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Differential Transform Method for Solving System of Linear Ordinary Differential Equations

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Abstract

Systems of ordinary differential equations (ODEs) are integral to modelling phenomena in engineering, science, and technology. This study aims to explore and apply the differential transform method (DTM) for solving systems of linear ODEs. DTM is a semi-analytical method that has gained popularity due to its simplicity and adaptability in handling complex systems and higher-order equations. This method simplifies complex problems by transforming differential equations into recursive relations. The results obtained through the DTM are then compared with the exact solutions derived using the Sawi transform method (STM) from previous research. STM, an analytical method, provides exact solutions and serves as a benchmark in this research due to its ability to yield precise values. This makes it an ideal reference for evaluating the accuracy of the DTM in solving such systems. Two numerical problems, which are system of first-order linear ODEs and system of second-order linear ODEs are solved using DTM. MATLAB was utilized for numerical implementation and graphical visualization. The software's user-friendly interface, combined with its powerful mathematical tools and efficient computational capabilities, makes it an ideal platform for solving such systems and visualizing the results effectively. Additionally, the systems is solved with various truncation orders to evaluate their impact on the accuracy of DTM. The findings emphasize that increasing the truncation order enhances the accuracy of DTM solutions, closely aligning them with the exact solutions. This comparative study underscores the potential of DTM as a reliable and efficient semi-analytical method for solving systems of linear ODEs in various applied fields.

1. Introduction

Numerous scientific and technological problems are demonstrated mathematically by systems of ordinary differential equations (ODEs) [1]. Such systems are prevalent across various fields, including chemical, ecological, biological, and engineering applications. For example, problems such as deflection in curved beams, three-layered beam analysis, aircraft control in space, chemical reaction circuit, and others [2]. Systems of ODEs is gaining increasing importance across various fields as advancements in technology and lifestyle. System of ODEs is a simultaneous set of equations that involves two or more dependent variables which depend on one independent variable. They are from differential equations that are classified as ODEs, partial differential equations (PDEs), fractional differential equations (FDEs), and more. Moreover, ODEs come in various forms, such as ODEs with constant coefficients and ODEs with variable coefficients. It also comprises both linear and nonlinear ODEs [3].

This study focuses mainly on solving systems of linear ODEs of the first and second order. Consequently, various methods for solving these equations have been developed. One of the methods employed for solving such systems is the differential transform method (DTM).

DTM is recognized as the most effective and powerful tool for solving various types of differential equations [4]. It is recognized as one of the top ten techniques for solving both linear and nonlinear problems [5]. Previous studies have confirmed its effectiveness in addressing a wide range of problems, with most research affirming that this method is reliable, efficient, and widely applicable for solving any type of equation [6]. Initially, DTM was introduced by Zhou in 1986 to solve initial value problems (IVPs) in electrical circuits. This semi-analytical method encompasses numerical and analytical approaches in solving integral, ordinary, partial, and differential equation systems [7]. It relies on computing the coefficients of the Taylor series representing the solution to the problem. By employing iteration, it derives the analytical Taylor series solution for differential equations and offers an approximate solution through a finite Taylor series [8]. Besides, DTM has been recognized as the best as it is free from rounding off errors and minimizes computer power [9].

However, DTM can become computationally expensive for higher-order terms and may be less accurate over large time spans or regions [5]. Instead, it generally provides a precise approximation of the actual solution within a small region. Besides, it also relies on a truncated series solution [10]. This truncated solution may not accurately reflect the actual behaviour of the problem. The truncation order significantly affects the accuracy of the DTM results, with higher truncation orders yielding greater accuracy and vice versa.

The results from DTM then compared with the exact solution derived using Sawi transform method (STM). STM, an analytical method, provides exact solutions and serves as a benchmark in this research due to its ability to yield precise values. This makes it an ideal reference for evaluating the accuracy of the DTM in solving such systems. STM is a relatively newer approach and is a useful and good tool for solving ODEs and partial differential equations (PDEs) [11]. This method was proposed by [12] to facilitate the process of solving ODEs and PDEs in the time domain. It can solve linear systems of ODEs, as in [3], and nonlinear systems of ODEs, as in [13]. Moreover, it can also solve systems of ODEs with constant coefficients, as proved by [12], and systems of ODEs with variable coefficients, as introduced by [14]. The STM is derived as a type of integral transformation originating from the Fourier integral [3]. This transform provides an analytical and exact solution to the problem without complicated calculations [14].

The primary objective of this research is to explore DTM for solving systems of linear ODEs. It will solve these systems using DTM to evaluate its effectiveness in obtaining approximations of the solutions. Additionally, the study will compare the results with the exact solution from STM from previous research. Besides, the study aims to analyse the performance of DTM in solving systems of linear ODEs at various truncation orders. This comprehensive approach seeks to analyse the strengths and limitations of DTM, offering valuable insights into its applicability and effectiveness in solving ODEs systems.

2. Research Methodology

This study focuses on developing and applying DTM for solving systems of ODEs. A brief explanation of STM, as it is used to obtain the exact solution. This study emphasizes solving first and second order systems of linear ODEs. The methodology involves theoretical formulation, computational implementation, and comparative analysis.

