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Nonlinear Wave Modulation In Prestressed Elastic Tube Filled With Viscous Fluid

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Abstract: From the previous studies, researchers were concerned about wave propagation in the artery. The research of wave modulation in the artery are somewhat limited since mathematical modelling is challenging to perform. The aim of this study is to investigate nonlinear wave modulation in a prestressed elastic tube filled with viscous fluid. In this research, the artery is assumed as thin-walled, long, and circularly cylindrical, the prestressed elastic tube with variable cross-section. Blood is considered an incompressible viscous fluid. For the reductive perturbation method (RPM), the stretched coordinates and asymptotic series were introduced in the dimensionless equations of tube and fluid. The RPM is applied to obtain a set of various orders of differential equations. The nonlinear Schrodinger (NLS) equation with variable coefficients is obtained by solving these differential equations as the nonlinear evolution equation in the corresponding mathematical model. The NLS equation with variable coefficients will be solved analytically. MATLAB's graphical output shows that the wave speed increases when the viscosity of fluid increases. This happened due to the resistance of the blood flow indicates which results in the reaction of the wave speed. Increasing in wave number leads to decreasing in wavelength. Other than that, the wave travels further in the expanding tube than the narrowing tube. Furthermore, the wave amplitude effected by the cross-section area of the tube. Waves with greater amplitude comes from a high disturbance of energy. For future study, it is suggested to explore in wave modulation. There are few suggestions which are nonlinear wave modulation of heterogenous fluid flow in thin elastic tube with variable cross-sectional area, nonlinear wave modulation of Newtonian fluid flow in thin viscoelastic tube with variable cross-sectional area and nonlinear wave modulation of viscous fluid flow in thin stenosed viscoelastic tube.

Keywords: Wave Modulation, Nonlinear Schrodinger Equation With Variable Coefficients, Viscous Fluid, Thin Wall Elastic Tube With Variable Cross-Section

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1 Introduction

In general, the human body consists of many biological systems to do a specific function to breathe and move. For instance, there are respiratory system, circulatory system, skeletal system, and digestive system. Ref. [1] stated that the circulatory system aids in blood circulation, which means it helps transport oxygen and nutrients to tissues in the body. Blood vessels in the body vary in size according to their functions. According to [2], the smallest blood vessels are capillaries, which only 5μm. It is only one cell thick that optimizes the diffusion of oxygen into the tissues. Many researchers have studied the wave modulation of fluid flow in an elastic tube such as [3], [4], [5] and [6]. In this research, blood is treated as an incompressible viscous fluid and the artery is a thin-walled, long, and prestressed elastic tube.

2 Equations of the tube and fluid

This section explains the equations of an incompressible viscous fluid-filled prestressed elastic tube. The equation of fluid is given as follow [3]

$$\frac{\partial w^*}{\partial t^*} + w^*\frac{\partial w^*}{\partial z^*} + \frac{1}{\rho_f}\frac{\partial P^*}{\partial t^*} - \bar{v}\left(\frac{8w^*}{r_f^2} + \frac{\partial^2 w^*}{\partial t^{*2}}\right) = 0, \tag{1}$$

Where, $r_f = r_0 + f^* + u^*$is the final radius after deformation occurred.

$$2\frac{\partial u^*}{\partial t^*} + 2w^*\left[f^{*\prime} + \frac{\partial u^*}{\partial z^*}\right] + (r_0 + f^* + u^*)\frac{\partial w^*}{\partial z^*} = 0, \tag{2}$$

where *w** is the mean of fluid speed, *t** is time. *z** is a coordinate that located on axis when the changes of radius maintain its value, *f**(*z**) is the function of a variable radius, ρ_f the mass density, *P** is the mean of fluid pressure, $\bar{v}$ is the viscosity for fluid flow, *u** is the function of displacement of the radius, and r_0 is the initial radius in the coordinate system.

The equation of motion of elastic tube in the radial direction could be written as follows [3]:

$$-\frac{\mu}{\lambda_2}\frac{\partial\Sigma}{\partial\lambda_2} + \mu R_0 \times \frac{\partial}{\partial z^*}\left\{\frac{\left(f^{*\prime} + \frac{\partial u^*}{\partial z^*}\right)}{\left[1 + \left(f^{*\prime} + \frac{\partial u^*}{\partial z^*}\right)^2\right]^{\frac{1}{2}}}\frac{\partial\Sigma}{\partial\lambda_1}\right\} + \frac{P_r^*}{H}(r_0 + f^* + u^*)$$

$$\times\left[1 + \left(f^{*\prime} + \frac{\partial u^*}{\partial z^*}\right)^2\right]^{\frac{1}{2}} = \rho_0\frac{R_0}{\lambda_z}\frac{\partial^2 u^*}{\partial t^{*2}}, \tag{3}$$

where

$$P_r^* = \left[1 + (f^{*\prime} + \frac{\partial u^*}{\partial z^*})^2\right]^{-1/2} \times \left[P^* + 4\mu_v\frac{(f^{*\prime} + \frac{\partial u^*}{\partial z^*})}{(r_0 + f^* + u^*)}w^*\right]$$

R_0 is the radius of the tube, Σ is the strain energy density function membrane, μ is the shear modulus of the material of the tube, λ_z represents the axial stretch of the tube, λ_2 is the circumference of curves, P_r^* is a force where it is developed from the reaction of the fluid, H is the thickness of the tube, and $\rho_0$is the tube's mass density. Both equations of tube and fluid using the function, u and depends on the same fast, and slow variables. Fast variables are *t* and *z* while slow variables are *ξ* and *τ*.

