Enhanced Knowledge in Sciences and Technology Vol. 2 No. 1 (2022) 001-010
© Universiti Tun Hussein Onn Malaysia Publisher's Office

EKST

Homepage: http://publisher.uthm.edu.my/periodicals/index.php/ekst
e-ISSN : 2773-6385

Risk Aversion under Fuzzy Set and System Environment

Shubanath Thejani Mohammed Sayeed Shafaraz[1], Kavikumar Jacob[1]*

[1]Department of Mathematics and Statistics, Faculty of Applied Sciences and Technology,
Universiti Tun Hussein Onn Malaysia, 84600 Pagoh, Johor, MALAYSIA

*Corresponding Author Designation

DOI: https://doi.org/10.30880/ekst.2022.02.01.001
Received 20 June 2021; Accepted 29 November 2021; Available online 1 August 2022

Abstract: Risk aversion play an important role in economics, finance, psychology and especially decision making. This research conducts a study using random data by proving Arrow-Pratt measures using Jensen-type operators. The methodology is based on the objectives of the study which are determining the risk aversion by using fuzzy numbers under utility function, proving Arrow-Pratt measures by using Jensen type operators and determining risk premium under fuzzy risk aversion. The methodology includes formulating utility functions under decision making, triangular and trapezoidal fuzzy number is introduced to reduce the complexity of determining the utility function of a decision maker, proving Pratt's theorem for possibilistic risk aversion associated with fuzzy number, a utility function and weighting function under Jensen type operators and also risk premium was set defining as a measure of risk aversion discovered and explain using the possibilistic expected value using Arrow-Pratt formula The results and discussion yields all the three objectives based on the methodology discussed. The main notions are the possibilistic risk premium and the possibilistic relative risk premium associated with a fuzzy number and a utility function using Arrow-Pratt theorems with Jensen type operators under risk aversion.

Keywords: Risk Aversion, Fuzzy Set And System Environment, Risk Premium, Utility Function, Jensen-Type Operators, Arrow-Pratt Measures

1. Introduction

Risk aversion is traditionally defined in the context of lotteries over monetary payoffs. However, one can also consider risk aversion when the outcomes of risky lotteries may not be measurable in monetary terms [15]. Zadeh [12] initiated possibility theory in 1978 as a way to approach risk aversion. Risk theory is developed traditionally in the context of probability theory. Fuzzy logic models are more convenient for incorporating different expert opinions and more adapted to cases with insufficient and imprecise data. Providing a framework in which experts' input and experience data can jointly assess the uncertainty and identify major issues [11]. The risk situation has been described by fuzzy numbers

**Corresponding author: kavi@uthm.edu.my*
2022 UTHM Publisher. All rights reserved.

and the notions and results on possibilistic risk aversion are expressed by a possibilistic indicator [6]. Risk aversion under fuzzy numbers is to be said that the main notions are the possibility risk premium and the possibility relative risk premium associated with a fuzzy number and a utility function. During an uncertainty in life, when we face some partial information, uncertainty theory could not be used as a model to determine the risk aversion. Hence, this study considers the fuzzy number as a tool to overcome this kind of lack of risk aversion theory.

2. Materials and Methods

2.1 Risk aversion using fuzzy numbers under utility function

2.1.1 The utility function is formulated

This research first considers the concept of utility based on the risk aversion, which is used to evaluate and compare the various situations against the risk when the insured or insurer taking decisions. The utility function $u(\cdot)$ for economic models is based upon the income and profit of the given individual. The utility function has to be expressed by any of the following functions (Quittard-Pinon 2003) [16] such as:

- Quadratic ⇒ $u(x) = a \cdot x - b \cdot x^2$
- Logarithmic ⇒ $u(x) = \ln(x)$
- Power ⇒ $u(x) = w^{\lambda},\ \lambda < 1$
- Exponential ⇒ $u(x) = -e^{-a \cdot x}$

Thus, this study considers the decision-maker attitude towards the risk aversion, so the decision-maker prefers the excepted value as follows

$$\lambda u(x) + (1-\lambda)u(y) \leq u(\lambda x + (1-\lambda)y), \forall \lambda \in [0,1] \qquad \textit{Eq. 1}$$

where x is the maximum gain with the probability λ, whereas the minimum gain is y with the probability $1-\lambda$ [18].