2.1 System of Ordinary Differential Equations (ODEs)

The general form of first order system of ODEs are

$$\begin{gathered} y_1' = f_1(t, y_1, y_2, \dots, y_\rho) \\ y_2' = f_2(t, y_1, y_2, \dots, y_\rho) \\ \vdots \\ y_\rho' = f_\rho(t, y_1, y_2, \dots, y_\rho) \end{gathered} \tag{1}$$

with initial conditions

$$y_1(t_0) = y_{1_0}, y_2(t_0) = y_{2_0}, \dots, y_\rho(t_0) = y_{\rho_0} \tag{2}$$

The general form of second order system of ODEs are

$$\begin{gathered} y_1'' = f_1(t, y_1, y_2, \dots, y_\rho) \\ y_2'' = f_2(t, y_1, y_2, \dots, y_\rho) \\ \vdots \\ y_\rho'' = f_\rho(t, y_1, y_2, \dots, y_\rho) \end{gathered} \tag{3}$$

with initial conditions

$$y_1(t_0) = y_{1_0}, y_2(t_0) = y_{2_0}, \dots, y_\rho(t_0) = y_{\rho_0} \tag{4}$$

$$y_1(t_1) = y_{1_1}, y_2(t_1) = y_{2_1}, \dots, y_\rho(t_1) = y_{\rho_1}$$

where

$$\boldsymbol{y_1(t_1) = y_1'(t_0)} \textbf{ and } \boldsymbol{y_2(t_1) = y_2'(t_0)}$$

2.2 Differential Transform Method (DTM)

According to [4], the transformation of the k^{th} derivative of a function $f(t)$ is defined as

$$Y(k) = \frac{1}{k!}\left[\frac{d^k}{dt^k}y(t)\right]_{t=t_0} \tag{5}$$

where

$Y(k)$ = The transformation function
$y(t)$ = The original function

The differential inverse transformation $Y(k)$ is defined as

$$y(t) = \sum_{k=0}^{\infty} Y(k)(t - t_0)^k \tag{6}$$

Table 1 shows a list of some fundamental operations performed by the differential transformation.

Table 1 Fundamental operations by differential transformation by [4]

No	Original function	Transformed function
1	$y(t) = g(t) \pm h(t)y(t)$	$Y(k) = G(k) \pm H(k)$
2	$y(t) = cg(t)$	$Y(k) = cG(k)$, where c is constant
3	$y(t) = y'(t)$	$Y(k) = (k+1)Y(k+1)$
4	$y(t) = y^m(t)$	$Y(k) = (k+1)(k+2)\cdots(k+m)Y(k+m)$
5	$y(t) = \sin(\alpha t + \beta)$	$Y(k) = \dfrac{\alpha^k}{k!}\sin\left(\dfrac{\pi k}{2} + \beta\right)$

2.3 Sawi Transform Method (STM)

According to [2], the Sawi transform of the function $\omega(t)$, $y \geq 0$ is given by

$$S\{\omega(t)\} = \frac{1}{\sigma^2}\int_0^\infty \omega(t)\, e^{-\left(\frac{1}{\sigma}\right)t} dt = T(\sigma), \quad \sigma > 0 \tag{7}$$

Table 2 shows the Sawi transformation of derivatives of some functions, meanwhile Table 3 presents Sawi transformations and inverse Sawi transformations of some fundamental functions.

Table 2 Sawi transformation of derivatives of some functions by [2]

No	Function of derivatives	Sawi transformation of derivatives
1	$\omega'(t)$	$\dfrac{1}{\sigma}T(\sigma) - \dfrac{1}{\sigma^2}\omega(0)$
2	$\omega''(t)$	$\dfrac{1}{\sigma^2}T(\sigma) - \dfrac{1}{\sigma^3}\omega(0) - \dfrac{1}{\sigma^2}\omega'(0)$
3	$\omega^{(\rho)}(t)$	$\dfrac{1}{\sigma^2}T(\sigma) - \sum_{k=0}^{\rho-1}\left(\dfrac{1}{\sigma}\right)^{\rho-(k-1)}\omega^{(k)}(0)$

Table 3 Sawi transformations and inverse Sawi transformations of some fundamental functions by [2]

No	Fundamental functions		Inverse Sawi transformation	
	$\omega(t)$	$S\{\omega(t)\} = T(\sigma)$	$T(\sigma)$	$\omega(t) = S^{-1}\{T(\sigma)\}$
1	1	$\frac{1}{\sigma}$	$\frac{1}{\sigma}$	1
2	t	1	1	t
3	t^2	2σ	σ	$\frac{t^2}{2!}$
4	e^{at}	$\frac{1}{\sigma(1-a\sigma)}$	$\frac{1}{\sigma(1-a\sigma)}$	e^{at}
5	$sin(at)$	$\frac{a}{1+(a\sigma)^2}$	$\frac{1}{1+(a\sigma)^2}$	$\frac{sin(at)}{a}$
6	$cos(at)$	$\frac{1}{\sigma(1+(a\sigma)^2)}$	$\frac{1}{\sigma(1+(a\sigma)^2)}$	$cos(at)$

3. Result and Discussion

This section discusses the results obtained for solving systems of linear ODEs using DTM. The results, computed with the aid of mathematical software MATLAB R2024b, are compared with the exact solution derived using STM from [3]. Two numerical problems from [3], consisting of system of first and second order linear ODEs, have been solved using DTM.