The following non-dimensional quantities are introduced at this stage [3]:

$$t^* = \left(\frac{R_0}{c_0}\right)t, \qquad z^* = R_0 z, \qquad u^* = R_0 u,$$

$$m = \frac{p_0 H}{p_f R_0}, \qquad w^* = c_0 w, \qquad f^* = R_0 f,$$

$$r_0 = R_0\lambda_\theta, \qquad P^* = p_f c_0^2 p, \qquad c_0^2 = \frac{\mu H}{p_f R_0},$$

$$\bar{v} = p_f c_0 R_0 \hat{v}, \tag{4}$$

By applying Eq. (4) into Eq. (1), (2) and (3) yield

$$\frac{\partial w}{\partial t}\frac{c_0^2}{R_0} + \frac{c_0^2}{R_0}w\left(\frac{\partial w}{\partial z}\right) + \frac{c_0^2}{R_0}\frac{\partial P}{\partial z} - \frac{c_0^2\hat{v}\rho_f R_0}{R_0^2}\left[-\frac{8w}{(\lambda_\theta + f + u)^2} + \frac{\partial^2 w}{\partial z^2}\right] = 0,$$

$$2\left(\frac{\partial w^*}{\partial w}\frac{\partial w}{\partial z}\frac{\partial z}{\partial z^*}\right) + 2w^*\left(f^{*\prime} + \frac{\partial u^*}{\partial u}\frac{\partial u}{\partial z}\frac{\partial z}{\partial z^*}\right) + (r_0 + f^* + u^*) \times \left(\frac{\partial w^*}{\partial w}\frac{\partial w}{\partial z}\frac{\partial z}{\partial z^*}\right) = 0,$$

$$p = \frac{m}{\lambda_z(\lambda_\theta + f + u)}\frac{\partial^2 u}{\partial t^2} + \frac{1}{\lambda_z(\lambda_\theta + f + u)}\frac{\partial \Sigma}{\partial \lambda_2} - \frac{1}{(\lambda_\theta + f + u)}\frac{\partial}{\partial z}$$
$$\times \left\{\frac{f' + \frac{\partial u}{\partial z}}{\left[1 + \left(f' + \frac{\partial u}{\partial z}\right)^2\right]^{\frac{1}{2}}}\frac{\partial \Sigma}{\partial \lambda_1}\right\} - 4\hat{v}\frac{f' + \frac{\partial u}{\partial z}}{(\lambda_\theta + f + u)}w. \tag{5}$$

where, λ_θ is the stretch ratio in the circumferential direction.

3 Nonlinear Wave Modulation

In this section, to study the non-linear wave modulation in a viscous fluid contained in a thin elastic tube with the variable cross-sectional area, the reductive perturbation method (RPM) is applied. Based on the boundary-value problem, the following type of stretched coordinates are introduced [3]:

$$\xi = \varepsilon(z - \lambda t), \quad \tau = \varepsilon^2 z, \tag{6}$$

where ξ is the wave profile and τ is the space. ε indicates the nonlinearity measurer's weakness with a small value and λ is the scale constant to be determined from the solution.

Since this study has a variable cross-section of tube, the order of $\hat{h}$ should be first-order, (ε), where $\hat{h}(\varepsilon, \tau) = \varepsilon h(\tau)$ [3]. The differential relations can be expressed as [3]:

$$\frac{\partial}{\partial z} \to \frac{\partial}{\partial z} + \varepsilon\frac{\partial}{\partial \xi} + \varepsilon^2\frac{\partial}{\partial \tau}, \qquad \frac{\partial}{\partial t} \to \frac{\partial}{\partial t} - \varepsilon\lambda\frac{\partial}{\partial \xi}. \tag{7}$$

The field quantities u, w and p are assumed can be expressed as asymptotic series in the following form [3]:

$$u = \varepsilon u_1 + \varepsilon^2 u_2 + \varepsilon^3 u_3 + \cdots$$
$$w = \varepsilon w_1 + \varepsilon^2 w_2 + \varepsilon^3 w_3 + \cdots$$
$$p = p_0 + \varepsilon p_1 + \varepsilon^2 p_2 + \varepsilon^3 p_3 + \cdots \tag{8}$$

where u, w, and p are the functions of fast variables and slow variables.

