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Forecasting Mortality Rates in Malaysia using ARIMA and Lee-Carter Models

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Abstract

Forecasting mortality rates is crucial for understanding demographic trends and describing Malaysian public health planning, such as optimising resource allocation and guiding evidence-based policy development to meet future healthcare demands. It is also an important technique that provides professionals with valuable insights, enabling them to predict trends and remain vigilant. The objectives in this study is applying ARIMA and Lee-Carter models to mortality rate in Malaysia, then comparing the accuracy between ARIMA and Lee-Carter models to identify the most reliable forecasting approach by analysing RMSE, MAE and MAPE and lastly is using the most suitable model to forecast mortality rate of Malaysia. The goal of the study is to determine the best model to forecast the mortality rate in Malaysia between the ARIMA model and the Lee-Carter model. This forecasted mortality could help researchers and policymakers understand the trend of mortality rates and enhance health initiatives in Malaysia. The methodology involves using ARIMA model by using Box-cox transformation and differencing for stabilising data meanwhile Lee-Carter model uses Singular Value Decomposition to capture trend of future rates. The mortality rate data in Malaysia for this study was obtained from the United Nations World Population Prospects and this study compares the ARIMA and Lee-Carter models using mortality data from 1950 to 2018, categorized into seven age groups which is infant, toddler, childhood, teenager, young adult, adult and elderly. In result, ARIMA (1, 1, 0) performs better than Lee-Carter model for most of the age groups, especially for the younger age groups.

1. Introduction

Forecasting mortality rates is an important technique in planning public health since it provides information about demographic changes and healthcare demands. Mortality trends inform the allocate resources and develop strategies to address public health challenges such as aging populations and the burden of non-communicable diseases [1]. Mortality rate also known as death rate is the incidence rate that calculates the frequency of deaths in each population during a specified period [2]. Based on World Health Organisation in 2016, mortality rate is the total number of deaths in a specific population (based on sex, age or other demographic factors), divided by the total number of individuals in that population and is usually expressed per 100,000 individuals for a given year, within a particular country, territory or geographic area.

In Malaysia, reliable mortality forecasts are critical for addressing obstacles associated with an aged population, enhancing healthcare planning, and regulating variable death trends. These early models provided foundational approaches to mortality analysis but were limited in their ability to account for stochastic variations over time. [3] choose ARIMA (1,1,4) as the best model to predict COVID-19 mortality rates in Malaysia, as provided better model fitting compared to ARIMA (1,1,3) despite this model has a slightly lower error value.

The Lee-Carter model, introduced in 1992, advanced mortality forecasting by incorporating both age-specific and temporal components through a log-bilinear model [4]. This method uses singular value decomposition (SVD) to analyse mortality trends and has been widely applied in demographic and actuarial studies [5]. However, its reliance on historical trends sometimes leads to overestimation, particularly in older age groups. In 2018, Halim and Ismail examined the mortality rate pattern in Malaysia and future projections using Lee-Carter model, valued for the simplicity, robustness and precision in forecasting age-specific mortality rates and addressing longevity risk. The findings emphasize applicability of the model to mortality data of Malaysia and highlight its potential for socio-economic improvements, while suggesting extensions with other stochastic tools like the Cairns-Blake-Dowd model for further study [6].

This study aims to apply ARIMA and Lee-Carter models to mortality rate data of Malaysia. Besides, this study also compares the accuracy of these models' using metrics such as Mean Absolute Error (MAE), Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE). Lastly, this study identifies the most reliable forecasting model for mortality rate in Malaysia.

The field of mortality forecasting has evolved significantly, with early methods relying on static models that lacked predictive capabilities for long-term trends. Models such as ARIMA and Lee-Carter models incorporate dynamic elements, including the ability to account for time-varying trends, age-specific sensitivity, and stochastic variations in mortality data, to better reflect age-specific mortality changes over time [7]. Forecasting mortality rates involves addressing challenges like data quality, changing demographic patterns, and the impact of external factors such as healthcare improvements and socio-economic shifts [8]. These challenges necessitate robust and adaptable modelling techniques to ensure reliable predictions.

2. Methodology

The mortality rate data came from United Nations World Population Prospects and the data period from 1950 to 2018, was divided into seven age groups, such as infants, toddlers, children, teenagers, young adults, adults and elderly. The dataset was then divided into training (1950–2010) and testing sets (2011–2018), with a total of 6969 data points. Software such as Microsoft Excel is used for preliminary analysis, meanwhile Minitab and Python were used for ARIMA modelling and Python is used for Lee-Carter modelling.