3.1 Example 1

Consider the following first order system of linear ODEs from [3].

$$\begin{aligned} y_1' &= -y_1 \\ y_2' &= y_1 - 2y_2 \\ y_3' &= 2y_2 \end{aligned} \tag{8}$$

where

$$y_1(0) = 1,\ \ y_2(0) = 0,\ \ y_3(0) = 0 \tag{9}$$

and given the exact solution derived from STM [3]

$$\begin{aligned} y_1(t) &= e^{-t} \\ y_2(t) &= e^{-t} - e^{-2t} \\ y_3(t) &= 1 + e^{-2t} - 2e^{-t} \end{aligned} \tag{10}$$

By using the theorem of DTM from the Table 1, we get

$$\begin{aligned} Y_1(k+1) &= \frac{1}{k+1}[-Y_1(k)] \\ Y_2(k+1) &= \frac{1}{k+1}[Y_1(k) - 2Y_2(k)] \\ Y_3(k+1) &= \frac{1}{k+1}[2Y_2(k)] \end{aligned} \tag{11}$$

with initial conditions

$$Y_1(0) = 1,\ \ Y_2(0) = 0,\ \ Y_3(0) = 0 \tag{12}$$

Substitute the values of $k = 0,1,2,\dots,9$ into (11) using initial conditions (12). The results are shown in Table 4.

Table 4 Differential transformation values of Example 1 for $\boldsymbol{k = 0, 1, 2, \dots, 9}$

k	$Y_1(k+1)$	$Y_2(k+1)$	$Y_3(k+1)$
0	1	0	0
1	-1	1	0
2	$\frac{1}{2}$	$-\frac{3}{2}$	1
3	$-\frac{1}{6}$	$\frac{7}{6}$	-1
4	$\frac{1}{24}$	$-\frac{5}{8}$	$\frac{7}{12}$
5	$-\frac{1}{120}$	$\frac{31}{120}$	$-\frac{1}{4}$
6	$\frac{1}{720}$	$-\frac{7}{80}$	$\frac{31}{360}$
7	$-\frac{1}{5040}$	$\frac{127}{5040}$	$-\frac{1}{40}$
8	$\frac{1}{40320}$	$-\frac{17}{2688}$	$\frac{127}{20160}$
9	$-\frac{1}{362880}$	$\frac{73}{51840}$	$-\frac{17}{12096}$

Then merge the term from Table 4, do the Taylor series until 10^{th} term.

$$
\begin{aligned}
y_1(t) &= 1 - t + \frac{1}{2}t^2 - \frac{1}{6}t^3 + \frac{1}{24}t^4 - \frac{1}{120}t^5 + \frac{1}{720}t^6 - \frac{1}{5040}t^7 + \frac{1}{40320}t^8 - \frac{1}{362880}t^9 + \cdots \\
y_2(t) &= t - \frac{3}{2}t^2 + \frac{7}{6}t^3 - \frac{1}{12}t^4 + \frac{31}{120}t^5 - \frac{7}{80}t^6 + \frac{127}{5040}t^7 - \frac{17}{2688}t^8 + \frac{73}{51840}t^9 + \cdots \\
y_3(t) &= t^2 - t^3 + \frac{7}{12}t^4 - \frac{1}{4}t^5 + \frac{31}{360}t^6 - \frac{1}{40}t^7 + \frac{127}{20160}t^8 - \frac{17}{12096}t^9 + \cdots
\end{aligned}
\tag{13}
$$

The numerical results obtained from DTM are presented in Table 5 to Table 7, along with the absolute error between the exact values from [3] and the DTM. Additionally, DTM is computed at different truncation orders to measure its accuracy in approximating the exact value. Table 5 shows the numerical solution for $y_1(t)$ at 10^{th} and 30^{th} order. In addition, Table 6 shows the numerical solution for $y_2(t)$ at 10^{th} and 30^{th} order.

Table 5 The numerical solution for $\boldsymbol{y_1(t)}$ at $\boldsymbol{10^{th}}$ and $\boldsymbol{30^{th}}$ order

t	Exact Value	DTM		Absolute Error	
		10^{th} order	30^{th} order	|Exact-DTM| 10^{th} Order	|Exact-DTM| 30^{th} Order
0.0	1.000000E+00	1.000000E+00	1.000000E+00	0.000000E+00	0.000000E+00
1.0	3.678794E-01	3.678792E-01	3.678794E-01	2.000000E-07	0.000000E+00
2.0	1.353353E-01	1.350970E-01	1.353353E-01	2.383000E-04	0.000000E+00
3.0	4.978707E-02	3.705357E-02	4.978707E-02	1.273350E-02	0.000000E+00
4.0	1.831564E-02	-1.922399E-01	1.831564E-02	2.105555E-01	0.000000E+00
5.0	6.737947E-03	-1.827105E+00	6.737947E-03	1.833843E+00	0.000000E+00
6.0	2.478752E-03	-1.065714E+01	2.478751E-03	1.065962E+01	1.000000E-09