By solving Eq. (5), the various order equations obtained are shown as following

$O(\varepsilon)$ order equation:

$$\frac{\partial w_1}{\partial t}+\frac{\partial p_1}{\partial z}=0,\qquad \frac{\partial u_1}{\partial t}+\frac{\lambda_\theta}{2}\frac{\partial \omega_1}{\partial z}=0,\qquad p_1=\frac{m}{\lambda_z\lambda_\theta}\frac{\partial^2 u_1}{\partial t^2}-a_0\frac{\partial^2 u_1}{\partial z^2}+\beta_1(u_1+h). \quad (9)$$

$O(\varepsilon^2)$ order equation:

$$\frac{\partial w_2}{\partial t}+\frac{\partial p_2}{\partial z}-\lambda\frac{\partial w_1}{\partial \xi}+\frac{\partial p_1}{\partial \xi}+w_1\frac{\partial w_1}{\partial z}=0,$$

$$\frac{\partial u_2}{\partial t}+\frac{\lambda_\theta}{2}\frac{\partial \omega_2}{\partial z}-\lambda\frac{\partial u_1}{\partial \xi}+\frac{\lambda_\theta}{2}\frac{\partial \omega_1}{\partial \xi}+\omega_1\frac{\partial u_1}{\partial z}+\frac{u_1}{2}\frac{\partial \omega_1}{\partial z}+\frac{h(\tau)}{2}\frac{\partial \omega_1}{\partial z}=0,$$

$$p_2=\frac{m}{\lambda_z\lambda_\theta}\left(\frac{\partial^2 u_2}{\partial t^2}-2\lambda\frac{\partial^2 u_1}{\partial t\partial \xi}\right)-\frac{m}{\lambda_z\lambda_\theta^2}u_1\frac{\partial^2 u_1}{\partial t^2}+\beta_2u_1^2+\beta_1u_2-a_0\frac{\partial^2 u_2}{\partial z^2}-2a_0\frac{\partial^2 u_1}{\partial z\partial \xi}$$
$$+\left(\frac{\alpha_0}{\lambda_\theta}-2\alpha_1\right)u_1\frac{\partial^2 u_1}{\partial z^2}-\alpha_1\left(\frac{\partial u_1}{\partial z}\right)^2+\left[-\frac{m}{\lambda_z\lambda_\theta^2}\frac{\partial^2 u_1}{\partial t^2}+2\beta_2u_1+\left(\frac{\alpha_0}{\lambda_\theta}-2\alpha_1\right)\frac{\partial^2 u_1}{\partial z^2}\right]$$
$$\times h(\tau)+\beta_2(h)^2. \quad (10)$$

$O(\varepsilon^3)$ order equation:

$$\frac{\partial w_3}{\partial t}+\frac{\partial p_3}{\partial z}-\lambda\frac{\partial w_2}{\partial \xi}+\frac{\partial p_2}{\partial \xi}+\frac{\partial p_1}{\partial \tau}+w_1\left(\frac{\partial w_2}{\partial z}+\frac{\partial w_1}{\partial \xi}\right)+w_2\frac{\partial w_1}{\partial z}+v\left(\frac{8}{\lambda_\theta^2}w_1-\frac{\partial^2 w_1}{\partial^2 z}\right)=0,$$

$$\frac{\partial u_3}{\partial t}+\frac{\lambda_\theta}{2}\frac{\partial \omega_3}{\partial z}-\lambda\frac{\partial u_2}{\partial \xi}+\frac{\lambda_\theta}{2}\frac{\partial \omega_2}{\partial \xi}+\omega_1\left(\frac{\partial u_2}{\partial z}+\frac{\partial u_1}{\partial \xi}\right)+\frac{\partial u_1}{\partial z}\omega_2+\frac{u_1}{2}\left(\frac{\partial \omega_2}{\partial z}+\frac{\partial \omega_1}{\partial \xi}\right)$$
$$+\frac{u_2}{2}\frac{\partial \omega_1}{\partial z}+\frac{h(\tau)}{2}\left(\frac{\partial \omega_2}{\partial z}+\frac{\partial \omega_1}{\partial \xi}\right)=0,$$