2.1.2 Fuzzy numbers

In order to reduce the complexity of determining the utility function of a decision-maker, this study applying for the triangular number and the trapezoidal fuzzy number to the concept of the individual utility level which will give us mathematical accuracy to human thinking.

Definition 3.1 Triangular Fuzzy Number

A triangular fuzzy number A is represented by the (a_1, a_2, a_3) is defined by the membership function [12]

$$\mu_A(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ \dfrac{x-a_2}{a_2-a_3} & a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases} \qquad \textit{Eq. 2}$$

where $[a_1, a_3]$ is the support range, and the point $(a_2, 1)$ is the normal.

Definition 3.2 Trapezoidal Fuzzy Number

A trapezoidal fuzzy number A is represented by (a_1, a_2, a_3, a_4) is defined by the membership function [10]:

$$\mu_A(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ \dfrac{x-a_4}{a_3-a_4}, & a_3 \le x \le a_4 \\ 0 & \text{otherwise} \end{cases} \qquad \textit{Eq. 3}$$

2.1.3 Model for determining the Utility Function

When the decision-maker choice is risk-averse, it is important to evaluate a certain equivalent or guarantee money by the use of trapezoidal fuzzy numbers which depends on decision-makers' initial wealth W_0 [5]. Mathematically, according to the risk-aversion, the utility of the final wealth expected value is

$$E[u(W_0 + x)]. \qquad \textit{Eq. 4}$$

Then the utility of the final expected wealth

$$u(W_0 + E(x)) \qquad \textit{Eq. 5}$$

is different from the expected value of the final wealth utility. Now, we can find the certainty equivalent (CE) as follows

$$CE = u^{-1}\{E[u(W_0 + x)]\} \qquad \textit{Eq. 6}$$

Adopt the Lixandroiu [4] steps as follows

Step 1: Draw the decision tree depends on the lottery and find the utility function of initial and final points.

Step 2: Determine the certainty equivalent value as a trapezoidal fuzzy number.

Step 3: Determine the utility of the certainty equivalent value. Then, determine the other values of the utility function and trace the concave curve of utility function to decide the decision maker is risk aversion.

2.2 Arrow-Pratt measures using Jensen-type operators

The agent is risk averse if she is willing to pay more than E_x for an insurance contract paying out the monetary equivalent of a random outcome X, regardless her initial wealth w. One may prove that the agent is risk averse if and only if the function u is concave, that is,

$$u(ax + (1-a)y) \ge au(x) + (1-a)u(y)$$
$$\text{for all } x, y \text{ and } 0 < a < 1 \qquad \textit{Eq. 7}$$

Moreover, if two agent have the same initial wealth and the i-th agent has twice differentiable utility function u_i , $i = 1, 2$, then the first of them is more risk averse (wants to pay not less than the other agent), if and only if $\alpha_1(x) \ge \alpha_2(x)$ for all x, where α_i(x) is the coefficient of absolute risk aversion of the i-th agent defined as:

$$\alpha(x) = -\frac{u''(x)}{u'(x)} \text{ for } u \in \{u_1, u_2\}. \qquad \textit{Eq. 8}$$

It is also called the Arrow-Pratt index.