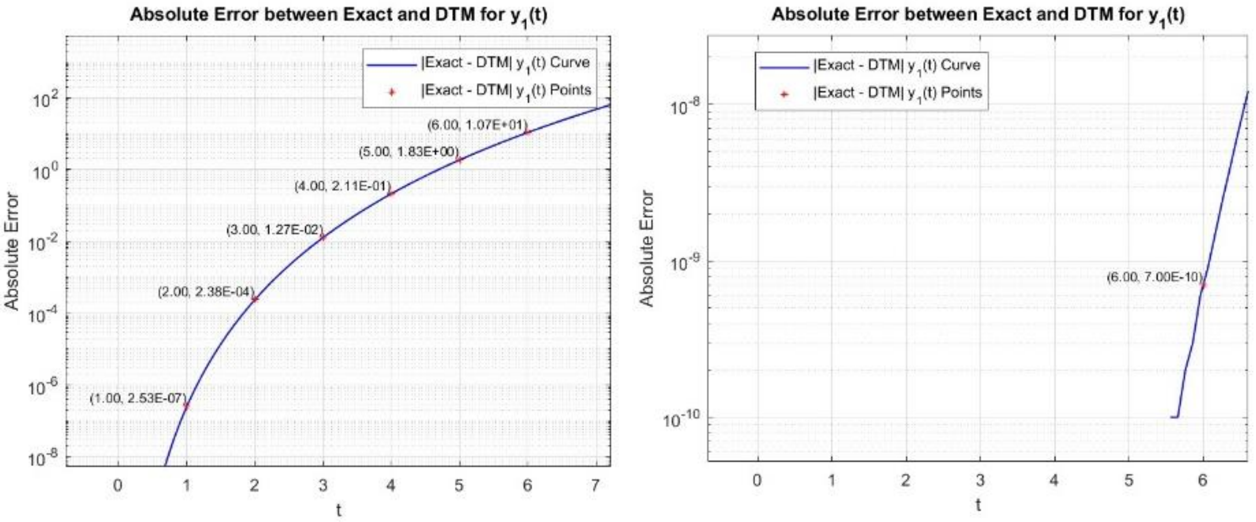
Table 6 The numerical solution for $\boldsymbol{y_2(t)}$ at $\boldsymbol{10^{th}}$ and $\boldsymbol{30^{th}}$ order

t	Exact Value	DTM		Absolute Error	
		10^{th} order	30^{th} order	|Exact-DTM| 10^{th} Order	|Exact-DTM| 30^{th} Order
0.0	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1.0	2.325442E-01	2.327822E-01	2.325442E-01	2.380000E-04	0.000000E+00
2.0	1.170196E-01	3.273369E-01	1.170196E-01	2.103173E-01	0.000000E+00
3.0	4.730832E-02	1.069420E+01	4.730832E-02	1.064689E+01	0.000000E+00
4.0	1.798018E-02	1.683203E+02	1.798388E-02	1.683023E+02	3.700000E-06
5.0	6.692547E-03	1.411318E+03	9.537593E-03	1.411311E+03	2.845046E-03
6.0	2.472608E-03	7.942971E+03	6.460231E-01	7.942969E+03	6.435505E-01

Table 7 The numerical solution for $\boldsymbol{y_3(t)}$ at $\boldsymbol{10^{th}}$ and $\boldsymbol{30^{th}}$ order

t	Exact Value	DTM		Absolute Error	
		10^{th} order	30^{th} order	|Exact-DTM| 10^{th} Order	|Exact-DTM| 30^{th} Order
0.0	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
1.0	3.995764E-01	3.993386E-01	3.995764E-01	2.378000E-04	0.000000E+00
2.0	7.476451E-01	5.375661E-01	7.476451E-01	2.100790E-01	0.000000E+00
3.0	9.029046E-01	-9.731250E+00	9.029046E-01	1.063415E+01	0.000000E+00
4.0	9.637042E-01	-1.671280E+02	9.637005E-01	1.680917E+02	3.700000E-06
5.0	9.865695E-01	-1.408490E+03	9.837245E-01	1.409477E+03	2.845000E-03
6.0	9.950486E-01	-7.931314E+03	3.514982E-01	7.932309E+03	6.435504E-01