2.1 Preliminary Analysis

In the preliminary analysis, the mortality rate dataset in 69 years is analysed for the data exploration to examine the historical mortality data to understand its structure, trend and patterns. Besides, calculations of mean, standard deviation and other summary statistics to get the sense of the central tendency and variability of the data.

2.2 Model Identification

ACF is used to identify significant lags for moving average (MA) components, while the PACF plots identifies significant lags for autoregressive (AR) components [9]. Three stages of Box-Jenkins modelling approach such as identification, estimation as well as diagnosis and forecasting are aligned with ARIMA structure. ACF and PACF plots can further help in identifying the appropriate model, which discussed in Table 1 [10].

Table 1 Model Identification from ACF and PACF plots

ACFs	PACFs	Model
Decay to zero with exponential pattern	Cuts off lag p	AR (p)
Cuts off after lag q	Decay to zero with exponential pattern	MA (q)
Decay to zero with exponential pattern	Decay to zero with exponential pattern	ARMA (p, q)
Cuts off after q	Cuts off after lag p	AR (p) or MA (q)

Meanwhile for Lee-Carter model, the age specific mortality rates are log-transformed to stabilise variance. Singular Value Decomposition (SVD) is then applied to the log-transformed matrix decomposing it. After estimating the parameters, a time series model is fitted to the k_t series, which is the general trend in mortality over time, to forecast future values [11].

2.3 Ljung-Box Test

Ljung-Box Test checks for the presence of serial autocorrelation up to a specified lag k. This test examines whether errors show independence and identical distribution (iid) or display a more complex pattern. Its focus is on assessing whether the autocorrelations of residuals differ from zero. Essentially, it serves as a test to determine if the model fits well. When the autocorrelations of residuals are relatively low, it suggests that the model adequately represents the data, indicating a significant lack of fit. In order to run the Ljung-Box test, statistic Q must be calculated as Equation (1):

$$Q(m) = n(n+2)\sum_{j=1}^{m}\frac{{r_j}^2}{n-j} \quad (1)$$

where n is the sample size, r_j is the autocorrelation at lag j and m is the number of lags being tested.

2.4 ARIMA Model

ARIMA model needs differencing and ACF and PACF analysis for model identification. The mortality rate data was divided into training (1950-2010) and testing (2011-2018) sets. The ARIMA model follows in the Eq. (2).

$$y_t = \phi_1 y_{t-1} + \varepsilon_t \quad (2)$$

where ϕ_1 is autoregressive coefficient for the first lag, y_t is the forecast value at time t, y_{t-1} is the value at the previous year and ε_t is error term at a time, *t*.

Furthermore, model identification is a crucial step of building a ARIMA forecasting model. It involves understanding the structure of the time series data by evaluating the ACF and PACF plots. The ACF plot shows the relationship between the current values and its lag values. For ARIMA, its objectives are to identify the Autoregressive (AR) component. By observing at the ACF plot, the significant lags reflect the point in time where previous observations are associated with the current observation. The decreasing of correlations occurs as the lag increases provides information about the order of the AR component. The PACF plot complements the ACF plot by showing the correlation between the current observation and its lags, omitting intermediate lags. PACF helps to detect the AR component by providing insights into the relationship between the current observation and previous observations at specific lags.

2.5 Lee-Carter Model

The Lee-Carter model utilized Singular Value Decomposition (SVD) to identify age-specific mortality trends [12]. The model is expressed as in Eq. (3).

$$\ln m_{x,t} = a_x + b_x k_t + \varepsilon_{x,t} \quad (3)$$

where $m_{x,t}$ is the central death rate at age *x* in year *t*, a_x is the average pattern of mortality by age across years , k_t shows the index of the level of mortality at time *t*, b_x is the relative speed of change at each age and $\varepsilon_{x,t}$ is the residual at age *x* and time *t*.