2.3 Risk premium under risk aversion

The risk premium ρX (associated with the random variable X and the utility function u) is defined by the identity

$$E\big(u(X)\big) = u(E(X) - \rho X) \quad \textit{Eq. 9}$$

Let us assume that u is twice differentiable, strictly concave and increasing. Then,

$$\rho x = -\frac{1}{2}\sigma^2 x \, \frac{u''(E(X))}{u'(E(X))} \quad \textit{Eq. 10}$$

where $\sigma^2 x$ is the variance of X

The Arrow-Pratt index (= the coefficient of absolute risk aversion) associated with a utility function u is introduced by the equality:

$$\alpha(x) = \frac{u''(x)}{u'(x)} \; for \; all \; x \in \mathbb{R} \quad \textit{Eq. 11}$$

The risk premium ρx can be expressed in terms of the Arrow-Pratt index and of two probabilistic indicators (expected value and variance). Thus the Arrow-Pratt index can be viewed as a measure of the risk aversion of the agent represented by the utility function u.

3. Results and Discussion

3.1 Risk aversion by using fuzzy numbers under utility function

3.1.1 Risk aversion under utility function

Individual A has the opportunity to participate in two gambles. In the first, a referee will flip a coin, and if it lands heads, Individual A will receive an old rare coin (Individual A is a coin collector). In the second, the referee will flip a different coin, and if it lands tails, Individual A will receive a nice pair of shoes. Individual A believes both coins to be fair. Now a trickster comes along and offers Individual A sort of an insurance: for a few cents, the trickster will rig the game so that the first coin determines both outcomes – if it lands heads, Individual A gets the rare coin, and if it lands tails, Individual A gets the shoes. Therefore, Individual A is guaranteed to receive some prize. Individual A values the two goods independently in the sense that having one does not add to or decrease from the value of having the other. He decides that the trickster's deal is worthwhile and it would be nice to guarantee that he gets something no matter what. So, he decides to pay a few cents to rig the game. We can represent his options schematically as follows:

Table 1: Offers between two deals

	HH	HT	TH	TT
Deal 1	Coin	Coin and shoes	Nothing	Shoes
Deal 2	Coin	Coin	Shoes	Shoes

Individual A prefers deal 2 compared to deal 1 and this seems very much reasonable as many of the players will have similar preferences. However, standard decision theory [9] rules this out as an absurd decision.

Individual B values small amount of money, receiving RM 50 is just the same whether individual B starts with RM 0 or RM 50, and feels similarly about all small increments of money. It can be said that individual B values money linearly: every ringgit received is worth as much to as the previous ones, at least for amounts of money less than RM 200. Individual B prefers RM 50 to a coin flip between RM 0 and RM 100. If individual B takes the former, then RM 50 would be obtained, and the possibility of getting RM 100 is not enough to make up for the for the possibility of obtaining RM 0. Individual B would rather take RM 50 as a guarantee money than take that chance. These preferences also might seem appealing to many people, and are at least understandable. But standard decision theory cannot represent individual B's preferences, and judges like Individual A to be an irrational decision.

Finally, in a classic example due to Maurice Allais [2], commonly known as the Allais paradox, people are presented with a choice between X_1, X_2, X_3 and X_4, where the gambles are as follows:

X_1: RM 5,000,000 with probability 0.1, RM 0 otherwise.

X_2: RM 1,000,000 with probability 0.11, RM 0 otherwise.

X_3: RM 1,000,000 with probability 0.89, RM 5,000,000 with probability 0.1, RM 0 otherwise.

X_4: RM 5,000,000 with probability 1.

People tend to choose X_1 over X_2, and X_4 over X_3: in the first pair, the minimum amount that one stands to walk away with is the same for either gamble, and there is not much difference in one's chances of winning some money. However, X_1 yields higher winnings; in the second pair, however, the minimum amount that X_4 yields is a great deal higher than the minimum that X_3 yields. Again, these preferences are understandable (most people express them), but standard decision theory cannot accommodate them, and, again, must judge the decision to be absurd or irrational.

