Enhanced Knowledge in Sciences and Technology Vol. 1 No. 2 (2021) 143-151

EKST

Homepage: http://publisher.uthm.edu.my/periodicals/index.php/ekst
e-ISSN : 2773-6385

A Study On Predator-Prey Model For Competitive Corporation

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DOI: https://doi.org/10.30880/ekst.2021.01.02.017
Received 25 May 2021; Accepted 11 July 2021; Available online 29 July 2021

Abstract: Predator-prey models have been used for years to model animal populations. In recent years, researchers started applying to economic-related problem. The purpose of this research is to examine the competitive corporations in Malaysia. This paper examines the basic Lotka-Volterra predator-prey model and two predators and one prey model. The numerical dynamical behavior of the model by regression techniques for estimating parameters of the predator-prey model has been discussed. The obtained results must be significant for the further analysis. The logistic equation is dealing with the possibility of predicting long-term development based on logistic curves. Finally, Hopf Bifurcation will be used to analysis the equilibrium of the approximate Lotka-Volterra equations.

Keywords: Lotka-Volterra model, Logistic Growth, Hopf Bifurcation

1. Introduction

Predation is a biological interaction in which a predator kills its prey and consumes it. Nonetheless, predator–prey models are applicable not only for environmental and resources economic problems but also to other economic fields. From the point of view of economic, the appearance of the other corporation can make harvesting more or less easy because the harvesting prices could also be littered with the presence of another corporation. The predator and prey relationship drives to a source of provisional convenience with a lower dimension of predators to prey in harvesting prey for a country. In literature, some studies have to use the predator-prey model to evaluate any behaviour circumstances of the economic, including but not restricted for competitiveness such as in stock market of South Korean [1] and to investigate whether the performance of large companies that located in United States such as Target and Walmart, perchance modelled by various predator and prey models [2]. This research considers the basic Lotka–Volterra predator-prey model with two-predator and one-prey which assumption corporations in Malaysia. Further, this project will discuss the numerical dynamical behaviour of the model by regression techniques for estimating parameters of the predator-prey model for two predators and one prey by using Lotka-Volterra. As the market becomes increasingly

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competitive, customer demands are becoming more complex, customized and multifaceted, leading to shortened product life cycles and difficulties of product development. The Lotka–Volterra equation usually represents a simple non-linear model for the complex relationship between two populations where one population gains at the expenditure of the other hand.

2. Materials and Methods

2.1 Basic Lotka–Volterra Predator–Prey Model

According to [2], a model possible to be constructed as a differential equations system of the Lotka–Volterra. This takes into consideration the time−dependent growth of a species whose population will represent by function (t). Then $\frac{dx}{dt}$ represents as the change in the population of prey, x, as t changes, and $\frac{dy}{dt}$ represents the change in the population of predator, y, as t changes. The basic Lotka−Volterra is shown as

$$\frac{dx}{dt} = bx - pxy \qquad \frac{dy}{dt} = dxy - ry \tag{1}$$

2.2 Two–predator, One–prey Model

The two–predator, one–prey model is a variation on the basic predator–prey model Lotka–Volterra that accounts for a situation in which two predator populations are present and both predate as their primary source of food on a single prey species. According to the study of [2], the simplest system of equations modelling this type of behaviour is as follows:

$$\frac{dx}{dt} = ax - bxy - cxz \qquad \frac{dy}{dt} = dxy - ey \qquad \frac{dz}{dt} = fxz - gz \tag{2}$$

2.3 Parameter Estimation Techniques

In this research, the dynamical behaviour of the predator–prey model for corporation competitive by using parameter estimation techniques. Subsequently, to approximate the parameters, regression analysis by using Excel Software will be used, and decide whether the model is sufficiently optimistic to use Matlab Software to further estimate data parameters. For the results of Excel to be considered sufficiently important, it is enforced with two conditions which are the approximation of the parameter of Excel should be $p < 0.05$, and the term of interaction should be non-zero. The analysis of statistical will be conducted after the data have been fitted to decide whether the findings significant. Then, further analysis can be performed when the model is significant.