The a_x values in the Lee-Carter model are calculated as the average of the natural logarithm of the mortality rates, $\ln m_{x,t}$ over time. The b_x and k_t values are estimated using Singular Value Decomposition (SVD) [13]. To ensure a unique solution, certain constraints are applied: the a_x values are set equal to the time-averaged $\ln m_{x,t}$, the b_x values are constrained to sum to 1, and k_t values are constrained to sum to zero. Equation of k_t is shown in Eq. (4), where d is the average annual change in k_t and e_t is uncorrelated errors.

$$k_t = k_{t-1} + d + e_t \quad (4)$$

2.6 Predictive Accuracy Measurement

In this study, accuracy measurement is used to assess the performance in forecasting of ARIMA and Lee-Carter models. The accuracy metrics in forecasting such as Mean Absolute Percentage Error (MAPE), Mean Absolute

Error (MAE) and Root Mean Square Error (RMSE) is applied to compare the accuracy of the models [14]. More accurate forecasting models have lower values of MAPE, MAE and RMSE. The equations for the MAE, MAPE and RMSE are shown in Eq. (5), Eq. (6) and Eq. (7) respectively.

$$MAE = \frac{1}{2}\sum_{t-1}^{n}\left|\frac{A_t - F_t}{A_t}\right| \tag{5}$$

$$MAPE = \frac{1}{2}\sum_{t-1}^{n}\left|\frac{A_t - F_t}{A_t}\right| \times 100\% \tag{6}$$

$$RMSE = \sqrt{\sum_{t-1}^{n}\frac{(A_t - F_t)^2}{n}} \tag{7}$$

where A_t is the actual value at period *t*, F_t is the predicted value at period *t*, and *n* is the number of observations. An MAPE value below 10% indicates a highly accurate forecast. A value of MAPE between 10% and 20% signifies a good forecast. Values between 20% and 50% suggest a reasonable forecast, whereas a MAPE over 50% indicates poor accuracy. This interpretation is summarized in Table 2 [15].

Table 2 Interpretation of the MAPE Values

MAPE	Interpretation of Forecast Accuracy
Below 10%	Highly Accurate Forecasting
10% to 20%	Good Accurate Forecasting
20% to 50%	Reasonable Accurate Forecasting
50% and above	Inaccurate Forecasting

3. Time Series Plot

The time series plot for mortality rate in Malaysia for different age groups starting from 1950 to 2018 which for total 69 years. The plot has been plotted as shown in Fig.1.

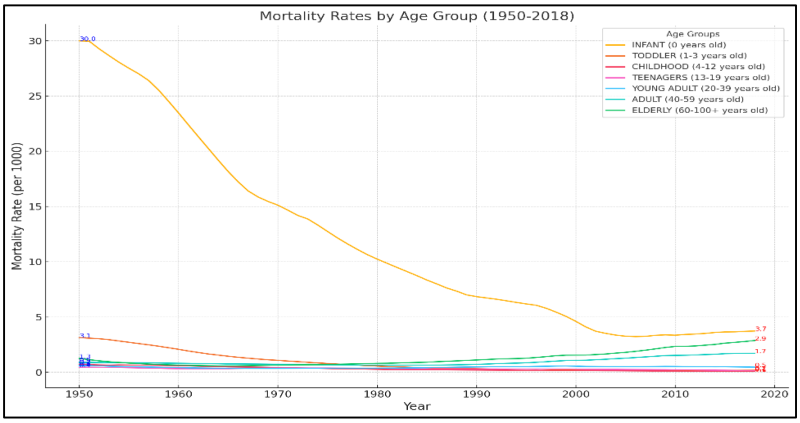

Fig. 1 Time series plot mortality rate in Malaysia for different age groups in Malaysia

Most of the age groups show a decline mortality rate over time, showing that there is an improvement in healthcare and living conditions. Notably, infant and toddler mortality rates decreased significantly, showing the impact of advancements in early childhood care. In contrast, for older adults and the elderly, the mortality rate trends initially declined but began increasing in recent decades, potentially due to age-related diseases becoming more prominent as life expectancy increases.

3.1 Preliminary Analysis

The calculated means and standard deviations are shown in Table 3. As descriptive statistics, the calculated mean and standard deviation help to understand the mortality rate data in Malaysia from 1950 till 2018.