3.1.2 Risk aversion using fuzzy numbers

A recent research proposes using the concept of expected value of a fuzzy number developed in [18], which for a fuzzy number $\tilde{a}$, we symbolize as $EV[\tilde{a},\beta]$. This value can be obtained by introducing the decision-maker risk aversion with the parameter β, where $0 \leq \beta \leq 1$:

$$EV[\tilde{a},\beta] = (1-\beta)\int_0^1 \underline{a}(\alpha)d\alpha + \beta\int_0^1 \overline{a}(\alpha)d\alpha \qquad \textit{Eq. 12}$$

So, if $\tilde{a}$ is the triangular fuzzy number (a, l_a, r_a):

$$EV[\tilde{a},\beta] = a - \frac{l_a}{2} + \frac{\beta}{2}(l_a + r_a) \qquad \textit{Eq. 13}$$

The fuzzy numbers constitute a class of possibilistic distributions with remarkable properties and with important applications. In this section, a series of definitions and results on the expected value $E_f(A)$ and the variance $Var_f(A)$ of a fuzzy number A is recalled. Two propositions on the expected value are given (Propositions 4.1 and 4.2) which was used in proving the main results of the section in chapter 3 under section 3.4. The main contribution of the section is the introduction of a new possibilistic indicator the variance $Var_f^*(A)$ for which two calculation formulae are established. $Var_f^*(A)$ is a possibilistic variance different from $Var_f(A)$ and it is used in evaluating the possibilistic risk aversion [20].

Let A be a fuzzy number such that for any $\gamma \in [0,1]$, the γ-level set $[A]^\gamma = [a_1(\gamma)a_2(\gamma)]$ is non-degenerate $\big(a_1(\gamma) \neq a_2(\gamma)\big)$.

The notion of central value introduced below allows us to define and study the expected value and variance of a fuzzy number.

The central value of $[A]^\gamma$ is the real number

$$centre([A]^\gamma) = \frac{1}{a_2(\gamma) - a_1(\gamma)}\int_{a_1(\gamma)}^{a_2(\gamma)} x\,dx = \frac{a_1(\gamma) + a_2(\gamma)}{2} \qquad \textit{Eq. 14}$$

If $g : \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function then $g(A)$ is defined by the Zadeh sup-min extension principle [13]

$$g(A)(y) = \begin{cases} \sup A(x) \\ g(x) = y \\ 0 \end{cases} \quad if\ there\ exist\ x \in \mathbb{R}\ such\ that\ g(x) = y, \qquad Eq.\ 15$$

If $g : \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function then the central value of the γ-level set of $g(A)$ is defined by

$$centre(g[A]^{\gamma}) = \frac{1}{a_2(\gamma) - a_1(\gamma)} \int_{a_1(\gamma)}^{a_2(\gamma)} g(x)dx \qquad Eq.\ 16$$

A non-negative and monotone increasing function $f: [0,1] \rightarrow \mathbb{R}$ is a weighting function if it satisfies the normality condition $\int_0^1 f(\gamma)\, d\gamma = 1$.

The expected value of a fuzzy number A with respect to a weighting function f is defined by

$$E_f(A) = \int_0^1 centre([A]^{\gamma}) f(\gamma)\, d\gamma \int_0^1 \frac{a_1(\gamma) + a_2(\gamma)}{2} f(\gamma)\, d\gamma \qquad Eq.\ 17$$

If $f(\gamma) = 2\gamma$ for $\gamma \in [0,1]$ then $E_f(A)$ is exactly the possibilistic mean value $\bar{M}(A)$ introduced in [8].

If $g : \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function then the expected value of $g(A)$ with respect to a weighting function f is defined by

$$\begin{aligned} E_f\big(g(A)\big) &= \int_0^1 centre([g(A)]^{\gamma}) f(\gamma)\, d\gamma \\ &= \int_0^1 \left(\frac{1}{a_2(\gamma) - a_1(\gamma)} \int_{a_1(\gamma)}^{a_2(\gamma)} g(x)dx \right) f(\gamma)\, d\gamma \end{aligned} \qquad Eq.\ 18$$

g is interpreted as a utility function, and $E_f\big(g(A)\big)$ as the possibilistic expected utility.

