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Solving Mixed Convection Boundary Layer Flow with Convective Boundary Condition using bvp4c Solver

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Abstract

This study examines mixed convection boundary layer flow toward a vertical plate with a convective boundary condition. The goals include employing similarity transformations to convert the controlling partial differential equations (PDEs) into ordinary differential equations (ODEs), and using the shooting method with bvp4c solver in MATLAB to acquire numerical solutions as well as comparing the results to those of previous study. It is found that the temperature and velocity profiles have been significantly affected by governing parameters, such as the mixed convection parameter and Prandtl number, as demonstrated by numerical simulations. It has been noted that a higher value of the convective parameter γ results in greater temperature and velocity gradients at the surface, raising the surface shear stress and the rate of heat transfer. The results closely resemble those of previous study, demonstrating the method's correctness and the bvp4c solver's resilience when dealing with mixed convective flow issues. This work provides a solid numerical framework for further thermofluid system research and advances our understanding of mixed convection dynamics.

1. Introduction

In fluid mechanics and heat transfer, mixed convection boundary layer flow is a crucial subject where natural (driven by buoyancy) and forced (driven by external forces) convection mechanisms interact. In many technical applications, such as reactor temperature control, electronic device cooling systems, and environmental processes, it is especially important. The study of boundary layers [1] initially proposed, offers a basis for comprehending the dynamics of heat transmission and flow in these systems.

Convective boundary conditions are an essential component of thermal analysis since surface temperatures in real-world situations are rarely constant and instead rely on heat flow. These boundary conditions, which are controlled by Newton's law of cooling, specify heat transfer between a solid surface and the fluid around it, allowing precise simulation of actual heat exchange processes. The fundamental understanding of the interaction between buoyancy and forced flow on vertical and inclined surfaces was established by early studies conducted by [2] and [3]. The foundation for comprehending the behaviour of the boundary layer in mixed convection settings was established by these investigations.

Solving complicated partial differential equations (PDEs), which are frequently nonlinear and analytically difficult, is necessary for the mathematical modelling of mixed convective boundary layer flow. The conservation of mass, momentum, and energy in the boundary layer [1]. These partial differential equations (PDEs) can be simplified and made acceptable for numerical methods by employing similarity transformations to convert them into a system of ordinary differential equations (ODEs).

In order to acquire approximate solutions for these equations, numerical approaches are essential, especially in situations where traditional analytical methods are not feasible. The bvp4c solver in MATLAB is a popular tool for resolving ordinary differential equation boundary value problems (BVPs). By using adaptive mesh refinement in conjunction with the collocation approach, the solver can accurately handle complex circumstances, boundary singularities, and nonlinearities. For the analysis of mixed convection flows, it is a dependable option due to its efficient solution of such problems.

The mixed convection boundary layer flow toward a vertical plate under convective boundary conditions is investigated in this study. The main goals are to use similarity transformations to convert the governing PDEs into ODEs, obtain numerical solutions for the ODEs using MATLAB's bvp4c solver, and validate and compare the results with the findings of the previous study by [4], since [4] used shooting technique in Maple software. Besides, [5] and [6] obtained dual solutions for a similar problem of mixed convection, but near a stagnation-point considering porous plate/medium. The present investigates how velocity and temperature profiles are affected by important characteristics, such as the mixed convection parameter, Prandtl number, and convective parameter. The study also determines the physical relevance of dual solutions found in opposing flows. This work advances our knowledge of mixed convection dynamics and demonstrates the reliability of numerical approaches in solving practical thermofluid problems by utilizing contemporary computational tools and cross-referencing results with previous studies.

2. Problem Formulation

A two-dimensional steady boundary layer flow towards a vertical plate immersed in a viscous fluid of ambient temperature T_∞ is considered as shown in Fig. 1. The external velocity is prescribed as $u_e(x)=a\sqrt{x}$, where a is a constant. The left side of the plate is assumed to be heated by convection from hot fluid at temperature T_f, giving a heat transfer coefficient h_f.

