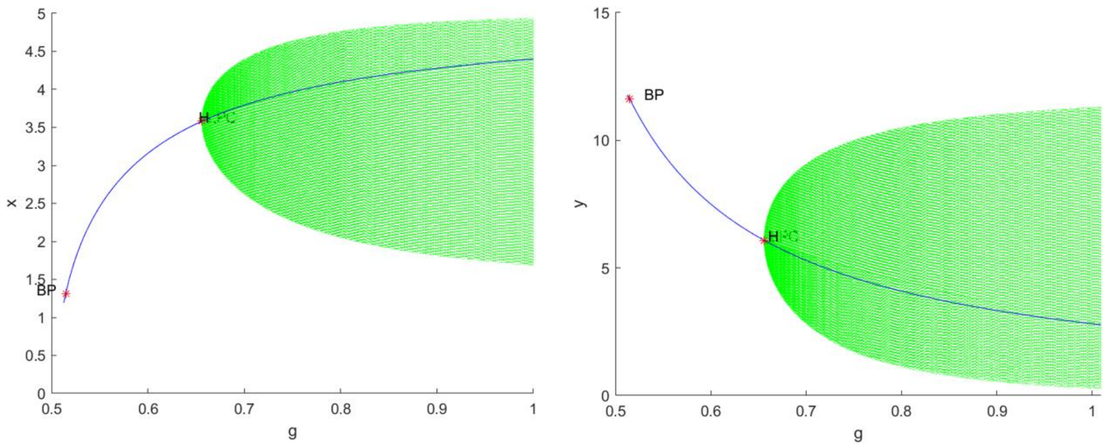
Table 1 *The set of parameter values from [8]*

Parameter	Value
r	2.0
k	5.0
R	0.8
L	0.6
μ	0.1
m	0.5
a	5.0
b	5.0
e	0.18
q	0.1
E	0.01

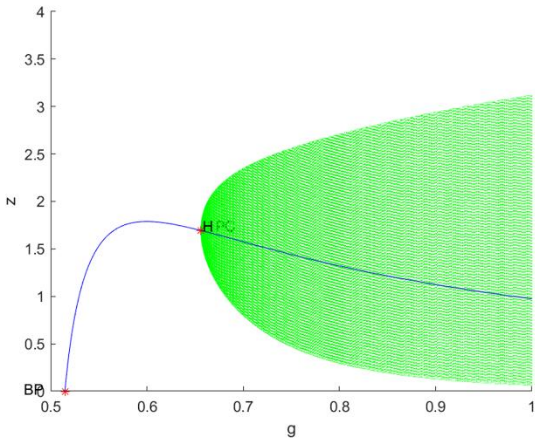
3.2 Crop-pest-predator model without pest harvesting

This study extended the analysis to determine the time series plot with different g values for the mathematical model (1). The consumption rate of pests by predators, g served as the bifurcation parameter in the bifurcation diagram. Therefore, the bifurcation diagrams of the system (1), as shown in Fig. 1, were generated by MATCONT.

Fig. 1 illustrates the bifurcation diagrams of model (1) with respect to the rate of consumption of pests by predators,g. The diagrams indicate the presence of transcritical bifurcation at $g = 0.515$ and a Hopf bifurcation occurs at $g = 0.656$. The equilibrium point of a model (1) may exhibit stable behaviour for $0.5 < g < 0.655$, but becomes unstable when $g > 0.655$. Hence, three different g values are chosen to simulate the time series plots of system (1), which are $g = 0.51$, $g = 0.63$, and $g = 0.90$. These g values were selected to analyse the system behaviour before the transcritical bifurcation point at $g = 0.515$, between the transcritical bifurcation point and the Hopf bifurcation point at $g = 0.656$, and beyond the Hopf bifurcation point. These cases demonstrated how the crop-pest-predator system exhibit distinct dynamic behaviour.