2.4 Hopf Bifurcation

Next, the existence of Hopf bifurcation in the economic dynamical system's policy will be investigated. According to [3], by considering the general second-order system as:

$$\frac{dx}{dt} = \dot{x} = F(x, y, \lambda) \qquad \frac{dy}{dt} = \dot{y} = G(x, y, \lambda) \tag{3}$$

The problem is solving by using Jacobian Matrix as shown:

$$\boldsymbol{J}(x_0, y_0) = \begin{bmatrix} b - py & -px \\ dy & dx - r \end{bmatrix} \tag{4}$$

2.5 Logistic equation

Logistic equation is dealing with the possibility of predicting long-term development on the basis of logistic curves. Matlab software will be used in order to stimulate the result and analyse the result of prediction for the structure of economic growth for the corporation. In the research [4], the formula of the logistic equation is shown as

$$\frac{dN}{dt} = rN(1 - \frac{N}{K}) \tag{5}$$

The exact solution for equation (5) is

$$N(t) = \frac{N_0 K e^{rt}}{[K + N_0(e^{rt}) - 1)]} \tag{6}$$

where r is the growth rate and K is carrying capacity. There is a certain value K in Eq.6 which makes zero on a bracket term. Three important features of the logistic development are:

1. $\lim_{t \to \infty} N(t) = K$, the population will exceed their carrying capacity.
2. The relative growth rate, $\frac{1}{N}\frac{dN}{dt}$, declines linearly with population growth increasing.
3. The population at the point of inflection (where growth rate is maximum),N_{inf}, is exactly half the carrying capacity.

2.6 Data Observation

The collection of data is explained as the procedure for the collection, measurement and analysis of accurate research insights using standard validated techniques. Every data sets used are approachable to the public. Except where stated otherwise, all data taken from Bursa Malaysia website (www.bursamalaysia.com). The data gained from Bursa Malaysia website are based on the monthly stock volume from the company for the range three months. This research only examine the success of competitive corporations as measured by indicators of market share by assuming few competitive corporations in Malaysia. The parameter used in this study are b as the growth of prey in the absence of predator activity, p as the effect of predator predation on prey, d as the growth of predator in perfect condition and r is the decreasing of predator from natural causes.

3. Results and Discussion

3.1 Formulation for predator-prey model for two-predator and one prey

3.1.1 Leslie-Gower predator-prey model

The dynamical behaviour of three populations a Leslie-Gower predator-prey model have presented, where two predators competing on one prey can be written as:

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - \beta_1 x y_1 - \beta_2 x y_2$$

$$\frac{dy_1}{dt} = y_1 r_1 \left(1 - \frac{y_1}{x}\right) - \alpha_1 y_1 y_2$$

$$\frac{dy_2}{dt} = y_2 r_2 \left(1 - \frac{y_2}{x}\right) - \alpha_2 y_1 y_2 \tag{7}$$

where x, y_1 and y_2 are correlate respectively to the population of prey, the first predator population and the second predator population at each instant of time. Now, the model can be written in non-dimensional form to minimize the number of parameters, by chosen:

$$x = kx, y_1 = ky, y_2 = kz, a = \frac{\beta_1 k}{r}, b = \frac{\beta_2 k}{r}, s = \frac{r_1}{r}, \alpha = \frac{a_1 k}{r}, p = \frac{r_2}{r}, \beta = \frac{a_2 k}{r}.$$

The form of system (7) can be simplified as:

$$\begin{aligned} \frac{dx}{dt} &= x(1-x) - axy - bxz \\ \frac{dy}{dt} &= sy\left(1 - \frac{y}{x}\right) - \alpha yz \\ \frac{dy_2}{dt} &= pz\left(1 - \frac{z}{x}\right) - \beta yz \end{aligned} \tag{8}$$

where x, y and z compose population density of prey population, the first predator population and the second predator population at time, respectively.