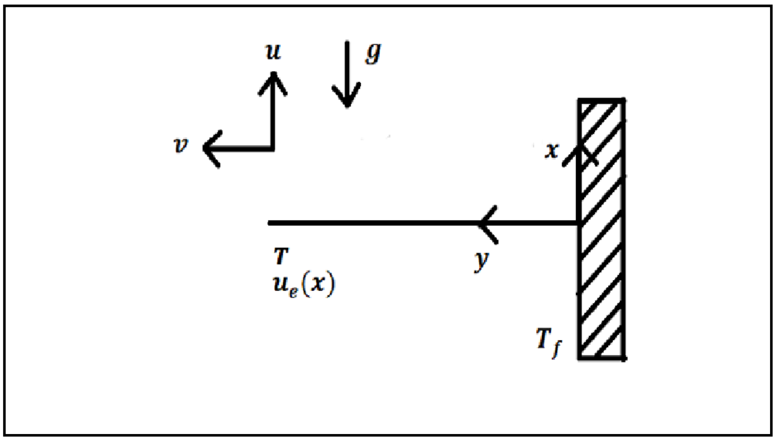


Fig. 1 Physical model of two-dimensional steady boundary layer flow towards a vertical plate [6]

The coordinate system is defined so that the y-axis is perpendicular to the plate, signifying the normal direction, while the x-axis runs down the plate, signifying the streamwise direction. The symbols for the velocity components are $v(y)$ in the y-direction and $u(x)$ in the x-direction. Viscosity causes a boundary layer to form when the fluid travels along the plate, resulting in velocity gradients close to the surface.

The continuity equation, momentum equation, and energy equation which explain mass conservation, fluid motion under the influence of buoyancy, and heat transfer, respectively are the governing equations for this flow. The buoyancy term $g\beta(T-T_\infty)$ in the momentum equation emphasizes the role of natural convection, which, depending on the temperature differential, can either support or impede the external flow. The governing equations are as follow [4]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{du_e}{dx}+v\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where u and v are the velocity components along the x-direction and y-direction respectively, T is the fluid temperature in the boundary layer, g is the acceleration due to gravity, a is the thermal diffusivity, β is the thermal expansion coefficient, and v is the kinematic viscosity. The boundary conditions can be expressed as [7]:

$$u=0,\quad v=0,\quad -k\frac{dT}{dy}=h_f\left(T_f-T_w\right)\ \text{at}\ y=0$$
$$u\rightarrow u_e,\quad T\rightarrow T_\infty,\ \text{at}\ y\rightarrow\infty \tag{4}$$

where k is the thermal conductivity of the fluid, T_w is the plate temperature, and $T_f>T_w>T_\infty$

The non-linear partial differential equations of continuity equation (1), momentum equation (2), and energy equation (3) are transformed into ordinary differential equations. By following [7-8], the similarity transformation are as follows:

$$n=\left(\frac{u_e}{v_x}\right)^{\frac{1}{2}}y=\left(\frac{a}{v}\right)^{\frac{1}{2}}x^{-\frac{1}{4}}y$$

$$\psi=\left(vxu_e\right)^{\frac{1}{2}}f(\eta)=(av)^{\frac{1}{2}}x^{\frac{3}{4}}f(\eta)$$

$$\theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty}\rightarrow T=\left(T_f-T_\infty\right)\theta(\eta)+T_\infty \tag{5}$$

where η is the similarity variable, f is the dimensionless stream function, θ is the dimensionless temperature, and φ is the stream function defined as $u=\frac{\delta\varphi}{\delta y}$ and $\nu=-\frac{\delta\varphi}{\delta x}$. we obtain the following nonlinear ordinary differential equations:

$$f'''+\frac{3}{4}ff''-\frac{1}{2}f'^2+\frac{1}{2}+\lambda\theta=0 \tag{6}$$

$$\frac{1}{Pr}\theta''-\frac{3}{4}f\theta'=0 \tag{7}$$

Which are subject to the boundary conditions by [7-8]:

$$f'(\eta)\rightarrow 1,\qquad \theta(\eta)\rightarrow 0\qquad \eta\rightarrow\infty.$$
$$f(0)=0,\qquad f'(0)=0\qquad \theta'(0)=-\gamma\left[1-\theta(0)\right], \tag{8}$$