(a) (b)



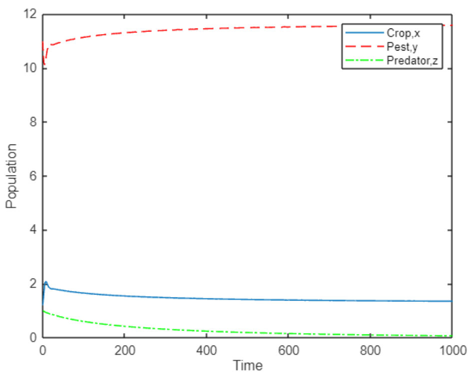
(c)

Fig. 1 *Bifurcation diagrams of the system (1) for (a) crops, x vs g;(b) pests, y vs g;(c) predators, z vs g*

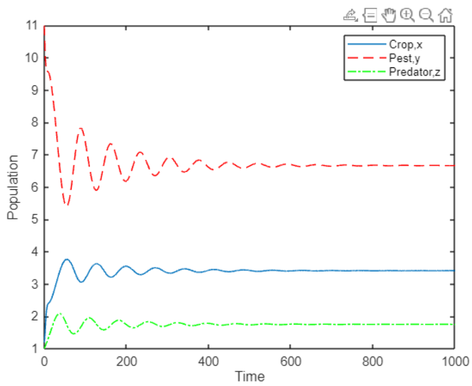
Fig. 2 represents the time series plot for system (1) with three different values of g. In Fig. 2(a), the population dynamics of crops, pests, and predators experience slight changes before achieving equilibrium at $(x, y, z) = (1.316, 11.626, 0)$. The predator population goes extinct due to the low consumption rate of pests by predators, which limits their survival. The pest population starts at a high level and then stabilises at a maximum after a slight increase. This can impact the crop population, which initially rises slightly, then declines and finally stabilises at lower levels due to significant consumption pressure exerted by the pests.

The system undergoes minor oscillations and then reaches equilibrium at $(x, y, z) = (3.413, 6.667, 1.754)$ in Fig. 2(b). The predator population persists, which indicates that the predators able to sustain their presence in the system. As predators apply more pressure on pests, the pest population reduces significantly but stabilises at a lower level compared to the previous case. The crop population experiences slight variations before stabilising at a higher value due to a lower pest population. Therefore, the system (1) has achieved state of equilibrium in which all three populations coexist.

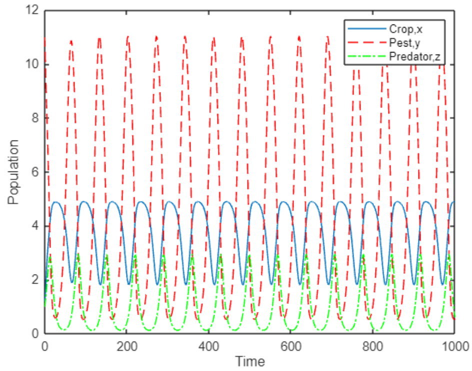
Fig. 2(c) shows the populations move away from the equilibrium points, $(x, y, z) = (4.279, 3.333, 1.124)$ and the system exhibits continuous oscillations over time. Predator populations can survive and regulate the pest population because they consume pests at a high rate. However, the system does not stabilise at a stable equilibrium, but instead oscillates continually because of the dynamic predator-prey interactions that characterise behaviour beyond the Hopf bifurcation point.