3.1.2 3-Dimensional predator-prey Lotka-Volterra

It is supposed that the two predator species compete entirely exploitatively with no intervention between rivals, the growth rate of the prey species is logistic or linear in the absence of predation, respectively, and the functional response of predator is linear. Then, the model is shown as:

$$\begin{aligned} \frac{dS(t)}{dt} &= S(t)\left(r_3 - \frac{1}{k}S(t) - b_1 x_1(t) - b_2 x_2(t)\right) \\ \frac{dx_1(t)}{dt} &= x_1(t)\left(-r_1 + a_1 S(t)\right) \\ \frac{dx_2(t)}{dt} &= x_2(t)\left(-r_2 + a_2 S(t)\right) \end{aligned} \tag{9}$$

where $x_i(t)$ for $i = 1,2$ represents the population density of the i −th predator at the time t, $S(t)$ represents the population density of the prey at the time t, $r_3 > 0$ is the inherent rate of growth of the prey, $K > 0$ is the carrying capacity of the prey, which describes the abundance of resources of prey. $K > \infty$, the increase boundless of prey, which implies that the growth rate of the prey species is linear in the existence of predation; $b_i > 0$ is the effect of the i −th predation on the prey, $r_i > 0$ is the natural death rate of the i −th predator in the absence of prey, a_i is the efficiency and the propagation rate of the i −th predator in the presence of prey.

3.1.3 Modified predator-prey Lotka-Volterra

The model based on the modified of two dimensional Lotka-Volterra predator-prey model where includes logistic growth model of two species, a carrying capacity of the prey, a carrying capacity of the predator and a predatory factor. The modified three dimensional Lotka-Volterra predator-prey model also uses a nonlinear system of equations:

$$\begin{aligned} \dot{x} &= ax - bx^2 - cxy - dxz \\ \dot{y} &= -ey - fxy \\ \dot{z} &= -gz - hzx + ixz + jyz \end{aligned} \tag{10}$$

The system of differential equation in equation (10) modelling the population dynamics of a predator y, a scavenger z, and the prey x, where a is the growth rate of x, b is connected to the carrying capacity of x, c is the rate of change of the x due to the existence of y, d is the rate of change of x due to the existence of z, e is the natural death rate of y, f is the rate of change of y due to the existence of x, g is the natural death rate of z, i is the rate of change of z due to the existence of x and finally, j is the rate of change of z due to the existence of y. By using a change of coordinates system, the system of equation (10) can be modified into system

$$\begin{aligned} \dot{x} &= ax - bx^2 - xy - xz \\ \dot{y} &= -by - xy \\ \dot{z} &= -cz - dzx + exz + fyz \end{aligned} \tag{11}$$

The Lotka-Volterra predator prey model with a scavenger, equation (11) can exhibit the possible population trends when there is interaction between a predator, a prey and a scavenger population.

3.2 Dynamical behaviour for corporation competitive

3.2.1 Predator-prey Lotka-Volterra model

For first competitive corporation, the relationship between Sime Darby Property Berhad and IOI Properties Group Berhad were examined. By using an Excel Software, utilizing IOI Properties Group Berhad as the prey population and Sime Darby Property Berhad as the predator population, after approximating the parameters on the set of data, it is found that the $p - value$ is 0.018139 which is less than 0.05. The interaction terms for the one-predator, one-prey are calculated as non-zero, and, additionally, the model is significant and this implies there is interaction between the two corporations, and thus the model is truly an interactive predator prey model. The system of differential equation with the parameter from the Excel result is shown as:

$$\frac{dx}{dt} = 19131x - 0.2683xy \tag{12}$$

$$\frac{dy}{dt} = 0.3338xy - 30435y \tag{13}$$

From the equation (12), it is shown that the economic growth for IOI Properties Group Berhad (prey) in perspective of number of customer is increases 19131 customers in the absence of interaction with Sime Darby Property Berhad (predator). The value of 0.2683 is shown as the rate of change effect of the IOI Properties Group Berhad (prey) when there is predation from the Sime Darby Property Berhad (predator). From the equation (13), the value of 0.3338 is shown as the rate of change over the time of the effect from the predation of Sime Darby Property Berhad towards IOI Properties Group Berhad. It is shown that the number of customer for the Sime Darby is decreases as 30435 customers. Next, since the model is significant, thus, this study can use Matlab Software for further analysis.