$$p_3=\frac{m}{\lambda_z\lambda_\theta}\frac{\partial^2 u_3}{\partial t^2}-a_0\frac{\partial^2 u_3}{\partial z^2}+\beta_1u_3-2\lambda\frac{m}{\lambda_z\lambda_\theta}\frac{\partial^2 u_2}{\partial t\partial \xi}-2a_0\frac{\partial^2 u_2}{\partial z\partial \xi}$$
$$+\left(\lambda^2\frac{\alpha_0}{\lambda_\theta\lambda_z}-\alpha_0\right)\frac{\partial^2 u_1}{\partial \xi^2}+\frac{m}{\lambda_z\lambda_\theta^2}\left(2\lambda u_1\frac{\partial^2 u_1}{\partial t\partial \xi}-u_1\frac{\partial^2 u_2}{\partial t^2}-u_2\frac{\partial^2 u_1}{\partial t^2}\right)$$
$$+\frac{m}{\lambda_z\lambda_\theta^3}u_1^2\frac{\partial^2 u_1}{\partial t^2}+\beta_3u_1^3+2\beta_2u_1u_2-2a_0\frac{\partial^2 u_1}{\partial z\partial \tau}$$
$$+\left(\frac{3}{2}\alpha_0-3\gamma_1\right)\left(\frac{\partial u_1}{\partial z}\right)^2\frac{\partial^2 u_1}{\partial z^2}+\left(\frac{1}{\lambda_\theta}\alpha_0-2\alpha_1\right)\left(u_1\frac{\partial^2 u_2}{\partial z^2}+u_2\frac{\partial^2 u_1}{\partial z^2}\right)$$
$$-2\alpha_1\frac{\partial u_1}{\partial z}\left(\frac{\partial u_2}{\partial z}+\frac{\partial u_1}{\partial \xi}\right)+2\left(-\alpha_2+2\frac{\alpha_1}{\lambda_\theta}-\frac{\alpha_0}{\lambda_\theta^2}\right)u_1^2\frac{\partial^2 u_1}{\partial z^2}$$
$$+\left[\frac{m}{\lambda_z\lambda_\theta^2}\left(2\lambda\frac{\partial^2 u_1}{\partial t\partial \xi}-\frac{\partial^2 u_2}{\partial t^2}\right)+2u_1\frac{m}{\lambda_z\lambda_\theta^3}\frac{\partial^2 u_1}{\partial t^2}+3\beta_3u_1^2+2\beta_2u_2\right.$$
$$\left.+\left(\frac{1}{\lambda_\theta}\alpha_0-2\alpha_1\right)\frac{\partial^2 u_2}{\partial z^2}+2\left(\frac{1}{\lambda_\theta}\alpha_0-2\alpha_1\right)\frac{\partial^2 u_1}{\partial z\partial \xi}+\left(-\alpha_2+\frac{\alpha_1}{\lambda_\theta}\right)\left(\frac{\partial u_1}{\partial z}\right)^2\right.$$

$$+2u_1\left(-\alpha_2+2\frac{\alpha_1}{\lambda_\theta}-\frac{\alpha_0}{\lambda_\theta^2}\right)\frac{\partial^2 u_1}{\partial z^2}\Bigg]h(\tau)+\left(-\alpha_2+\frac{\alpha_1}{\lambda_\theta}\right)\left(\frac{\partial u_1}{\partial z}\right)^2 u_1$$
$$+\left[\frac{m}{\lambda_z\lambda_\theta^3}\frac{\partial^2 u_1}{\partial t^2}+3\beta_3 u_1+\left(-\alpha_2+2\frac{\alpha_1}{\lambda_\theta}-\frac{\alpha_0}{\lambda_\theta^2}\right)\frac{\partial^2 u_1}{\partial z^2}\right]h^2(\tau)+h^3(\tau)\beta_3. \tag{11}$$

Where $\alpha_0, \alpha_1, \alpha_2, \beta_0, \ldots, \beta_3$ and γ_1 are defined by

$$\alpha_0=\frac{1}{\lambda_\theta}\frac{\partial\Sigma}{\partial\lambda_z},\qquad \alpha_1=\frac{1}{2\lambda_\theta}\frac{\partial^2\Sigma}{\partial\lambda_\theta\partial\lambda_z},\qquad \alpha_2=\frac{1}{2\lambda_\theta}\frac{\partial^2\Sigma}{\partial\lambda_\theta\partial\lambda_z}\qquad \gamma_1=\frac{\lambda_z}{2\lambda_\theta}\frac{\partial^2\Sigma}{\partial\lambda_z^2}$$
$$\beta_0=\frac{1}{\lambda_z\lambda_\theta}\frac{\partial\Sigma}{\partial\lambda_\theta},\qquad \beta_1=\frac{1}{\lambda_z\lambda_\theta}\frac{\partial^2\Sigma}{\partial\lambda_\theta^2}-\frac{\beta_0}{\lambda_\theta},\qquad \beta_2=\frac{1}{2\lambda_z\lambda_\theta}\frac{\partial^3\Sigma}{\partial\lambda_\theta^3}-\frac{\beta_1}{\lambda_\theta},\qquad \beta_3=\frac{1}{6}\frac{\partial^4\Sigma}{\partial\lambda_\theta^4}-\frac{\beta_2}{\lambda_\theta}.$$

Solving the Eq. (9), (10) and (11) give the following partial differential equation (PDE), which is the nonlinear Schrodinger (NLS) equation with variable coefficient

$$i\frac{\partial U}{\partial\tau}+\mu_1\frac{\partial^2 U}{\partial\xi^2}+\mu_2|U|^2U+i\mu_3 h\frac{\partial U}{\partial\xi}+[\mu_4 h^2+(\mu_5 h'-\mu_6 h)\xi+i\mu_7]U=0.$$