The physical quantity of interest is the skin friction coefficient and the local Nusselt number which are respectively defined as

$$C_f=\frac{\tau\omega}{\frac{\rho U^2}{2}}$$

$$Nu_x=\frac{xy_w}{k\left(T_f-T_\infty\right)} \tag{9}$$

3. Numerical Method

It is often necessary to convert a nonlinear second-order differential equation, such as those controlling mixed convective boundary layer flow, into a system of first-order ODEs in order to solve it because MATLAB's bvp4c solver can only handle first-order systems. The dependent variable (such temperature or velocity) is represented by one variable, and its first derivative is represented by another. This is achieved by introducing new variables. Two equations make up the system of first-order ODEs that results from this transformation: one represents the derivative of the dependent variable, and the other identifies the nonlinear function that governs the system's behaviour. Two equations make up the system of first-order ODEs that results from this transformation: one identifies the nonlinear function that governs the system's behaviour. Derivation below is coding based on this problem and writing in a system of differential equations $\frac{\partial y}{\partial x}$:

$$f(\eta) = y(1)$$

$$f'(\eta) = y(2)$$

$$f''(\eta) = y(3)$$

$$f'''(\eta) = -\frac{3}{4}y(1)\,y(3) + \frac{1}{2}\left[y(2)\right]^2 - \frac{1}{2} - Ly(4)$$

$$\theta(\eta) = y(4)$$

$$\theta'(\eta) = y(5)$$

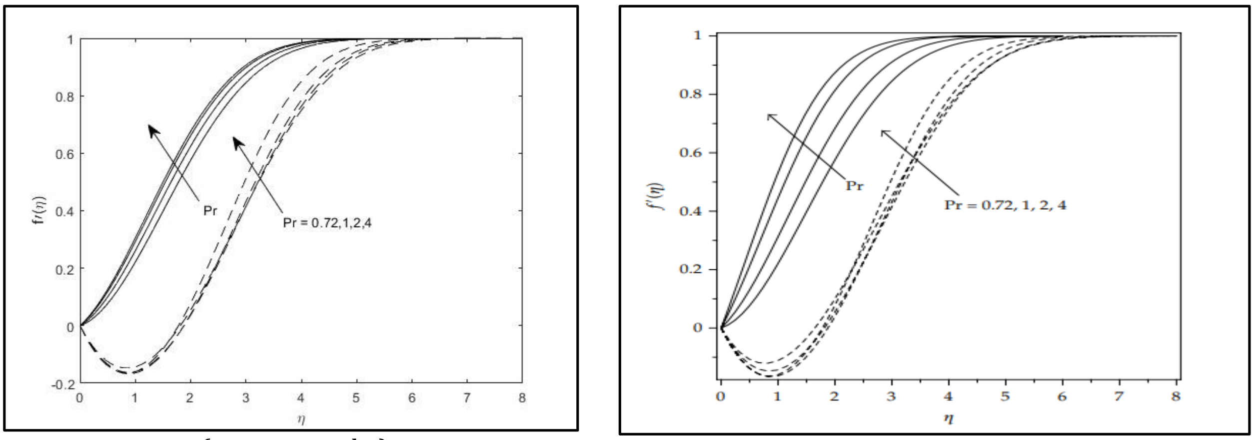
$$\theta''(\eta) = \Pr\left[-\frac{3}{4}y(1)\,y(5)\right]$$

4. Results and Discussion

This study is carried out using similarity transformation, the partial differential equations (PDEs) for momentum and energy that regulate mixed convection boundary layer flow have been converted into ordinary differential equations (ODEs). The system becomes simpler and more suited to numerical solutions as a result of this modification. The shooting technique in the MATLAB bvp4c solver has been used to numerically solve the converted ODEs and the boundary conditions. In order to ensure precise computation of the non-linear differential equations under the specified boundary conditions, the shooting technique reformulates the boundary value problem into an initial value problem. The numerical results of the current study are compared to those of [4], who resolved the related issue, in order to validate the findings. The comparison highlights similarities and differences by looking at the temperature and velocity patterns graphically. This study sheds light on how the flow velocity and thermal profiles are affected by mixed convection parameters, Prandtl number, and convective boundary conditions. This study clarifies the effects of Prandtl number, convective boundary conditions, and mixed convection parameters on the flow velocity and temperature profiles.