(a)



(b)



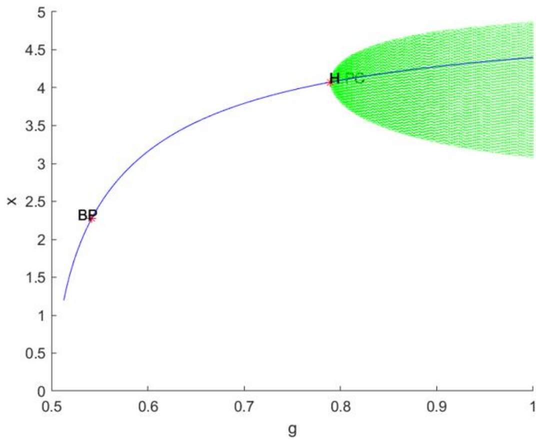
(c)

Fig. 2 *Time series plots of the system (1) for (a)* $g = 0.51$;*(b)* $g = 0.63$;*(c)* $g = 0.90$

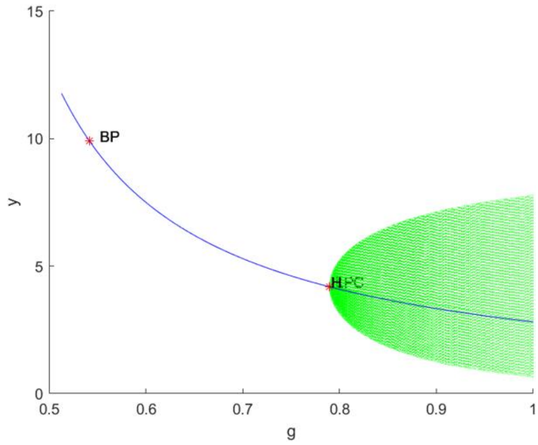
3.3 Crop-Pest-Predator Model with Pest Harvesting

The mathematical model developed by [8] is modified by incorporating pest harvesting control into the pest population to investigate the dynamics of crop-pest-predator model. The bifurcation parameter of this analysis is set to be the consumption rate of pests by predators, denoted by g. The parameter values listed in Table 2 were used with the assumption that $p = 5$ for this analysis. The bifurcation diagrams have been obtained using MATCONT software.

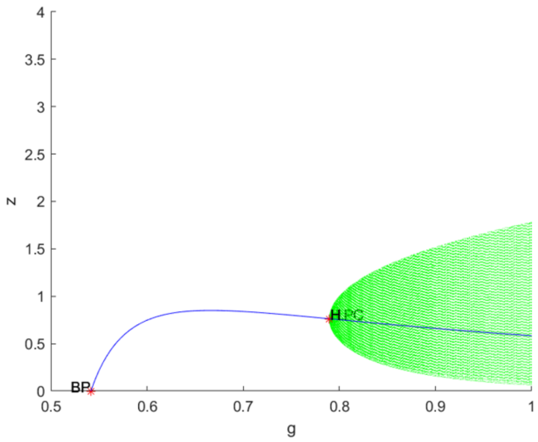
Fig. 3 demonstrates the bifurcation diagrams of the model (2), which includes the pest harvesting controls to the pest population with respect to the consumption rate of pests by predators, g. The bifurcation points in model (2) occur at higher values of g compared to the previous model (1). In particular, the transcritical bifurcation occurs at $g = 0.542$, whereas the Hopf bifurcation occurs at $g = 0.790$. This could indicate that the stability thresholds are influenced by the effect of pest harvesting in the system. The presence of pest harvesting controls appears to delay the appearance of both the transcritical and Hopf bifurcations. Therefore, the system (2) maintains stability across a wider range of g values. The values of $g = 0.51$, $g = 0.63$, and $g = 0.90$ were used to plot the time series graphs.