Where U is unknown function, and $\mu_1, \mu_2, \ldots, \mu_7$ are the variable coefficients shown as the following:

$$\mu_1=\frac{\left(\frac{\omega}{k}-\lambda\right)\left(3\frac{\omega}{k}-\lambda\right)+\frac{1}{2}\left(\left(\frac{m}{\lambda_z}\right)k^2\lambda^2-\alpha_0\lambda_\theta k^2\right)}{(\alpha_0\lambda_\theta k^3+\frac{2\omega^2}{k})},$$

$$\mu_2=\frac{1}{\left(\alpha_0\lambda_\theta k^3+\frac{2\omega^2}{k}\right)}\left\{-\frac{16\omega^2}{\lambda_\theta^2}+\frac{4\omega\lambda k}{\lambda_\theta^2}+\frac{3}{2}\left(\beta_3\lambda_\theta-\frac{\beta_1}{\lambda_\theta}\right)k^2\right.$$
$$+\left(\alpha_2\lambda_\theta-\frac{5\alpha_1}{2}\right)k^4-\lambda_\theta k^6\left(\frac{3\alpha_0}{4}-\frac{3\gamma_1}{2}\right)+\left[k^2\lambda_\theta\beta_2+\frac{5}{2}\beta_1k^2+3\alpha_1k^4\lambda_\theta\right]\times$$
$$\left.\frac{\left[\frac{3\omega^2}{\lambda_\theta}+\frac{4\omega\lambda}{\lambda_\theta k}+3\lambda_\theta\alpha_1k^2\right]}{3[(\lambda_\theta\beta_1k^2-2\omega^2)]}+2\omega k\left[\Phi_0+\frac{2\lambda}{\lambda_\theta}\right]\frac{-\frac{\lambda^2}{\lambda_\theta}+\frac{4\lambda\omega}{\lambda_\theta k}+\lambda_\theta\beta_2+\lambda_\theta\alpha_1k^2}{\lambda^2-\frac{\lambda_\theta\beta_1}{2}}\right\},$$

$$\mu_3=\frac{2\alpha_1\lambda_\theta k^3+\left(\frac{3}{2}\omega-2\lambda\omega\right)\Phi_0}{\left(\alpha_0\lambda_\theta k^3+\frac{2\omega^2}{k}\right)},$$

$$\mu_4=\frac{1}{\alpha_0\lambda_\theta k^3+\frac{2\omega^2}{k}}\left[\frac{\omega k(\lambda_\theta\Phi_0+2\lambda)\beta_2}{\lambda^2-\frac{\lambda_\theta\beta_1}{2}}-k^2\Phi_0^2+k^2\left(\frac{3}{2}\lambda_\theta\beta_3+\beta_2+\frac{\beta_1}{\lambda_\theta}\right)+\frac{1}{2}\alpha_0\lambda_\theta k^4\right],$$

$$\mu_5=\frac{1}{\left(\alpha_0\lambda_\theta k^2+\frac{2\omega^2}{2}\right)}\times\left\{2\omega k\Phi_0+\frac{\omega k(\lambda_\theta\Phi_0+2\lambda)(\beta_1-\lambda_\theta\Phi_0)}{\lambda^2-\frac{\lambda_\theta\beta_1}{2}}\right\},$$

$$\mu_6=\frac{8v\omega k(\lambda_\theta\Phi_0+2\lambda)\Phi_0}{\left(\alpha_0\lambda_\theta k^2+\frac{2\omega^2}{2}\right)},$$

$$\mu_7 = \frac{v\omega\left(\frac{8}{\lambda_\theta^2} + k^2\right)\Phi_0}{\left(\alpha_0\lambda_\theta k^2 + \frac{2\omega^2}{2}\right)}, \tag{12}$$

where k is number of wave, ω is the angular frequency, v is the fluid viscosity and

$$\Phi_0 = \frac{\left(\lambda_\theta\beta_2 + \frac{\beta_1}{2}\right)k^2 + \lambda_\theta\alpha_1 k^4}{2\omega k}.$$

4 Results and Discussion

In the previous section, the coefficient μ_7 describes the dissipation resulting from the viscosity of the fluid, coefficients μ_3, μ_4, μ_5 and μ_6 contribute the variable radius of the tube. Therefore, proposed solution of the NLS equation with variable coefficients of the following term:

$$U = a(\tau)V(\zeta)e^{i[\phi(\tau)\xi - \Omega(\tau)]}, \qquad \zeta = \alpha(\tau)[\xi - \varphi(\tau)]. \tag{13}$$

where $a(\tau), V(\zeta), \alpha(\tau), \varphi(\tau), \phi(\tau)$ and $\Omega(\tau)$ are the given as following

$$V(\zeta) = \operatorname{sech}(\zeta), \qquad a(\tau) = a_0 e^{-2\mu_7\tau},$$

$$\alpha(\tau) = \left[\frac{\mu_2}{2\mu_1}\right]^{\frac{1}{2}}(a_0 e^{-2i\mu_7\tau}), \qquad \phi(\tau) = K + A\left[\mu_5\tau - \mu_6\frac{\tau^2}{2}\right],$$