4.1 Velocity Profiles

The velocity profiles for different values of the Prandtl number (Pr) are shown in Fig. 2 and results closely resemble with [4] findings. The comparison demonstrates a continuous trend, with dual solutions seen for the opposing flow ($\lambda<0$) and velocity increasing with greater Pr. According to the profiles, the boundary layer in the first solution is thinner, while the boundary layer in the second solution is thicker. The trustworthiness of the numerical method employing the bvp4c solver in MATLAB is confirmed by these results, which nearly match those published in [4].



(current results) [4]

Fig. 2 Velocity profiles $f'(\eta)$ for various values of Prandtl number, Pr when $\gamma = 1$ and $\lambda = -1.2$

4.1.1 Effect of Mixed Convection Parameter (λ)

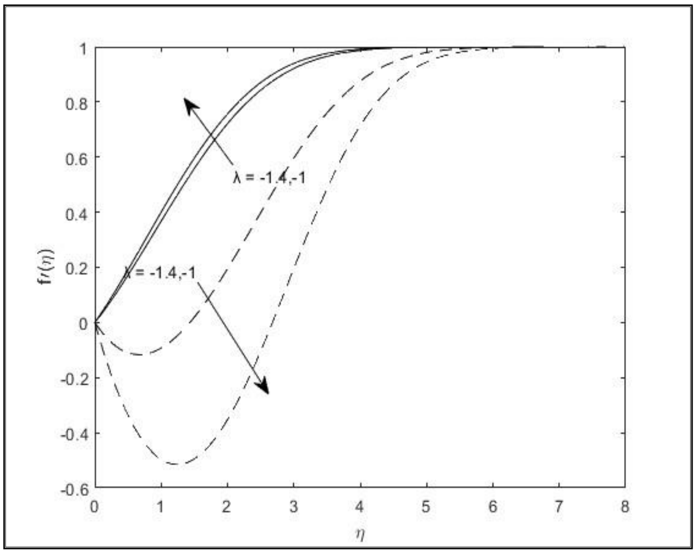


Fig. 3 Velocity profiles $f'(\eta)$ for some values of the buoyancy parameter λ when $\gamma = 1$ and Pr = 1.

From Fig. 3, in opposing flows (λ<0), the forced convection is offset by the buoyant force. As a result, the flow velocity inside the boundary layer decreases. Dual solutions, one stable (physically realizable) and the other unstable, may emerge for specific critical values of λ. As the fluid slows down close to the surface, opposing fluxes frequently produce thicker boundary layers.

4.1.2 Effect of Convective Parameter (γ)

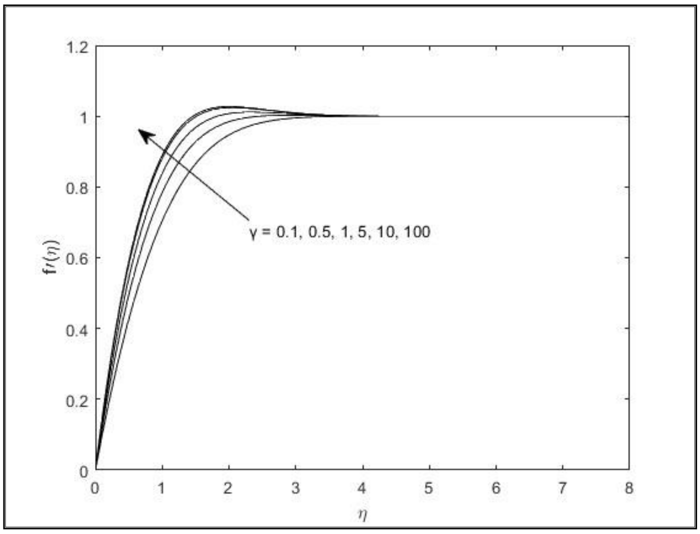


Fig. 4 Velocity profiles $f'(\eta)$ for some values of the γ when λ = 1 and Pr = 1