(a)

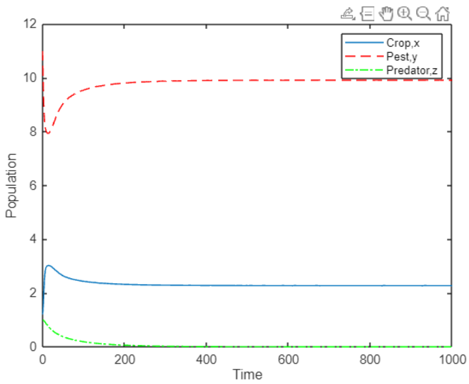


(b)

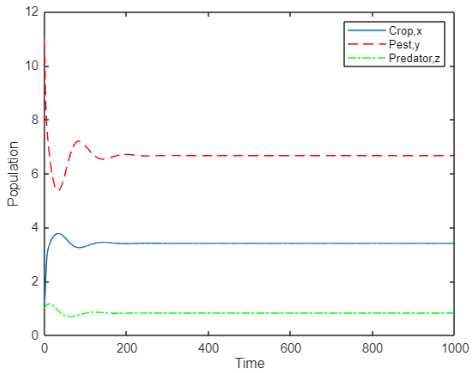


(c)

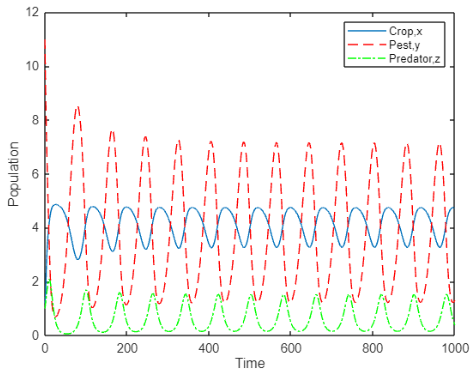
Fig. 3 *Bifurcation diagrams of the system (2) for (a) crops,* $\boldsymbol{x}$ *vs* $\boldsymbol{g}$*;(b) pests,* $\boldsymbol{y}$ *vs* $\boldsymbol{g}$*;(c) predators,* $\boldsymbol{z}$ *vs* $\boldsymbol{g}$



(a)



(b)



(c)

Fig. 4 *Time series plots of the system (2) for (a)* $g = 0.51$*;(b)* $g = 0.63$*;(c)* $g = 0.90$

Fig. 4 illustrates the time series plots of the system (2) with pest harvesting control, corresponding to three distinct values of g. In Fig. 4(a) clearly demonstrates that the system experiences little changes before reaching an equilibrium at $(x, y, z) = (2.273, 9.908, 0)$. The low rate of pest consumption, which makes it difficult for predators to survive, causes the predator population to go extinct. Nonetheless, the crop population stabilises at a higher level compared to the previous model, as pest harvesting effectively reduces the pest population.

Fig. 4(b) shows that the populations of crops, pests, and predators undergo minor oscillations and converge to an equilibrium at $(x, y, z) = (3.413,\ 6.667,\ 0.828)$. The dynamics of system (2) demonstrate a state of relative stability as the populations remain close to their equilibrium points over time. Hence, the crop, pest, and predator populations can coexist in this scenario.

Fig. 4(c) depicts that system (2) exhibits a persistent oscillations cycle with the populations moving away from the equilibrium points, $(x, y, z) = (4.279,\ 3.333,\ 0.661)$. The high consumption rate of pests by predators effectively limits the growth of pest populations. Initially, the pest population declines and then enters a sustained oscillatory cycle, which triggers corresponding oscillations in the crop and predator populations due to their interactions.

4. Conclusion

This study mainly focused on the interactions between crop-pest-predator models with variable consumption rates of pests by predators and pest harvesting. Both mathematical models were analysed using numerical and stability techniques to explore how changes in the key parameters affect the overall system dynamics. The first model is the system proposed by [8], while the second model incorporated pest harvesting as an additional control measure.

The consumption rate of pests by predators, g plays a vital role in determining the stability of the crop-pest-predator model. When g across a certain threshold, the system shows a transition from stable behaviour to unstable behaviour. According to the results, it can be concluded that the crop-pest-predator model may become stable up to a critical point as predators consume more pests. However, the system may become unstable if the rate of pest consumption by predators exceeds the threshold value. Excessive pest consumption rate by predators may contribute to an unstable state in the system.