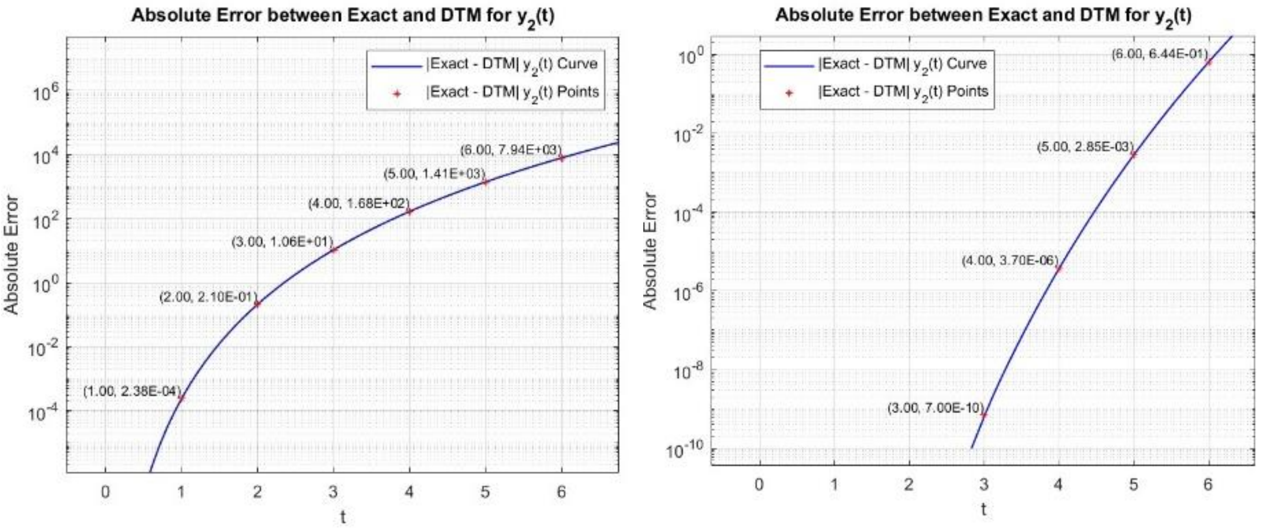
Table 7 shows the numerical solution for $y_3(t)$ at 10^{th} and 30^{th} order. The results are also visualized in graphical form using MATLAB R2024b. Here, Fig. 1 shows the absolute error between the exact value and DTM of $y_1(t)$ at 10^{th} and 30^{th} order.



(a) (b)

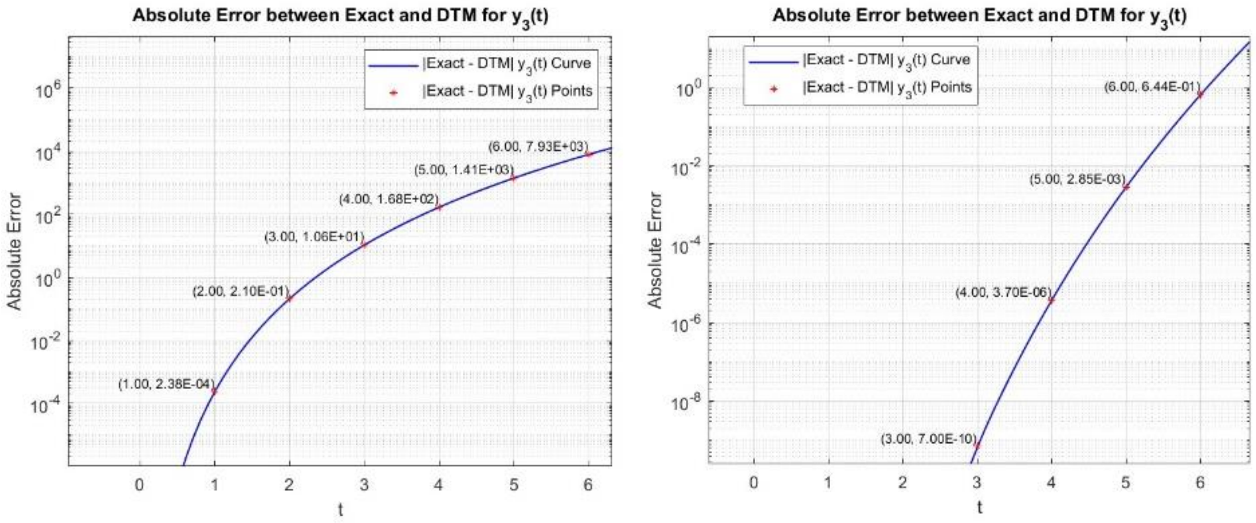
Fig. 1 Absolute error between the exact value and DTM of $\boldsymbol{y_1(t)}$ at (a) $\boldsymbol{10^{th}}$ order; (b) $\boldsymbol{30^{th}}$ order

Fig. 2 shows the absolute error between the exact value and DTM of $y_2(t)$ at 10^{th} and 30^{th} order. In addition, Fig. 3 shows absolute error between the exact value and DTM of $\boldsymbol{y_3(t)}$ at $\mathbf{10^{th}}$ and $\mathbf{30^{th}}$ order.



(a) (b)

Fig. 2 Absolute error between the exact value and DTM of $\boldsymbol{y_2(t)}$ at (a) $\mathbf{10^{th}}$ order; (b) $\mathbf{30^{th}}$ order



(a) (b)

Fig. 3 Absolute error between the exact value and DTM of $\boldsymbol{y_3(t)}$ at (a) $\mathbf{10^{th}}$ order; (b) $\mathbf{30^{th}}$ order

Fig. 1(a), Fig. 2(a), and Fig. 3(a) show the result of $\boldsymbol{y_1(t)}$, $\boldsymbol{y_2(t)}$, and $\boldsymbol{y_3(t)}$ at the $\mathbf{10^{th}}$ orders, respectively. These graphs illustrates that the $\mathbf{10^{th}}$ order approximation shows a progressively increasing error as t increases, with relatively large error magnitudes compared to the higher-order approximation. In contrast, Fig. 1(b), Fig. 2(b), and Fig. 3(b) depict the results of the $\mathbf{30^{th}}$ order approximation, which demonstrate significantly smaller absolute errors.