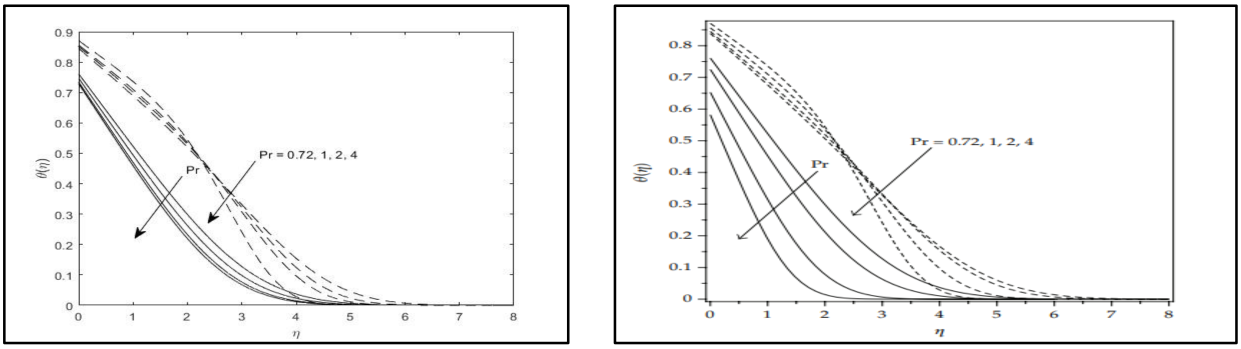
The efficiency of convective heat transport at the surface is represented by the convective parameter (γ). By altering the temperature gradients, which in turn affect the buoyancy forces, it has a substantial effect on the velocity distribution.

Heat transfer from the surface to the fluid is more effective when the value of γ is higher as shown in Fig. 4. Stronger temperature gradients close to the surface are the result, and in mixed convection flows, this amplifies the buoyancy effects. As a result, the boundary layer's thickness decreases and its internal velocity rises.

Heat transmission from the surface is less efficient when (γ) is small. This results in thicker boundary layers and slower velocity profiles by lessening the impact of buoyant forces. In severe situations when $\gamma \approx 0$, the flow gets closer to a regime of forced convection.

4.2 Temperature Profiles

The identical values of the mixed convection parameter λ are used to examine the temperature profiles $\theta(\eta)$. The findings indicate improved heat transfer at the surface when the thermal boundary layer thins as λ increases. For opposing flows (λ < 0), dual solutions are seen, much like the velocity profiles. Greater heat transfer rates result from a thinner thermal boundary layer at higher Prandtl numbers (Pr). These results are consistent with the temperature profiles shown by [4], who found that steeper temperature gradients are likewise produced by greater values of the convective parameter.



(current results) [4]

Fig. 5 Temperature profiles $\theta(\eta)$ for some value of Prandtl number Pr when γ = 1 and λ = -1.2

4.2.1 Effect of Mixed Convection Parameter (λ)

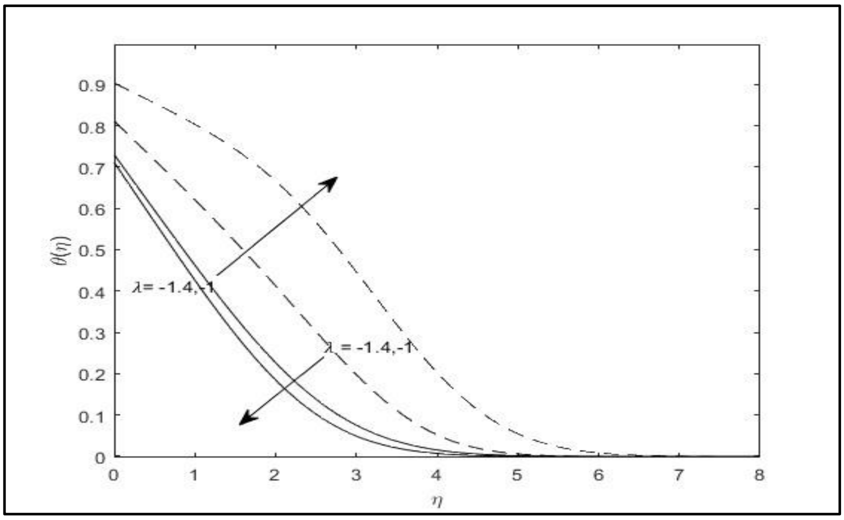


Fig. 6 Temperature profiles $\theta(\eta)$ for some values of the buoyancy parameter λ when γ = 1 and Pr = 1.