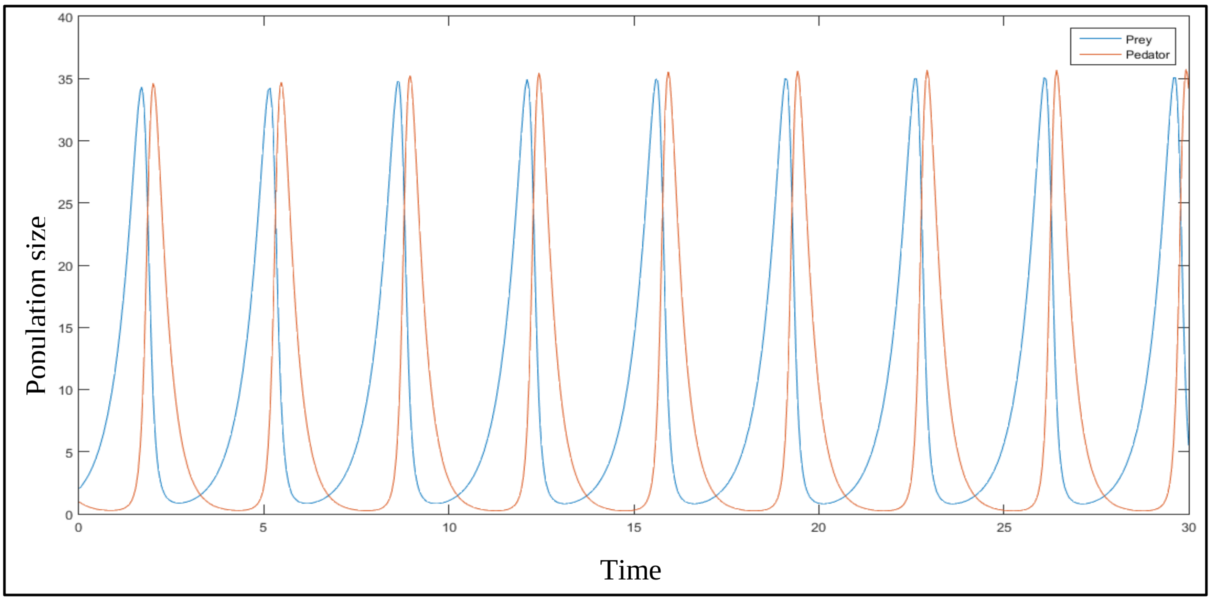

Figure 1: Predator prey interaction graph for IOI Properties Group Berhad and Sime Darby Property Berhad

From the Figure 1, it can clearly see from the graph that a great enough increasing in the number customer of Sime Darby Property Berhad (predator) leads to a decrease number of customer for IOI Properties Group Berhad.

For next competitive corporation, this study examines between AEON CO. (M) BHD and Parkson Holdings Berhad. After approximating the parameters on the set of data, by utilizing Parkson Holdings Berhad as the prey population and AEON CO. (M) BHD as the predator population, it is found that the $p - value$ is 0.055737 which is more than 0.05. However, even if the interaction terms for the one-predator, one-prey are calculated as non-zero, and thus the model is not an interactive predator-prey model and the model is not significant. For the data ranges tested, this implies that AEON CO. (M) BHD's (predator) success does not impact Parkson Holdings Berhad's (prey) success.