3.2 Proving Arrow-Pratt measures using Jensen-type operators

The following four conditions on a pair of (increasing, twice differential) Von Neumann-Morgenstern utility functions $u_a(\cdot)$ and $u_b(\cdot)$ are equivalent:

(1) $u_a(\cdot)$ is a concave transformation of $u_b(\cdot)$, in essence, $u_a(x) = \rho\big(\, u_a(x)\big)$ for some (necessarily increasing) concave function $\rho(\cdot)$.

(2) The Arrow-Pratt coefficients of absolute risk aversion satisfy the inequality

$$-\frac{u_a''(x)}{u_a'(x)} \geq \frac{u_b''(x)}{u_b'(x)} \text{ for all } x \qquad Eq.\ 19$$

(3) If k_a and k_b are such that $u_a(\, k_a) = \mathbb{E}_F\, u_a(x)$ and $u_b(\, k_b) = \mathbb{E}_F\, u_b(x)$ for some distribution $F(\cdot)$, then $k_a \leq k_b$.

(4) Suppose that $u_a(\cdot)$ and $u_b(\cdot)$ are concave. If r is known and $r > 0$, x is uncertain with $\mathbb{E}_F(x) > r$ and probability $(x < r) > 0$, and α_a and α_b respectively solve

$$\max_{0 \leq \alpha \leq I} \mathbb{E}_F u_a\big((I - \alpha)r + \alpha x\big) \qquad Eq.\ 20$$

and

$$\max_{0 \leq \alpha \leq I} \mathbb{E}_F u_b\big((I - \alpha)r + \alpha x\big) \qquad Eq.\ 21$$

then $\alpha_a \leq \alpha_b$.

Risk aversion is plainly related to concavity, hence to $u_a''(\cdot)$ and $u_b''(\cdot)$. But these, unlike the Arrow-Pratt coefficients $-\frac{u_a''(\cdot)}{u_a'(\cdot)}$ and $-\frac{u_b''(\cdot)}{u_b'(\cdot)}$, are not invariant to increasing linear transformations, and therefore cannot be linked as closely to the behavioural conditions (3) and (4) as the theorem's conclusion requires.

PROOF.

1. (1) ⟺ (2)

$$u_a(x) \equiv \rho\big(u_b(x)\big),$$
$$u_a'(x) \equiv \rho'\big(u_b(x)\big)u_b(x),$$
$$u_a''(x) \equiv \rho'(\cdot)u_b''(x) + \rho''(\cdot)\big(u_b''(x)\big)^2 \qquad \text{Eq. 22}$$

Thus,

$$-\frac{u_a''(x)}{u_a'(x)} \equiv \frac{-\rho(\cdot)u_b''(x) - \rho''(\cdot)\big(u_b'(x)\big)^2}{\rho'(\cdot)u_b'(x)} \equiv -\frac{u_b''(x)}{u_b'(x)} - \frac{\rho''(\cdot)}{\rho'(\cdot)}u_b'(x) \geq -\frac{u_b''(x)}{u_b'(x)} \qquad \text{Eq. 23}$$

because $\rho(\cdot)$ is increasing and concave. This proves (1) ⟹ (2). However to prove vice versa, (1) ⟸ (2), note that since $u_a(\cdot)$ and $u_b(\cdot)$ are both increasing functions of one variable, they can always be related, as in (1), by an increasing transformation $\rho(\cdot)$. (2) then shows that $\rho(\cdot)$ must be concave.

2. (1) ⟹ (3) depends on an important lemma known as Jensen's Inequality, which is true "in general" , but is proven here assuming twice differentiability:

If $f(y)$ is a concave function of one variable, then $\mathbb{E}f(y) \leq f(y)$.