The implementation of pest harvesting control, denoted by pE, has a significant impact on the system's stability. When pest harvesting controls are incorporated into the model, the range of stability increases, and the amplitude of oscillations in the populations of both pests and predators decreases. These findings provide insight into the possibility of integrated pest management strategies.

In conclusion, the modified model (2) demonstrates a wider range of stability and delays the occurrence of transcritical and Hopf bifurcation points. Pest harvesting improved system stability and minimised the amplitude of oscillations in crops, pests, and predator populations. These findings help to improve the understanding of how ecological systems respond to changes in predation rates and harvesting operations, with practical implications for agricultural pest management.

Acknowledgement

The authors would like to thank the Faculty of Applied Sciences and Technology, Universiti Tun Hussein Onn Malaysia for its support.

Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design:*** *Liu Qiu Yan;* ***analysis and interpretation of results:*** *Liu Qiu Yan, Hamizah Mohd Safuan;* ***draft manuscript preparation:*** *Liu Qiu Yan, Hamizah Mohd Safuan. All authors reviewed the results and approved the final version of the manuscript.*

Appendix A: MATLAB coding

```
syms x y z r k R L m g a b e q E lambda
mu = evalin(symengine, '`µ`');

eq1 = r*x*(1-x/k)-(R*x*y)/(a+x)-q*E*x
eq2 = (L*R*x*y)/(a+x)-mu*y-(g*y*z)/(b+y)
eq3 = (m*g*y*z)/(b+y)-e*z

J=jacobian([eq1,eq2,eq3],[x,y,z])
sol=solve([eq1,eq2,eq3],[x,y,z]);
sol1=simplify([sol.x(1),sol.y(1),sol.z(1)])
sol2=simplify([sol.x(2),sol.y(2),sol.z(2)])
sol3=simplify([sol.x(3),sol.y(3),sol.z(3)])
sol4=simplify([sol.x(4),sol.y(4),sol.z(4)])
sol5=simplify([sol.x(5),sol.y(5),sol.z(5)])
```

```
sol6=simplify([sol.x(6),sol.y(6),sol.z(6)])

E1=subs(J,[x,y,z],sol1)
EE1=E1-[lambda 0 0;0 lambda 0;0 0 lambda]
det(EE1)
CharEq1=collect(det(EE1),lambda)
solE1=solve(CharEq1,lambda)

E2=simplify(subs(J,[x,y,z],sol2))
EE2=E2-[lambda 0 0;0 lambda 0;0 0 lambda]
det(EE2)
CharEq3=collect(det(EE2),lambda)
solE22=simplify(solve(CharEq3,lambda))
solE2=solve(CharEq3,lambda)

E3=subs(J,[x,y,z],sol3)
EE3=E3-[lambda 0 0;0 lambda 0;0 0 lambda]
det(EE3)
CharEq3=collect(det(EE3),lambda)
solE3=solve(CharEq3,lambda)

E4=subs(J,[x,y,z],sol4)
EE4=E4-[lambda 0 0;0 lambda 0;0 0 lambda]
det(EE4)
CharEq4=collect(det(EE4),lambda)
solE4=solve(CharEq4,lambda)

E5=subs(J,[x,y,z],sol5)
EE5=E5-[lambda 0 0;0 lambda 0;0 0 lambda]
det(EE5)
CharEq5=collect(det(EE5),lambda)
solE5=solve(CharEq5,lambda)

E6=subs(J,[x,y,z],sol6)
EE6=E6-[lambda 0 0;0 lambda 0;0 0 lambda]
det(EE6)
CharEq6=collect(det(EE6),lambda)
solE6=solve(CharEq6,lambda)

plotODETimeGraphValue()

function plotODETimeGraphValue()
    r = 2.0;
    k = 5.0;
    R = 0.8;
    L = 0.6;
    mu = 0.1;
    g = 0.51;
    m = 0.5;
    a = 5.0;
    b = 5.0;
    e = 0.18;
    q = 0.1;
    E = 0.01;

    function dKdt = odefun(~, K)
        x = K(1);
        y = K(2);
        z = K(3);
        dxdt = r*x*(1-x/k)-(R*x*y)/(a+x)-q*E*x;
        dydt = (L*R*x*y)/(a+x)-mu*y-(g*y*z)/(b+y);
        dzdt = (m*g*y*z)/(b+y)-e*z;
        dKdt = [dxdt; dydt; dzdt];
end

    K0 = [1.2; 11; 1];
    tspan = [0 1000];
    [t, sol] = ode45(@odefun, tspan, K0);
    plot(t, sol(:, 1), '-', t, sol(:, 2), 'r--', t, sol(:, 3), 'g-.');
    legend('Crop,x', 'Pest,y', 'Predator,z');
    xlabel('Time');
    ylabel('Population');
end
```