3.2.2 Two-predator and one prey model

Furthermore, this study also considered a two-predator and one prey model. For this model, by utilizing Axiata Group Berhad, Maxis Berhad and Digi.Com Berhad's monthly stock volume as the two predator populations and one prey population. After approximating the parameters on the set of data by using an Excel Software, by making use of Digi.Com Berhad as the prey population and Axiata Group Berhad and Maxis Berhad as the first and second predator populations, respectively, it is found that the $p - value$ is 0.018139 which is less than 0.05 and the interaction terms for the two-predator, one-prey model are calculated as non-zero, and, additionally, the model is significant. The system of differential equation with the parameter from the Excel result is shown as:

$$\frac{dx}{dt} = 12718x - 0.00203xy - 1.320032xz \tag{14}$$

$$\frac{dy}{dt} = 2.669144xy - 22648y \tag{15}$$

$$\frac{dz}{dt} = 0.45044xz - 3612z \tag{16}$$

From the equation (14), it is shown that the economic growth of Digi.com Berhad (prey) in perspective number of customer is increases as 12718 customers in the range three months in the absence of interaction with Axiata Group Berhad (first predator) and Maxis Berhad (second predator).

The rate of change of the Digi.Com Berhad (prey) due to the presence of Axiata Group Berhad as the first predator is shown as 0.00203. The rate of change of Digi.Com Berhad due to the presence of Maxis Berhad as the second predator is shown as 1.320032.

From the equation (15), it is shown that Axiata Group Berhad (first predator) have lose their customer as 22648 customers are going out from continue using the services from Axiata Group Berhad (first predator). The value of 2.669144 is shown as the rate of change of Axiata Group Berhad (first predator) due to the predation towards Digi.Com Berhad (prey).

From the equation (16), the number of customer that using the services from Maxis Berhad (second predator) is decrease as 3612 customers. The value of 0.45044 is the rate of change of Maxis Berhad (second predator) of the predation towards Digi.Com Berhad (prey). From the result, it can clearly see that both Axiata Group Berhad (first predator) and Maxis Berhad (second predator) lose their customer without any present or interaction with other company.

3.2.3 Logistic Equation

Recall back from the result between the IOI Properties Group Berhad (prey) and Sime Darby Property Berhad, since the Lotka-Volterra model is significant and it is possible to perform further analysis which is Logistic Equation. In economic, the logistic function can be used to illustrate the progress of the diffusion of an innovative through its life cycle.