Fig. 6 is referred used to examines the influence of λ on the temperature distribution when γ=1 and Pr =1. As λ increases, the buoyancy force becomes stronger, leading to increase convective heat transfer and a thinner thermal boundary layer. In opposing flow conditions, heat transfer efficiency decreases because buoyancy resists the external flow, leading to a thicker thermal boundary layer.

4.2.2 Effect of Convective Parameter (γ)

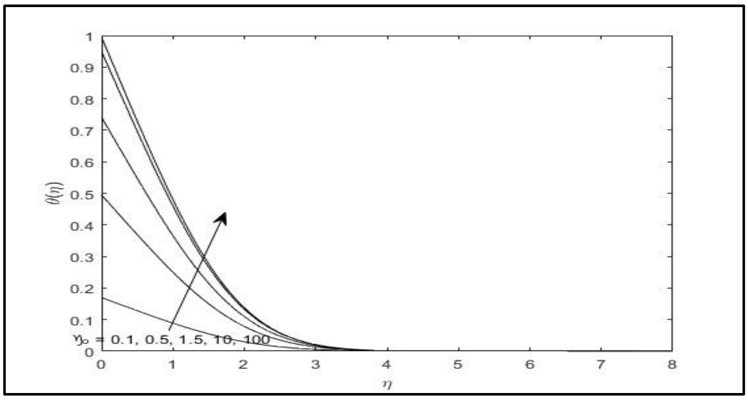


Fig. 7 Temperature profiles $\boldsymbol{\theta(\eta)}$ for some values of γ when λ = 1 and Pr = 1.

Fig. 7 shows the impact of the convective parameter (γ) on temperature profiles when Pr=1 and λ=1. Higher values of γ lead to stronger surface heat transfer, causing steeper temperature gradients near the plate and a thinner thermal boundary layer. When γ is small, heat transfer from the plate to the fluid is less efficient, resulting in a thicker thermal boundary layer and slower temperature decay. The effect of γ is significant in mixed convection, as it dictates how efficiently heat is transferred from the solid surface to the surrounding fluid.

5. Conclusion

This study focused on solving the mixed convection boundary layer flow problem with a convective boundary condition using numerical approach and comparing the results with those obtained by [4]. The main objectives of the research have been successfully achieved. A similarity transformation has been used to convert the governing partial differential equations (PDEs) for mixed convective boundary layer flow into ordinary differential equations (ODEs). This simplified the numerical solution of the equations and validated the consistency of the mathematical model.

Next, the bvp4c solver in MATLAB has been used to numerically solve the ODEs, employing a shooting strategy. The outcomes showed how well the solver handled boundary value problems with convective and nonlinear boundary conditions. It is found that the temperature and velocity profiles have been significantly affected by governing parameters, such as the mixed convection parameter and Prandtl number, as demonstrated by numerical simulations. It has been noted that a higher value of the convective parameter γ results in greater temperature and velocity gradients at the surface, raising the surface shear stress and the rate of heat transfer. The accuracy of the numerical method used has been confirmed by a thorough comparison of the results of the current study with those of Aman and Ishak [4]. For varying parameter values, both investigations displayed steady trends in the temperature and velocity profiles. Dual solutions for opposing flows ($\lambda < 0$) are seen to develop. Comparing it to the shooting approach employed by Aman and Ishak [4], this study shows that the bvp4c solver is a dependable and effective tool for addressing mixed convection boundary layer flow problems with convective boundary conditions.

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Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design:*** *Mohd Azhar Alpad, Fazlina Aman;* ***analysis and interpretation of results:*** *Mohd Azhar Alpad, Fazlina Aman;* ***draft manuscript preparation:*** *Mohd Azhar Alpad, Fazlina Aman. All authors reviewed the results and approved the final version of the manuscript.*

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