3.3 Risk premium under Risk Aversion

In what follows, by deriving Arrow's risk premium when probability distortion takes place, that is when normalized decision weights, $\omega(p)$, rather than objective probabilities, p, are employed. Based on [15] and [8], decision weights are employed Arrow's risk-measure increases relative to the case where the objective probabilities are employed. This increase may be quite significant, and a positive risk premium $\left(p > \frac{1}{2}\right)$ is obtained "in the small" even if $U'' = 0$, and may be also obtained with $U'' >$ 0 [20].

Based on the Rothschild and Stiglitz [14] definition of risk. One agent is weakly more risk averse than another if he always chooses the less risk two alternatives whenever the other agent does. As shown previously, these two notions of risk aversion are identical under expected utility theory so they will always identify the same utility functions as more risk averse.

If one utility function is more concave than another then it demands a larger risk premium for bearing any risk in its entirety. But it does not follow that a more concave utility function always demands a larger risk premium for moving from one prospect to a Rothschild-Stiglitz [14] more risk prospect. This can be illustrated with the following example:

There are two risky projects. The first $\tilde{m}$, has two outcomes with probability p of paying 20 and a probability $1 - p$ of paying 0. The second project $\tilde{n}$, has the same probability $1 - p$ of paying 0 and a probability $p/2$ of paying 25 and 15. According to [13], the second project is riskier than the first because the added variation of ± 5 in $\tilde{n}$ when $\tilde{m} = 20$, is conditionally means zero-noise [9].

The risk premium that an investor would pay to give up $\tilde{n}$ in favour of $\tilde{m}$ is the solution to $\mathbb{E}[u(\tilde{n})] \equiv \mathbb{E}[u(\tilde{m} - \pi)]$. The figure shows the risk premium for two exponential utility functions with risk aversions $a = 0.05$ and 0.09. The premiums are plotted against the probability p of the higher outcome. The more risk averse utility function demands a higher risk premium than the less risk averse utility only for $p > 57.9\%$.

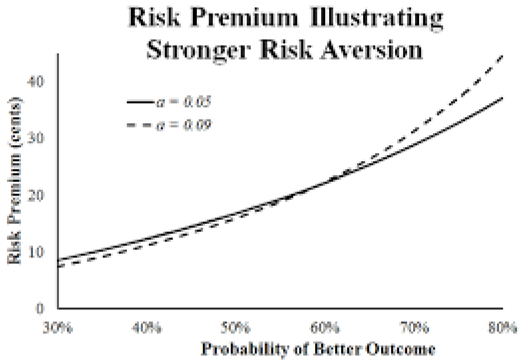

Figure 1: Risk Premiums Illustrating Stronger Risk Aversion

This figure illustrates that Arrow-Pratt risk is insufficient for an increase in risk to require a larger risk premium [10]. The risk premium here provides only partial insurance. It protects against the risk ± 5 risk, but not against the risk of getting 0 instead of 20. The Arrow-Pratt [10] theorem is only applicable for complete insurance.

This problem can also be analysed just as the simple Arrow-Pratt [9 and 11] problem. Because $\tilde{n}^d = \tilde{m} + \tilde{\varepsilon}$ with $\mathbb{E}[\tilde{\varepsilon}|m] = 0$, a Taylor expansion gives

$$\mathbb{E}[u(\tilde{m} - \pi_{n\to m})] \equiv \mathbb{E}[u(\tilde{n})] = \mathbb{E}[u(\tilde{m} + \tilde{\varepsilon})]$$

$$\mathbb{E}[u(\tilde{m})] - \pi_{n\to m}\mathbb{E}[u'(m)] \approx \mathbb{E}\big[\mathbb{E}[u(\tilde{m} + \varepsilon)|m]\big] \qquad \textit{Eq. 24}$$

$$\approx \mathbb{E}\left[u(\tilde{m}) + u'(\tilde{m})\mathbb{E}[\tilde{\varepsilon}|\tilde{m}] + \frac{1}{2}u''(\tilde{m})\mathbb{E}[\tilde{\varepsilon}^2|\tilde{m}]\right] \Rightarrow \pi_{n\to m} \approx \frac{\mathbb{E}[-u''(\tilde{m})var[\tilde{\varepsilon}|\tilde{m}]}{2\mathbb{E}[u'(\tilde{m})]}$$