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Table 3 Analysis of Mortality Rate of Different Age Groups in Malaysia

	Descriptive statistic						
Age Groups	Infant	Toddler	Childhood	Teenager	Young Adult	Adult	Elderly
Size	69	69	69	69	69	69	69
Mean	11.7559	0.8936	0.3130	0.3177	0.4593	0.9538	11.7559
Standard deviation	8.4884	0.9079	0.1986	0.0730	0.0747	0.3346	8.4884

Table 3 presents both infant and elderly age group have same mean value (11.7559), suggesting they are similar in this dataset, likely showing demographic transitions such as an aging population or high birth rates. Besides, they also have same standard deviation, which is relatively high, indicating non-consistent around the mean in these age groups over the dataset. The lowest mean in these age groups is childhood and teenager age group. The lowest standard deviations can be seen in teenager and young adult age group, showing a very consistent distribution around mean.

3.2 ARIMA Model

The ARIMA model was used to forecast mortality rates for all age groups, using previous yearly data to forecast future mortality rates. Table 4 shows the ARIMA model parameters and Ljung-Box Chi-Square statistics.

Table 4 ARIMA Model Parameters and Ljung-Box Chi-Square Statistics

Model / Age Group	ARIMA					
	Coefficient	*p*-value	Ljung-Box Chi-Square Statistics			
			Lags 12	Lags 24	Lags 36	Lags 48
Infant	AR 1: 0.9733	0.000	0.972	0.897	0.923	0.964
Toddler	AR 1: 0.9916 AR 1: 1.193 AR 2: -0.205	0.000 0.000 0.119	0.176 14.02	0.765 16.38	0.798 0.628	0.821 0.524
Childhood	AR 1: 0.9231	0.000	0.610	0.979	0.995	0.756
Teenager	AR 1: 0.8529	0.000	0.621	0.835	0.702	0.739
Young Adult	AR 1: 0.8296	0.000	0.695	0.953	0.909	0.057
Adult	AR 1: 0.9222	0.000	0.278	0.666	0.766	0.638
Elderly	AR 1: 0.9979	0.000	0.067	0.390	0.733	0.872

Based on Table 4, all age groups use first-order autoregressive parameter (AR 1) to describe the time series. The coefficient indicates the strength of independence on the previous year data. A higher coefficient (closer to 1) shows a strong persistence of the trend; this can be identified by looking at the coefficient of all the age groups. For *p*-values, all coefficients of ARIMA (1,1,0) have the value of 0.000, indicating the AR 1 parameters are statistically significant across all age groups. For toddler group, AR 2 shows a non-significant *p*-value (0.119) which is higher than significance level of 5% in the coefficient and needed to be excluded.

Based on Fig 2., the testing values of younger age group (infant, toddler and childhood) attached to the actual values, which shows the accuracy of the testing value is high. Meanwhile, the testing sets older age group (teenager, young adult, adult and elderly) does not fit to the actual values. Most of the age group have the testing sets that showing decreasing trend except for adult and elderly age groups which showing increasing trend.