3.2 Example 2

Consider the following second order system of linear ODEs [3].

$$\begin{gathered} y_1''(t)+5y_1(t)-2y_2(t)=0 \\ -2y_1(t)+2y_2(t)+y_2''(t)=3sin(2t) \end{gathered} \tag{14}$$

with initial condition

$$y_1(0)=0, y_1'(0)=0, y_2(0)=1, y_2'(0)=0 \tag{15}$$

and given the exact solutions derived from STM [3]

$$\begin{aligned} y_1(t) &= \frac{2}{5}\left[cos(t) - cos\left(\sqrt{6}\,t\right)\right] + \frac{4}{5}sin(t) - 2cos(t)sin(t) + \frac{\sqrt{6}}{5}sin\left(\sqrt{6}\,t\right) \\ y_2(t) &= \frac{1}{5}\left[cos\left(\sqrt{6}\,t\right) + 4cos(t) + 8sin(t)\right] - cos(t)sin(t) - \frac{\sqrt{6}}{10}sin\left(\sqrt{6}\,t\right) \end{aligned} \tag{16}$$

By using the theorem of DTM from Table 1, we get

$$\begin{aligned} Y_1(k+2) &= \frac{1}{(k+1)(k+2)}\left[-5Y_1(k) + 2Y_2(k)\right] \\ Y_2(k+2) &= \frac{1}{(k+1)(k+2)}\left[2Y_1(k) - 2Y_2(k) + 3\frac{2^k}{k!}sin\left(\frac{\pi}{2}k\right)\right] \end{aligned} \tag{17}$$

with initial conditions

$$Y_1(0) = 0, Y_1'(0) = 0, Y_2(0) = 1, Y_2'(0) = 0 \tag{18}$$

Substitute the values of $k = 0,1,2,\dots,9$ into (17) using initial conditions (18). The results are shown in Table 8.

Table 8 Differential transformation values of Example 2 for $\boldsymbol{k = 0, 1, 2, \dots, 9}$

k	$Y_1(k+1)$	$Y_2(k+1)$
0	0	1
1	0	0
2	1	-1
3	0	1
4	$-\frac{7}{12}$	$\frac{1}{3}$
5	$\frac{1}{10}$	$-\frac{3}{10}$
6	$\frac{43}{360}$	$-\frac{11}{180}$
7	$-\frac{11}{420}$	$\frac{4}{105}$
8	$-\frac{37}{2880}$	$\frac{13}{2016}$
9	$\frac{29}{10080}$	$-\frac{43}{15120}$

Then merge the term from Table 8, do the Taylor series until $\boldsymbol{10^{th}}$ term.

$$\begin{aligned} y_1(t) &= t^2 - \frac{7}{12}t^4 + \frac{1}{10}t^5 + \frac{43}{360}t^6 - \frac{11}{420}t^7 - \frac{37}{2880}t^8 + \frac{29}{10080}t^9 + \cdots \\ y_2(t) &= 1 - t^2 + t^3 + \frac{1}{3}t^4 - \frac{3}{10}t^5 - \frac{11}{180}t^6 + \frac{4}{105}t^7 + \frac{13}{2016}t^8 - \frac{43}{15120}t^9 + \cdots \end{aligned} \tag{19}$$

The numerical results obtained from DTM are presented in Table 9 and Table 10, along with the absolute error between the exact values from [3] and the DTM. Additionally, DTM is computed at various truncation orders to measure its accuracy in approximating the exact value. Table 9 shows the numerical solution for $y_1(t)$ at 10^{th} and 50^{th} orders. Meanwhile, Table 10 shows the numerical solution for $y_2(t)$ at 10^{th} and 50^{th} order.

Table 9 The numerical solution for $\boldsymbol{y_1(t)}$ at $\boldsymbol{10^{th}}$ and $\boldsymbol{50^{th}}$ order

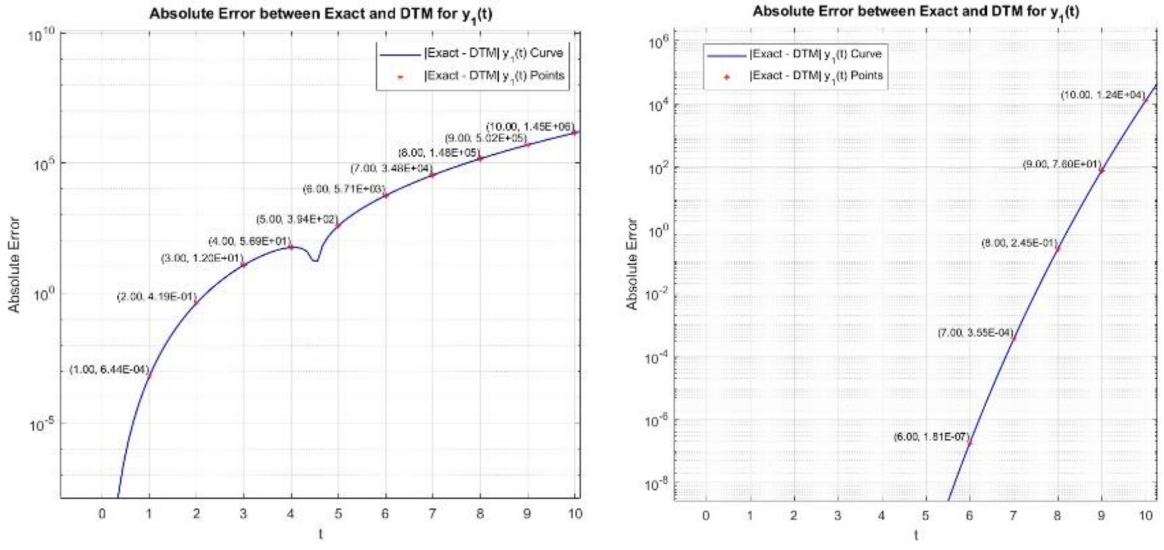
t	Exact Value	DTM		Absolute Error	
		10^{th} order	50^{th} order	|Exact-DTM| 10^{th} Order	|Exact-DTM| 50^{th} Order
0.0	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00