$$\varphi(\tau) = 2\mu_1\left(K\tau + A\mu_5\frac{\tau^2}{2} - A\mu_6\frac{\tau^3}{6}\right) + \mu_3 A\frac{\tau^2}{2},$$

$$\Omega(\tau) = \mu_1 K^2\tau + \frac{\mu_2 a_0^2}{4\mu_7}(e^{-4\mu_7\tau} - 1) + AK\left(\mu_1\mu_5\tau^2 + \mu_3\frac{\tau^2}{2}\right) + \frac{\tau^3}{3}\left(A^2\mu_1\mu_5^2 - AK\mu_1\mu_6 + A^2\mu_3\mu_5\right) - A^2\frac{\tau^4}{4}\left(\mu_1\mu_5\mu_6 + \frac{\mu_3\mu_6}{2}\right) - \mu_4 A^2 + \frac{A^2}{20}\mu_1\mu_6^2\tau^5. \tag{14}$$

Introducing functions $\phi(\tau), \varphi(\tau)$ and $\Omega(\tau)$ into equation Eq. (13), it gives

$$U = a_0 e^{-2\mu_7\tau}[\operatorname{sech}\zeta]e^{i[\eta]}.$$

with ζ and η are defined as

$$\zeta = \left(\frac{\mu_2}{2\mu_1}\right)^{1/2} a_0 e^{-2\mu_7\tau}\left[\xi - 2\mu_1\tau K - A\mu_1\mu_5\tau^2 + A\mu_1\mu_6\frac{\tau^3}{3} - \mu_3 A\frac{\tau^2}{2}\right], \tag{15}$$

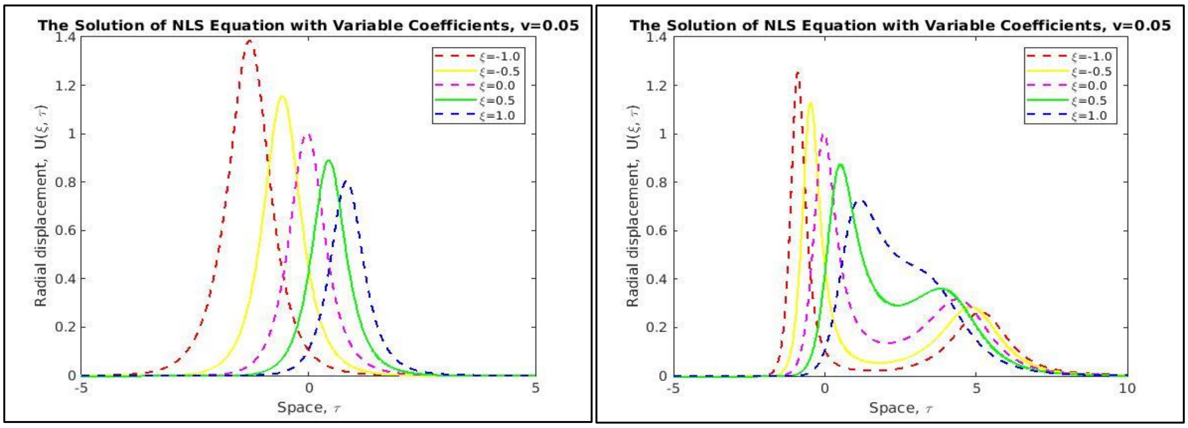
$$\eta = [\xi - \mu_1 K\tau]K + A\left(\mu_5\tau - \mu_6\frac{\tau^2}{2}\right)\xi - \frac{\mu_2 a_0^2}{8\mu_7}(e^{-4\mu_7\tau} - 1) - AK\left(\mu_1\mu_5 + \frac{\mu_3}{2}\right)\tau^2 - \frac{\tau^3}{3}\left(-\mu_4 A^2 + A^2\mu_1\mu_5^2 - AK\mu_1\mu_6 + A^2\mu_3\mu_5\right) + A^2\frac{\tau^4}{4}\left(\mu_1\mu_5\mu_6 + \frac{\mu_3\mu_6}{2}\right) - \frac{A^2}{20}\mu_1\mu_6^2\tau^5. \tag{16}$$