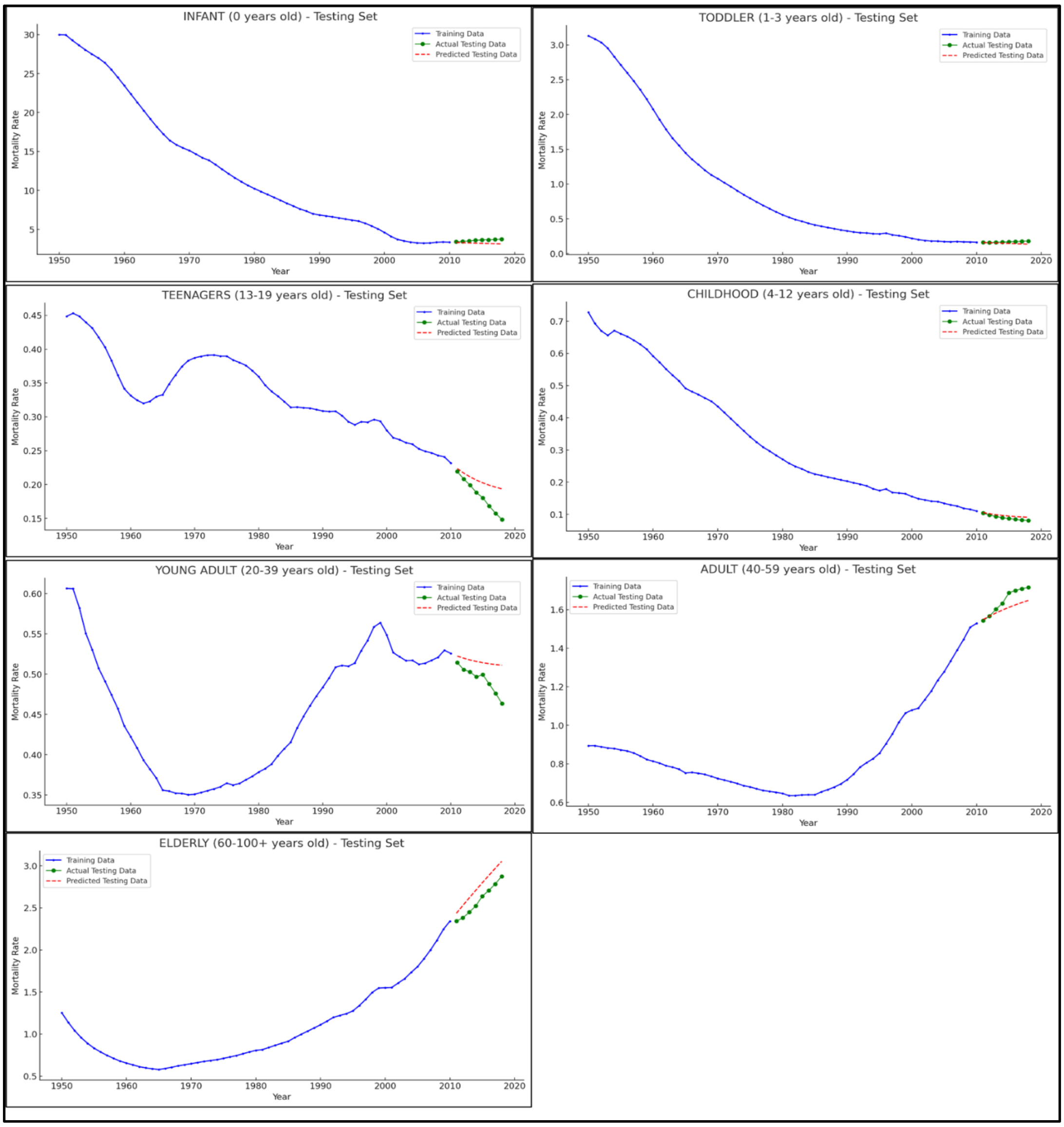

Fig. 2 Time series plot of mortality rate Malaysia by using ARIMA (1, 1, 0)

3.3 Lee-Carter Model

The Lee-Carter model was used to forecast mortality rates for all age groups, using historical data to estimate future mortality rates. Table 5 shows the parameters including coefficients and *p*-value of all age groups of Lee-Carter model.

Table 5 Lee-Carter Model Parameters

Model / Age Group	Lee-Carter Model	
	Coefficient	*p*-value
Infant	0.9928	0.000
Toddler	0.9888	0.000
Childhood	0.9919	0.000
Teenager	0.7846	0.000
Young Adult	0.1101	0.040
Adult	0.2585	0.000
Elderly	0.6251	0.000

Based on Table 5, which displays the Lee-Carter model parameters for various age groups, including coefficient and their *p*-values respectively. Infants, toddlers, and children have the highest coefficients, with values of 0.9928, 0.9888, and 0.9919 respectively, showing a strong significance in the model. Teenagers have a moderately high coefficient of 0.7846, which is lower than the previous age groups but still significant. Young adults exhibit a much lower coefficient of 0.1101 and the *p*-value of 0.040 suggests that the relationship is statistically significant at 5% significance level. Adults have a slightly higher coefficient of 0.2585, with a statistically significant *p*-value of 0.000. The elderly group with a coefficient of 0.6251 and a *p*-value of 0.000, also exhibits significant influence.