When $\tilde{m}$ is not random, it refers to the Arrow-Pratt result [see 9, 11, 15] where $\pi \approx \frac{1}{2}A(x)var[\tilde{\varepsilon}]$. When $\tilde{m}$ is random, with a constant conditional variance for $\tilde{\varepsilon}$, then

$$\pi_{n\to m} \approx \frac{1}{2}var[\tilde{\varepsilon}] \times {}^{\mathbb{E}[-u''(\tilde{m})]}\Big/_{\mathbb{E}[u'(\tilde{m})]} \qquad \textit{Eq. 25}$$

which is almost the same result.

However, when the conditional variance depends on $\tilde{x}$, as it does in this example, the results can be quite different. Risk premium can be re-expressed as

$$\pi \approx \frac{\mathbb{E}[-u''(\tilde{m})var[\tilde{\varepsilon}|\tilde{m}]}{2\mathbb{E}[u'(\tilde{m})]} = \frac{\mathbb{E}\big[var[\tilde{\varepsilon}|\tilde{m}]\big]}{2\mathbb{E}[u'(\tilde{m})]}\frac{\mathbb{E}[-u''(\tilde{m})var[\tilde{\varepsilon}|\tilde{m}]}{\mathbb{E}[var[\tilde{\varepsilon}|\tilde{m}]]} \qquad \textit{Eq. 26}$$
$$= \frac{1}{2}var[\tilde{\varepsilon}]\frac{\mathbb{E}[-u''(\tilde{m}) \times var[\tilde{\varepsilon}|\tilde{m}]/\mathbb{E}\big[var[\tilde{\varepsilon}|\tilde{m}]\big]}{\mathbb{E}[u'(\tilde{m})]}$$

The numerator is a weighted average of the second derivative of the utility function where the weights are the conditional variances at different values of m. For most utility functions, $-u''(m)$ decreases with m so if $var[\tilde{\varepsilon}|\tilde{m}]$ increases with m, the smaller values of $-u''(m)$ will be overweighted leading to a risk premium that is smaller than predicted by the Arrow-Pratt measure. When $-u''$ decreases rapidly, the overweighting can be great enough to decrease the risk premium when risk aversion rises.

4. Conclusion

The first discussion based on the first objectives was split into two parts, where the first is risk aversion under utility function and the second is risk aversion using fuzzy numbers. The first section incorporated some types of decision-makers which is related to utility theory. The second section was discussed based on the normal and convex fuzzy set. The possibilistic risk premium is also described by the use of fuzzy numbers. The next discussion is based on the second objective that mostly about proving the Arrow-Pratt theorem using Jensen-type operators where Von Neumann-Morgenstern utility functions were used. Apart from that, Lipchitz's continuity was briefly discussed and used in this section. The final discussion is based on the risk premium that was initiated from the idea of Arrow [1] and Pratt [10]. The discussion comprises with decision-making, market premium, decreasing absolute risk premium, which is discussed closely related to risk aversion. The proposed research is mainly focused on the concept of risk aversion, so throughout this study, the word 'risk aversion' is kept on repeated even when it is compared to the attitude of risk decision-makers.