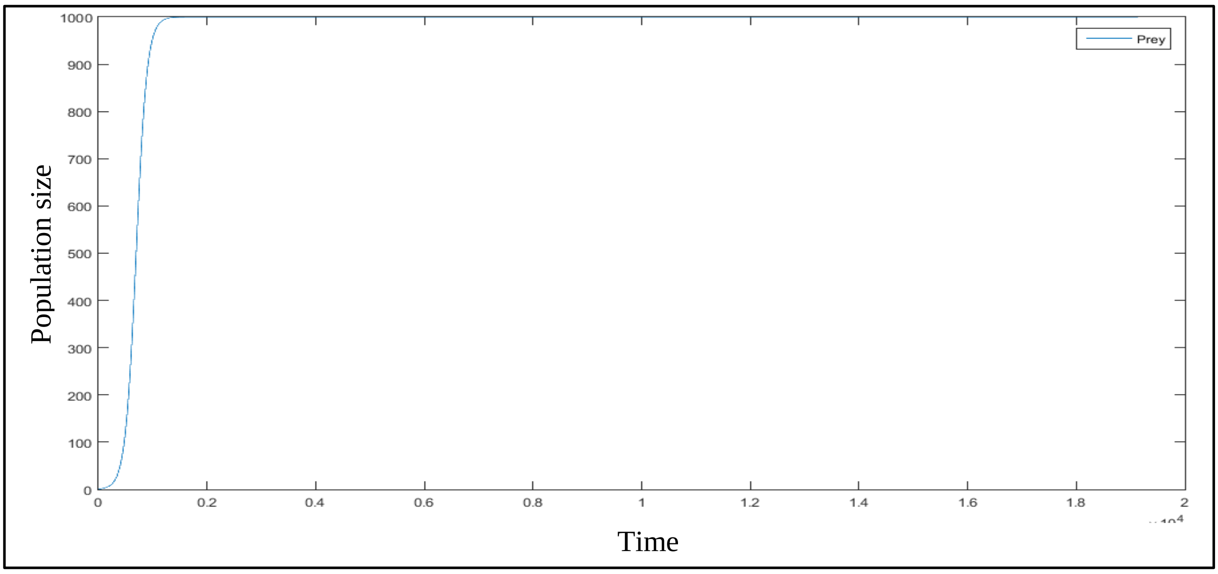

Figure 2: IOI Properties Group Berhad (Prey) logistic graph

In Figure 2, the IOI Properties Group Berhad (prey) grows at $K = 1000$ under the assumption that IOI Properties Group Berhad (prey) have a great service at all time, and will decrease till the Sime Darby Property Berhad (predator) effected them**.** The logistic curve allows to evaluate the future share of national economics in gross domestic product and to approximate the competitiveness of these economies. This lead to a period of industry growth for IOI Properties Group Berhad (prey).

Next, from Figure 3, the Sime Darby Property Berhad (predator) which act as a predator population in the system of Lotka-Volterra will grow exponentially over the time till reach its limited in predation activities. It is mean that the growth of an economy for Sime Darby Property Berhad (predator) is proportional to the current size of the economy.

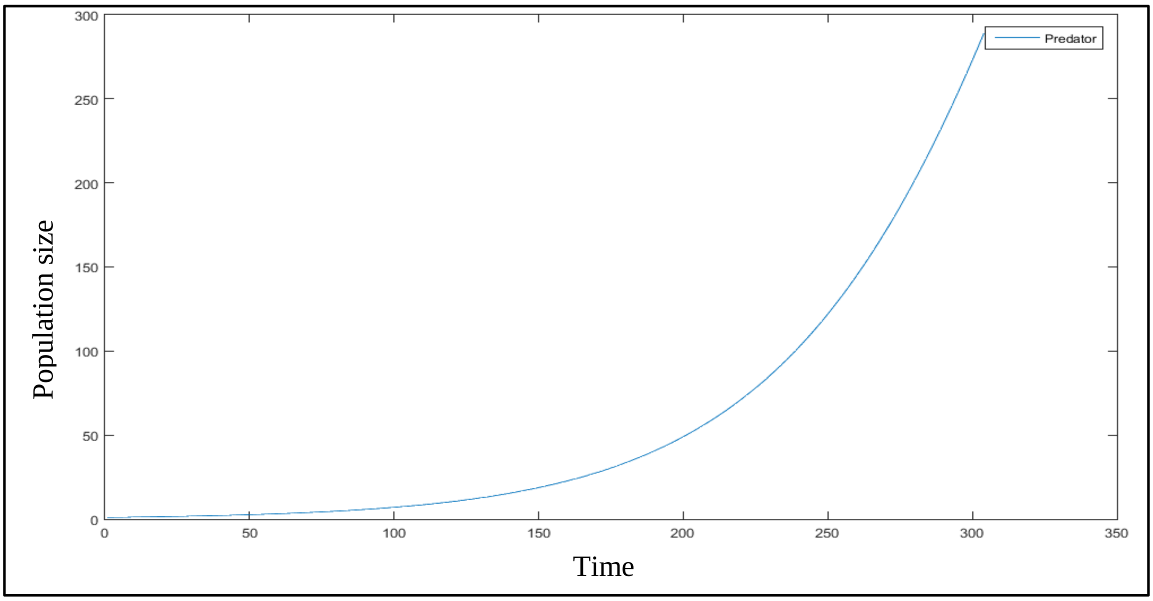

Figure 3: Sime Darby Property Berhad (Predator) logistic graph

4.3 Hopf Bifurcation analysis

For this section, the existence for fixed point will be shown and analysis of the stability for fixed points and the existence for Hopf Bifurcations were examined. From the equation of Lotka-Volterra model, it is assumed that the equation has an equilibrium (x_0, y_0) for $\boldsymbol{\lambda} = \boldsymbol{\lambda}_0$and that the Jacobian Matrix for the system is

$$\boldsymbol{J}(x,y) = \begin{bmatrix} 19131 - 0.2683y & -0.2683x \\ 0.3333y & 0.3333x - 30435 \end{bmatrix} \quad (17)$$