1.0	6.005947E-01	5.999504E-01	6.005947E-01	6.443000E-04	0.000000E+00
2.0	7.621833E-01	3.428571E-01	7.621833E-01	4.193262E-01	0.000000E+00
3.0	2.312369E-01	-1.181652E+01	2.312369E-01	1.204776E+01	0.000000E+00
4.0	-1.662395E+00	-5.856508E+01	-1.662395E+00	5.690269E+01	0.000000E+00
5.0	-6.430871E-01	3.937686E+02	-6.430871E-01	3.944117E+02	0.000000E+00
6.0	1.324634E+00	5.713714E+03	1.324633E+00	5.712389E+03	1.000000E-06
7.0	-5.962937E-01	3.484766E+04	-5.966486E-01	3.484826E+04	3.549000E-04
8.0	1.060152E+00	1.479394E+05	8.147154E-01	1.479383E+05	2.454366E-01
9.0	1.089073E+00	5.019399E+05	-7.489821E+01	5.019388E+05	7.598728E+01
10.0	-2.296860E+00	1.454068E+06	-1.239739E+04	1.454070E+06	1.239509E+04

Table 10 The numerical solution for $y_2(t)$ at 10^{th} and 50^{th} order

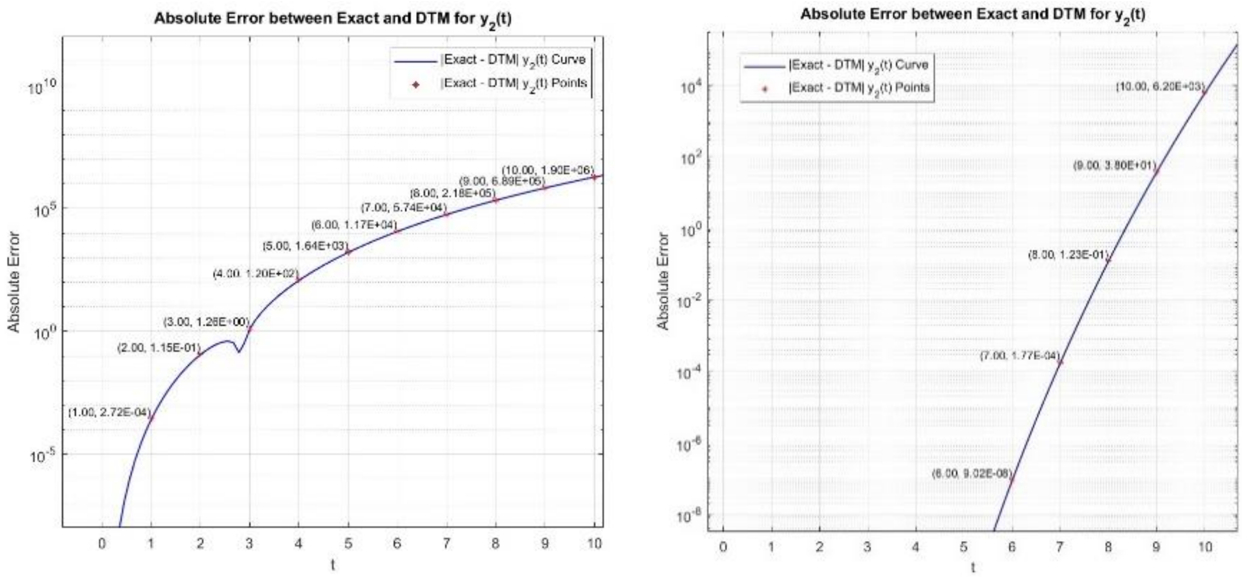
t	Exact Value	DTM		Absolute Error	
		10^{th} order	50^{th} order	|Exact-DTM| 10^{th} Order	|Exact-DTM| 50^{th}Order
0.0	1.000000E+00	1.000000E+00	1.000000E+00	0.000000E+00	0.000000E+00
1.0	1.013650E+00	1.013922E+00	1.013650E+00	2.720000E-04	0.000000E+00
2.0	1.778159E+00	1.893122E+00	1.778159E+00	1.149630E-01	0.000000E+00
3.0	-5.439554E-01	-1.804464E+00	-5.439554E-01	1.260509E+00	0.000000E+00
4.0	-2.325409E+00	-1.219376E+02	-2.325409E+00	1.196122E+02	0.000000E+00
5.0	-7.686217E-01	-1.642448E+03	-7.686217E-01	1.641679E+03	0.000000E+00
6.0	2.755954E-01	-1.173603E+04	2.755955E-01	1.173631E+04	1.000000E-07
7.0	1.375415E+00	-5.735179E+04	1.375592E+00	5.735317E+04	1.770000E-04
8.0	1.591044E+00	-2.176619E+05	1.713747E+00	2.176635E+05	1.227030E-01
9.0	1.195575E-01	-6.893554E+05	3.810718E+01	6.893555E+05	3.798762E+01
10.0	-1.691629E+00	-1.904998E+06	6.194590E+03	1.904996E+06	6.196282E+03