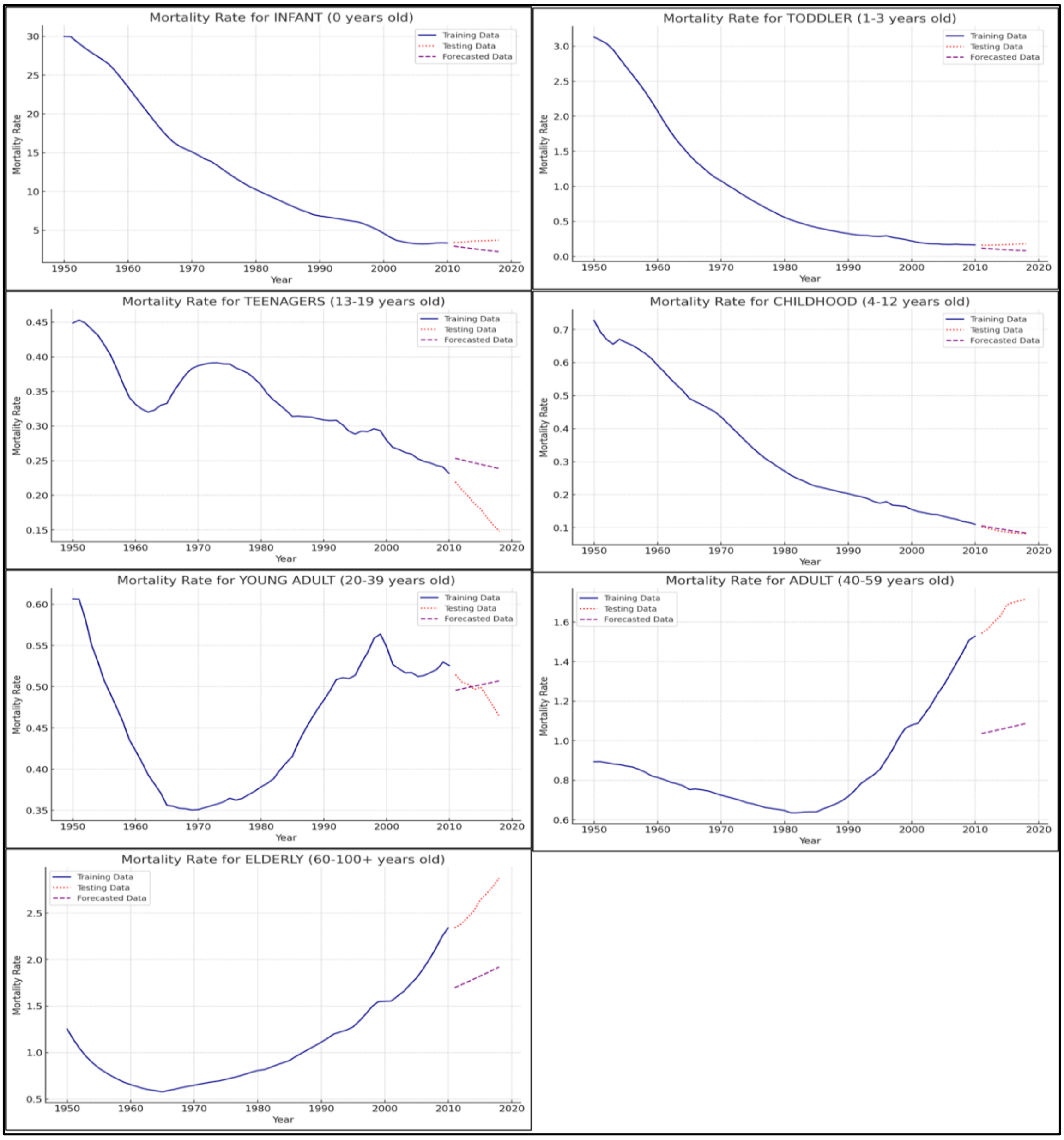

Fig. 3 Time series plot of mortality rate Malaysia by using Lee-Carter Model

Based on Fig 3., most of the testing sets does not align with the actual sets except for age group of childhood, showing that Lee-Carter model has lower accuracy in forecasting mortality rate of Malaysia than that of ARIMA (1, 1, 0). Most of the age groups showing increasing trend except for childhood and teenager age groups which showing decreasing trends.

Table 6 shows the forecast accuracy measurement of ARIMA of age groups of childhood, young adult, adult and elderly have the lowest value of MAPE (1.75%, 4.63%, 2.59 and 7.00% respectively) which is relatively accurate forecasting, while the age groups which are between 10% to 20 % (infant, toddler and teenager) can be said is good accurate forecasting. For Lee-Carter model, only childhood and young adult age groups achieved below 10% of MAPE value (5.22% and 3.28% respectively) which is highly accurate forecasting. However, the other age groups achieved the MAPE value that above 20%, which mean they are only reasonably accurate forecasting.