The carrier wave speed of the NLS equation with variable coefficients which obtained from Eq. (16). is given by

$$V_p = \frac{\partial\xi}{\partial\tau},$$

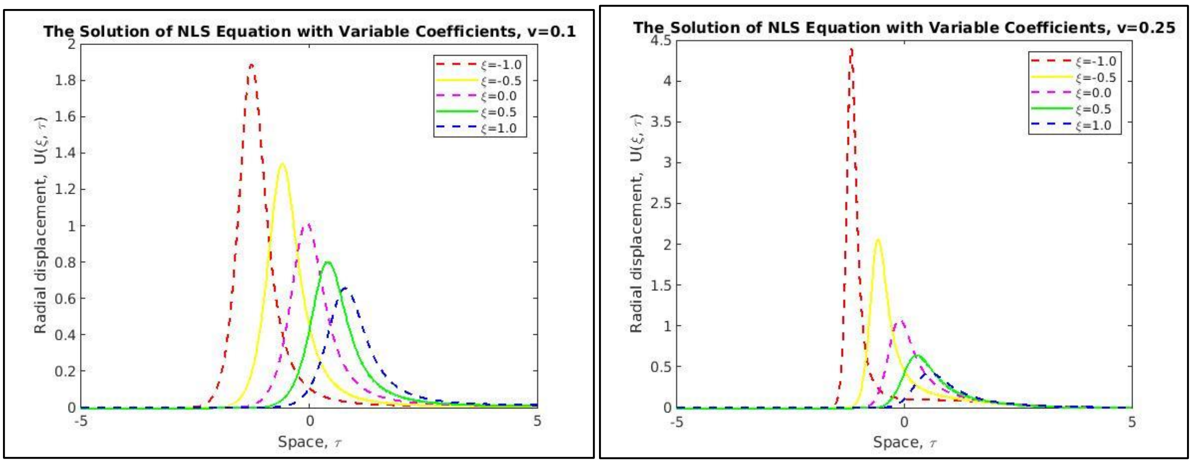
$$
\begin{aligned}
&= \frac{1}{K + A\mu_5\tau - \frac{1}{2}A(\mu_6\tau^2)}\Bigg[\mu_1 K^2 + 2AK\tau\left(\mu_1\mu_5 + \frac{\mu_3}{2}\right) - \frac{\mu_2 a_0^2}{2}e^{-4\mu_7\tau} + \frac{A^2}{4}\mu_1\mu_6^2\tau^4 \\
&\qquad - A^2\tau^3\left(\mu_1\mu_5\mu_6 + \frac{\mu_3\mu_6}{2}\right) + \tau^2\left(-\mu_4A^2 + A^2\mu_1\mu_5^2 - AK\mu_1\mu_6 + A^2\mu_3\mu_5\right)\Bigg] \\
&- \frac{(A\mu_5 - A\mu_6\tau)}{\left(K + A\mu_5\tau - \frac{1}{2}A\mu_6\tau^2\right)^2}\Bigg[\frac{\mu_2 a_0^2}{8\mu_7}\left(e^{-4\mu_7\tau} - 1\right) + AK\left(\mu_1\mu_5 + \frac{\mu_3}{2}\right)\tau^2 + \frac{A^2\mu_1\mu_6^2\tau^5}{20} + \mu_1K^2\tau \\
&\qquad + \frac{\tau^3}{3}\left(A^2\mu_1\mu_5^2 - \mu_4A^2 - AK\mu_1\mu_6 + A^2\mu_3\mu_5\right) - \frac{A^2\tau^4}{4}\left(\mu_1\mu_5\mu_6 + \frac{\mu_3\mu_6}{2}\right)\Bigg].
\end{aligned} \tag{17}
$$

The graphical outputs for radial displacement, the viscous effect of fluid, wave number, and wave speed are illustrated using MATLAB. In this research, the numerical value of α is 1.948 [7]. Other than that, the axial stretch, λ_z and λ_θ are assumed as 0.8 and 1.2, respectively.



(a) (b)

Figure 1: The solution of NLS equation with variable coefficients versus space, τ at v = 0.05, for (a) narrowing tube and (b) expanding tube respectively.



(a) (b)

Figure 2: The solution of NLS equation with variable coefficients versus space, τ for narrowing tube at different fluid viscosity, (a) v = 0.1 and (b) v = 0.25.

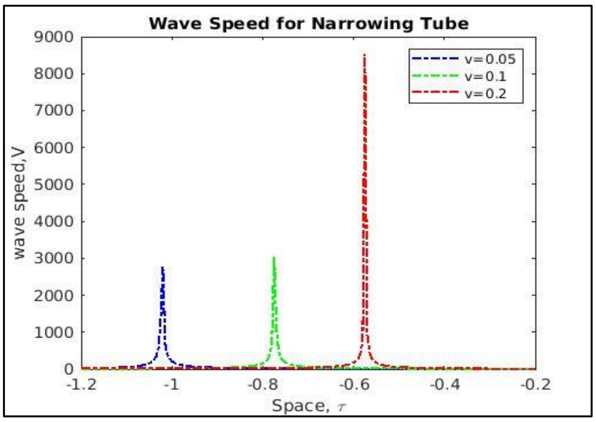
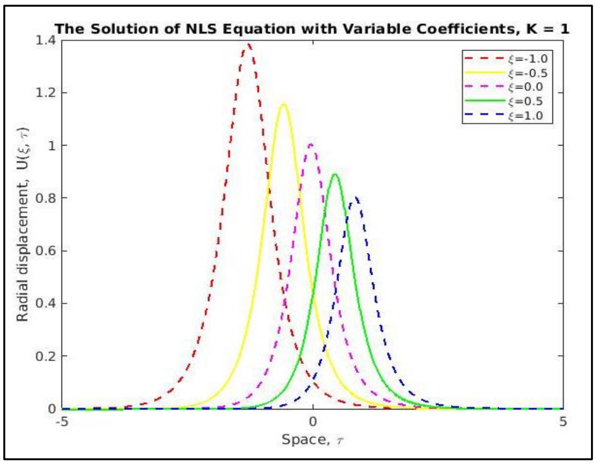
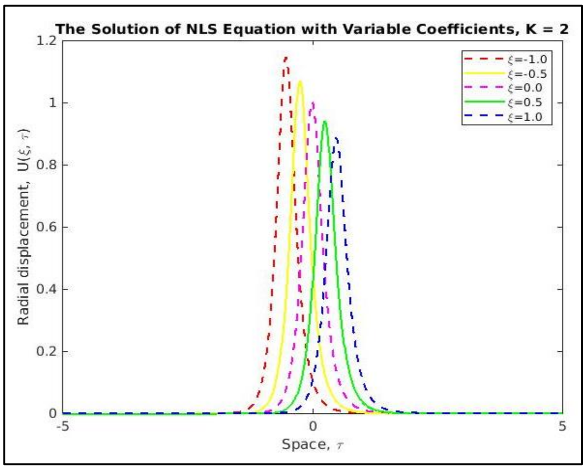