The results are also visualized in graphical form using MATLAB R2024b. Fig. 4 shows the absolute error between the exact value and DTM of $y_1(t)$ at 10^{th} and 50^{th} order.



(a) (b)

Fig. 4 Absolute error between the exact value and DTM of $y_1(t)$ at (a) 10^{th} order; (b) 50^{th} order



(a) (b)

Fig. 5 Absolute error between the exact value and DTM of $\boldsymbol{y_2(t)}$ at (a) $\mathbf{10^{th}}$ order; (b) $\mathbf{50^{th}}$ order

Fig. 5 shows the absolute error between the exact value and DTM of $y_2(t)$ at 10^{th} and 50^{th} order. Fig. 4(a) and Fig. 5(a) show the result of $\boldsymbol{y_1(t)}$ and $\boldsymbol{y_2(t)}$ at the $\mathbf{10^{th}}$ orders, respectively. These graphs illustrates that the $\mathbf{10^{th}}$ order approximation shows a progressively increasing error as t increases, with relatively large error magnitudes compared to the higher-order approximation. In contrast, Fig. 4(b) and Fig. 5(b) depict the results of the $\mathbf{50^{th}}$ order approximation, which demonstrate significantly smaller absolute errors.

4. Discussion

In examples 1 and 2, the DTM provides a numerical solution based on the Taylor series, with errors arising from truncation after a finite number of terms when compared to the exact solution. This is because DTM is a semi-analytical method that provides an approximate solution. However, the accuracy of DTM can be enhanced by incorporating more terms into the series. As the number of terms increases, the DTM solution converges more closely to the exact solution. This method is known as DTM(K), where a higher K results in a better approximation [7]. As K increases, it corresponds to considering more terms in the series, reducing the truncation error and leading to a more accurate solution.

Additionally, DTM offers a simpler and more straightforward approach, as it does not require complex calculations. This method is easy to understand and teach [7]. The DTM works by transforming a differential equation into a series of algebraic equations using a Taylor series expansion, which can be easily computed with recursive formulas. Besides, when dealing with higher-order systems, the DTM is easier to implement, as demonstrated in example 2. DTM simplifies the process by breaking down the problem into recursive steps and using Taylor series expansions, which can be easily computed and extended to higher-order systems.

The problems are solved using MATLAB, a widely preferred tool for handling systems of ODEs. MATLAB is an ideal tool for implementing DTM due to its simpler algorithm, which enables faster coding and computation.

5. Conclusion

This study successfully applied the DTM to solve systems of linear ODEs and compared the results with the exact solutions obtained using the STM from previous research. STM, an analytical method provided exact solutions, serving as a benchmark for evaluating the accuracy of DTM. Two numerical examples were solved using DTM: example 1 involved system of first-order linear ODEs, while example 2 involved system of second-order linear ODEs. MATLAB R2024b was utilized for numerical computations and graphical visualization.

Since DTM is an approximation approach and involves truncation errors, both examples were analysed using different truncation orders. This analysis was conducted to observe how increasing the number of truncated terms improves the accuracy of DTM. The results show that higher truncation orders yield solutions that are more closely aligned with the exact solutions. As the number of terms increases, the absolute error between the exact values and DTM decreases, approaching zero. This demonstrates that the accuracy of the DTM improved with higher truncation orders.

In conclusion, DTM showcases its simplicity and adaptability in addressing complex systems and higher-order equations. It proves to be an effective semi-analytical method that provides highly accurate approximations when an appropriate truncation order is chosen. However, higher-order terms of DTM require more computational work as more iterative steps are needed. Since this study addresses complex systems and involves higher-order equations, truncating to higher-order terms is necessary to improve the accuracy and approach the exact solution. Since we use MATLAB for the computations, the increased computational load is manageable. For educational purposes, the systems involved are typically simple, so the computational work remains appropriate for manual calculations.

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Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design:*** *Anis Eliesa Syareena Yaakub, Noor Azliza Abd Latif;* ***data collection:*** *Noor Azliza Abd Latif;* ***analysis and interpretation of results:*** *Anis Eliesa Syareena Yaakub, Noor Azliza Abd Latif;* ***draft manuscript preparation:*** *Noor Azliza Abd Latif, Norziha Che Him; All authors reviewed the results and approved the final version of the manuscript.*

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