Acknowledgement

The author, Shubanath Thejani binti Mohammed Sayeed Shafaraz, final year student of Faculty of Applied Science and Technology, University Tun Hussein Onn Malaysia am using this opportunity to express my gratitude to the University for helping me in this project and giving me a great opportunity to delve in this research paper. I am thankful for the aspiring guidance, invaluably constructive criticism and friendly advice during the project work. I am sincerely grateful to them for sharing their truthful and illuminating views on a number of issues related to the project. The author would also like to thank the Faculty of Applied Sciences and Technology, Universiti Tun Hussein Onn Malaysia for its support.

References

[1] A. Klinke and O. Renn, (2002). A new approach to risk evaluation and management: Risk-based, precaution-based, and discourse-based strategies. Risk Analysis: An International Journal, vol. 22(6), pp. 1071-1094.

[2] Allais Maurice (1953). "Criticisms of the postulates and axioms of the American School." In Rationality in Action: Contemporary Approaches, Paul K. Moser, ed. Cambridge University Press, 1990. (Reprint of 1953 original).

[3] C., Carlsson, & R. Fuller, (2012). Fuzzy reasoning in decision making and optimization (vol. 82). Physica.

[4] D. Lixandroiu, (2015). A model for determining the utility function using fuzzy numbers. Bulletin of the Transilvania University of Brasov Series V: Economic Sciences, vol. 8(57) (2), pp. 469-478.

[5] Fl. Gh. Filip, (2002). Decizie asistată de calculator.Decizii decidenti. Metode şi instrumente de bază: Editura Tehnică şi Editura Expert, Bucureşti.

[6] H. Kwakernaak, (1978). Fuzzy random variables-I. Definitions and theorems. Information Sciences, vol. 15(1), pp. 1-29.

[7] I. Georgescu, and J. Kinnunen, (2012). A probabilistic approach to possibilistic risk aversion. In 2012 7th International Conference on Computing and Convergence Technology (ICCCT), IEEE, pp. 342-345.

[8] J. J. Buckley, (1987). The fuzzy mathematics of finance, Fuzzy Sets and Systems, vol. 21(3), pp. 257-273.

[9] Jonathan Ingersoll (2015). Risk and Risk Aversion. Journal of Business, pp. 1-29.

[10] J.W. Pratt, (1964). “Risk Aversion in the Small and in the Large,” Econometrica vol. 32, pp. 122–136.

[11] K. Shang, and Z. Hossen, (2013). Using fuzzy multiple criteria decision-making approach to enhance risk assessment for metropolitan construction projects. International Journal of Project Management, vol. 31(4), pp. 602-614.

[12] L.A. Zadeh, (1978). Fuzzy sets as a basis for a theory of possibility, Fuzzy Sets and Systems 1(1), 3-28.

[13] M.Z., Ahmad, & M. K. Hassan, (2010). A new approach for computing Zadeh’s extension principle. MATEMATIKA: Malaysian Journal of Industrial and Applied Mathematics, vol. 26, pp. 71-81, 2010.

[14] M., Rothschild, & J. E. Stiglitz, (1970). Increasing risk: I.A definition. Journal of Economic theory, vol. 2(3), pp. 225-243.

[15] P. Blavatskyy, (2008). Risk aversion. IEW-Working Papers. IEWWP370, Institute for Empricial Research in Economics, University of Zurich.

[16] P. Majlender, (2004). A normative approach to possibility theory and decision support. TUCS Doctoral Dissertation (Turku Centre for Computer Science, Turku, 2004).

[17] Quittard-Pinon, François. (2003). Mathématiques financiers. Paris: Éditions EMS.

[18] R. Feron, (1976). Ensembles aleatoires flous, C.R. Acad. Sci. Paris, Ser. A, vol. 282, pp. 903-906.

[19] R. Fullér, (2000). Introduction to neuro-fuzzy systems (vol. 2). Springer Science & Business Media.

[20] Vahid Khatibi and Gholam Ali Montazer (2009). Intuitionistic Fuzzy Set vs. Fuzzy Set Application in Medical Pattern Recognition. Artificial Intelligence in Medicine vol. 47, pp. 43-52.