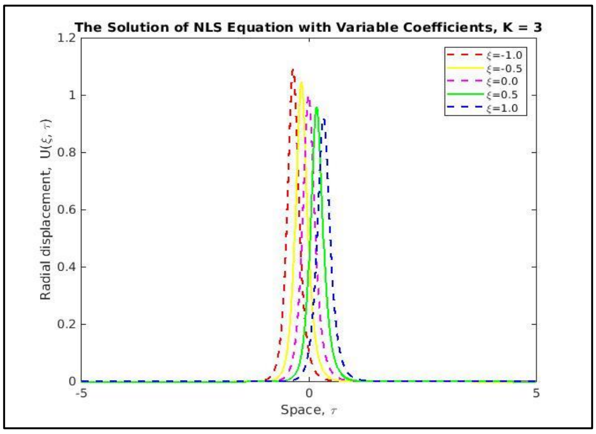
Figure 3: The wave speed, V of the NLS equation with variable coefficients at different fluid viscosity, v



(a)



(b)



(c)

Figure 4: The solution of NLS equation with variable coefficients at v = 0.05 using different wave number, (a) K = 1, (b) K = 2, (c) K = 3.

The radial displacement in Figure 1 (a) is decreasing when $A = -0.05$. However, the radial displacement in Figure 1 (b) is increasing when $A = -0.05$. Here, $A < 0$ represents the narrowing tube and $A > 0$ is the expanding tube. The decreasing of radial displacement is due to the presence of a dissipative term. The dissipative term appears when the blood is considered as an incompressible viscous fluid flow in the artery. Hence, the resistance for blood flows in the artery due to the presence of viscosity, v = 0.05. From Figure 2, the radial displacement increases due to an increase in fluid viscosity. This happened because of higher flow resistance.

Figure 3 shows that due to the fact when blood passes through the narrowing artery, the wave speed increased since the circumference of the artery is reduced, and the viscous effect of fluid is increased. Due to the viscous effect of fluid, the wave speed must be increasing in order to maintain an adequate flow of blood.

By comparing the graphs in Figure 4, changes of the bell-shaped graph are shown. The results show the wavelength for the solution of the NLS equation with variable coefficients throughout the narrowing tube are decreasing when the wave number increases. In addition, increasing wave number leads to decreasing in radial displacement and speed of travelling wave.

5 Conclusion

This study focused on nonlinear wave modulation of viscous fluid flow in the prestressed elastic tube with variable cross-section area. These dimensional equations of tube and fluid are converted into non-dimensionalized equations by introducing the non-dimensionalized quantities. Next, the RPM is employed in the dimensionless equations of tube and fluid to obtain various orders of the differential equations. The RPM covered the stretched coordinates and asymptotic series used. After that, the differential equations will be solved to get the nonlinear evolution equation, the NLS equation with variable coefficients. Then, the progressive wave solution is implemented in the NLS equation with variable coefficients to achieve the analytical solution for the NLS equation with variable coefficients. The graphical output generated by MATLAB discussed the effects of analytical solution on radial displacement, the viscous effect of fluid, wave number, and wave speed.

From the results, it shows that the wave in narrowing tube maintained its shape, and the amplitude was affected by the wave viscosity. In the expanding tube, the wave separates into two waves. This happened due to wave disruption. In an expanding tube, the wave travels longer than a wave in a narrowing tube. Other than that, the wavelength, radial displacement, and speed of travelling wave will be decreasing when the wave number increasing. The higher velocity of the wave, the higher the radial displacement. The wave speed is also affected by the fluid viscosity. This is because the high fluid viscosity has high resistance will make the fluid flows slower.

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