For first fixed point when estimated at the steady-state of $J(0,0)$ and the Jacobian matrix J is shown as:

$$\boldsymbol{J}(0,0) = \begin{bmatrix} 19131 & 0 \\ 0 & -30435 \end{bmatrix} \quad (18)$$

Then the corresponding eigenvalues of the Jacobian matrix are $\boldsymbol{\lambda}_1 = 19131$ and $\boldsymbol{\lambda}_2 = -30435$. The fixed point at the origin shown as saddle point since both of the eigenvalues are greater than zero. The stability of this fixed point is significance.

For the second fixed point, by estimating $\boldsymbol{J}$ at the next fixed point leads to

$$\boldsymbol{J}\left(\frac{30435}{0.3333}, \frac{19131}{0.2683}\right) = \begin{bmatrix} 0 & \frac{-0.2683(30435)}{0.3333} \\ \frac{19131(0.3333)}{0.2683} & 0 \end{bmatrix}$$
$$\boldsymbol{J}\left(\frac{30435}{0.3333}, \frac{19131}{0.2683}\right) = \begin{bmatrix} 0 & -24499.58 \\ 23765.7931 & 0 \end{bmatrix} \quad (19)$$

Hence, the eigenvalues of the matrix are $\boldsymbol{\lambda}_1 = i\sqrt{582251985.00}$ and $\boldsymbol{\lambda}_2 = -i\sqrt{582251985.00}$. As the both of the eigenvalues are purely imaginary and conjugate to each other's, this fixed point is elliptic, so the solutions are periodic and oscillating on a small ellipse around the fixed point. Usually, Hopf Bifurcation happen when the equilibrium stability adjusts due to the changing of parameter. When the combinations of parameter of this equilibrium is concentratedly observed, it can clearly see that the certainty of IOI Properties Group Berhad (prey) and Sime Darby Group Berhad (predator) should minimized the effect of competition while maximized the effecting of innovation and inhibition. Due to high creativity and less competition from each other, IOI Properties Group Berhad (prey) and Sime Darby Group Berhad (predator) can maintain a positive stable coexistence equilibrium.

4. Conclusion

The primary concern of the study is to examine the competitiveness relationship between IOI Properties Group Berhad and Sime Darby Property Berhad, between AEON CO. (M) BHD and Parkson Holdings Berhad, and between Axiata Group Berhad, Maxis Berhad and Digi.Com Berhad. Utilizing the daily data input in the range of three months from 1 January 2020 to 31 March 2020. The problems are solved by using parameter estimation techniques by the use of Excel Software to stimulate the result. From the simulation, the estimated coefficient in the interaction terms is non-zero with a $p - value < 0.05$. Since the result between Sime Darby Property Berhad (prey) and IOI Properties Group Berhad (predator) are significant, there is a possibility to perform further analysis.

Further, this study performs the logistic equation and Hopf bifurcation analysis. The logistic equation is performed by using Matlab Software. The result shows that the logistic curve for Sime Darby Property Berhad and exponentially curve for IOI Properties Group Berhad. Moreover, this study also investigated the presence of Hopf Bifurcation for the economic dynamical system. When performing equilibrium stability by Hopf bifurcation, the result shows that the damped oscillation of both IOI Properties Group Berhad (prey population) and Sime Darby Property Berhad (predator population) and is reaching a stable equilibrium state. Thus, it means that both companies can coexist in the market for a period of time.

Acknowledgement

Authors would like to thank the Faculty of Applied of Sciences and Technology, Universiti Tun Hussein Onn Malaysia for the support.

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