References

[1] M. Junaid and A. Gokce, "Global Agricultural Losses and Their Causes," *Bulletin of Biological and Allied Sciences Research*, vol. 9, no. 1, p. 66, Feb. 2024, doi: 10.54112/bbasr.v2024i1.66.

[2] S. Sharma, R. Kooner, and R. Arora, "Insect pests and crop losses," in *Breeding Insect Resistant Crops for Sustainable Agriculture*, Springer Singapore, 2017, pp. 45–66. doi: 10.1007/978-981-10-6056-4_2.

[3] A. S. Wusu and O. A. Olabanjo, "Dynamics of sustainable fisheries: a mathematical approach using Lotka-Volterra equations," *Annals of Mathematics and Computer Science*, vol. 19, pp. 59–67, Dec. 2023, doi: 10.56947/amcs.v19.225.

[4] C. M. Oliveira, A. M. Auad, S. M. Mendes, and M. R. Frizzas, "Crop losses and the economic impact of insect pests on Brazilian agriculture," *Crop Protection*, vol. 56, pp. 50–54, Feb. 2014, doi: 10.1016/j.cropro.2013.10.022.

[5] M. Ali, A. Abdullah Naif, and D. Huang, "Prey consumption and functional response of a phytoseiid predator, Neoseiulus womersleyi, feeding on spider mite, Tetranychus macfarlanei," *Journal of Insect Science*, vol. 11, no. 1, p. 167, Nov. 2011, Accessed: Dec. 10, 2024. [Online]. Available: https://doi.org/10.1093/jis/11.1.167

[6] S. Jana and T. K. Kar, "A mathematical study of a prey-predator model in relevance to pest control," *Nonlinear Dyn*, vol. 74, pp. 667–683, Nov. 2013, doi: 10.1007/s11071-013-0996-3.

[7] D. S. Mandal, S. Samanta, A. K. Alzahrani, and J. Chattopadhyay, "Study of a predator-prey model with pest management perspective," *J Biol Syst*, vol. 27, no. 3, pp. 309–336, Sep. 2019, doi: 10.1142/S021833901950013X.

[8] P. Panja, "Dynamics of a crop, pest and predator model in an agricultural system," *Results in Control and Optimization*, vol. 12, Sep. 2023, doi: 10.1016/j.rico.2023.100274.

[9] A. Suryanto and I. Darti, "Dynamics of Leslie-Gower Pest-Predator Model with Disease in Pest Including Pest-Harvesting and Optimal Implementation of Pesticide," *Int J Math Math Sci*, vol. 2019, p. 9, 2019, doi: 10.1155/2019/5079171.

[10] A. S. Purnomo, I. Darti, and A. Suryanto, "Dynamics of eco-epidemiological model with harvesting," in *AIP Conference Proceedings*, American Institute of Physics Inc., Dec. 2017. doi: 10.1063/1.5016652.

[11] D. G. Zill and M. R. Cullen, *Differential equations with boundary-value problems*. Cengage Learning, 2009.