Table 6 Comparison of Forecast Accuracy Measurement between ARIMA Model and Lee-Carter Model

Age Group \ Model	ARIMA (1,1,0)			Lee-Carter		
	MSE	RMSE	MAPE (%)	MSE	RMSE	MAPE (%)
Infant	0.3777	0.6146	10.38	1.1308	1.0634	27.77
Toddler	0.0007	0.0259	11.90	0.0052	0.0722	40.87
Childhood	0.0016	0.0398	1.75	0.00002	0.0048	5.22
Teenager	0.0006	0.0260	13.29	0.0042	0.0649	35.80
Young Adult	0.0006	0.0256	4.63	0.0004	0.0208	3.28
Adult	0.0028	0.0532	2.59	0.3421	0.5849	35.39
Elderly	0.0350	0.1870	7.00	0.6210	0.7880	30.02

3.4 Forecasting

The ARIMA model achieved lower MSE values than the Lee-Carter model for the infant, toddler, teenager, adult and elderly age groups. In contrast, the Lee-Carter model outperformed ARIMA with lower MSE values for the childhood and young adult age groups. The ARIMA obtained lower MAPE values for the infant, toddler, teenager, adult, and elderly age groups, whereas the Lee-Carter model achieved lower MAPE values for the childhood as well as young adult age groups. The most suitable model for forecasting mortality rate in Malaysia will rely on ARIMA (1, 1, 0) since it had the lower number of MSE, RMSE and MAPE, shows that the error of this model is lower than that of the Lee-Carter model. The next 10 years of the mortality rate is forecasted and is shown in Table 7.

Table 7 Mortality rate forecast for all age groups

Year	ARIMA (1,1,0)						
	Infant	Toddler	Childhood	Teenager	Young Adult	Adult	Elderly
2019	3.09100	0.132023	0.0758805	0.190597	0.510250	1.65812	3.19481
2020	3.06583	0.128654	0.0733850	0.188706	0.509648	1.66746	3.28861
2021	3.04134	0.125312	0.0710814	0.187094	0.509149	1.67608	3.38222
2022	3.01750	0.121999	0.0689551	0.185719	0.508735	1.68403	3.47564
2023	2.99430	0.118713	0.0669922	0.184546	0.508391	1.69137	3.56886
2024	2.97171	0.115456	0.0651804	0.183545	0.508106	1.69813	3.66189
2025	2.94974	0.112225	0.0635079	0.182692	0.507870	1.70436	3.75473
2026	2.92835	0.109022	0.0619640	0.181965	0.507674	1.71012	3.84738
2027	2.90753	0.105845	0.0605389	0.181344	0.507511	1.71542	3.93984
2028	2.88726	0.102696	0.0592234	0.180815	0.507376	1.72031	4.03211

Based on Table 7, most of the age groups showing a decreasing trend except for adult and elderly group which showing an increasing trend. The elderly group shows the highest mortality rate among the age groups, indicating aging population and age-related health issues meanwhile toddler and childhood age groups have the lowest mortality rate, showcasing the improvement of healthcare technology.

4. Conclusion

This study used and analysed two models to forecast mortality rates in Malaysia: the ARIMA model and the Lee-Carter model. ARIMA is a statistical model that analyses the connections between data points to forecast general time series, whereas the Lee-Carter model is specifically built for demographic forecasting, notably mortality rates. Both models aim to capture trends, but the Lee-Carter model includes age-specific elements, making it appropriate for actuarial and demographic applications.

This study provides researchers and policymakers with useful insights into how to improve public health in Malaysia by forecasting mortality patterns. Accurate forecasts allow for better resource allocation, identification of vulnerable age groups, and the implementation of targeted initiatives to reduce unnecessary fatalities. While both the ARIMA and Lee-Carter models were tested, complex mortality patterns provided obstacles for consistent performance across all age groups. However, the ARIMA model was identified as the most reliable forecasting method. Several restrictions influenced the accuracy of mortality rate forecasts. The information relatively limited the ability of models to predict trends during times of significant change. This limitation impacted the Lee-Carter model more severely, as it struggled to effectively capture age-specific mortality rates, resulting in misaligned projections across age groups. Future study could benefit from datasets with higher variability to improve forecasting accuracy.

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Conflict of Interest

Authors declare that there is no conflict of interest regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design:*** *Ting Sulung, Norhaidah Mohd Asrah;* ***data collection:*** *Ting Sulung;* ***analysis and interpretation of results:*** *Ting Sulung, Norhaidah Mohd Asrah;* ***draft manuscript preparation:*** *Ting Sulung. All authors reviewed the results and approved the final version of the manuscript.*

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