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Solitary Waves in Fluid Filled Elastic Tube with Variable Radius

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Abstract: In this study, the solitary wave modulation in an inviscid fluid filled in an elastic tube with variable radius is studied**.** The artery is considered as a thin walled and pre-stressed elastic tube with variable radius and the blood is treated as an inviscid fluid. Reductive perturbation method is utilized in the long wave approximation and the various orders of differential equations are obtained. The differential equations are then solved and reduced to nonlinear evolution equation which is the variable coefficient Nonlinear Schrodinger (NLS) equation. After looking a progressive wave type of solution to the nonlinear evolution equation, the graphical outputs are studied and discussed. The results shown that the wave maintained its symmetrical bell-shaped curve propagates to the right as time going. The amplitude of wave remain unchanged when it modulated over the time. This is due to no resistance for blood flowing as the blood in this study is considered as an inviscid fluid.

Keywords: Wave Modulation, Nonlinear Schrodinger Equation With Variable Coefficient, Inviscid Fluid, Thin Wall Elastic Tube With Variable Cross-Section

1. Introduction

In general, blood flow refers to the movement of blood through a vessel, tissue or organ. Blood flow is initiated by the contraction of the ventricles of the heart. An artery acts as a blood vessel which carries blood from the heart to other parts of the body. Blood movement in arteries is led by an unsteady flow phenomena. Under varying hemodynamic conditions, the arteries are living organs that can adapt to and change. In order to develop mathematical model for blood flow in artery, various methods have been applied in the past decades. Porenta, and Young [1] developed a mathematical model of arterial blood flow by using finite-element method. The model of the equations are altered into a system of algebraic equations that can be solved on a high speed digital computer to yield volume rate of flow as functions of time and arterial position. Besides, Demiray [2] has examined the propagation of nonlinear waves by assuming the arteries as a thin walled, prestressed thin elastic tube and the blood as an inviscid fluid. The propagation of waves was studied by utilizing the extended Poincare'-Lighthill-Kuo (PLK) perturbation method. It has been observed that the head-on collision of two solitary waves is elastic and

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they maintain the initial properties after the collision. On the other hand, Demiray and Bakirtas [3] have studied the propagation of weakly non-linear waves in a inviscid fluid filled elastic tube by treating the artery as a tapered,thin walled, long and circularly conical prestressed elastic tube .They obtained the Korteweg-de Vries equation with a variable coefficient as the governing equation for their mathematical model which admits a solitary wave type solution with changing wave speed. Referring to the previous studies, most studies considered the artery as circularly conical, tapered and prestressed elastic tube rather than the artery as with variable radius.Therefore, in this study, the modulation of solitary wave is examined by assuming the artery as a prestressed, thin walled, elastic tube with variable radius and the blood as an incompressible inviscid fluid. Reductive perturbation method will be applied in this study in order to find out the mathematical model for wave modulation in inviscid fluid contained in elastic tube with variable radius.

2. Equations of the tube and fluid.

This section explains the equations of an incompressible inviscid fluid-filled prestressed elastic tube. The equations of fluid are given as follow [4]:

$$\frac{\partial w^*}{\partial t^*} + w^* \frac{\partial w^*}{\partial z^*} + \frac{1}{\rho_f} \frac{\partial P^*}{\partial t^*} = 0, \qquad Eq.\ 1$$

$$2\frac{\partial u^*}{\partial t^*} + 2w^* \left[f^{*\prime} + \frac{\partial u^*}{\partial z^*} \right] + (r_0 + f^* + u^*) \frac{\partial w^*}{\partial z^*} = 0, \qquad Eq.\ 2$$

w^* is the mean of fluid speed, t^* is time. z^* is a coordinate that located on axis when the changes of radius maintain its value, $f^*(z^*)$ is the function of a variable radius, ρ_f the mass density, P^* is the mean of fluid pressure, $\bar{v}$ is the viscosity for fluid flow, u^* is the function of displacement of the radius, and r_0 is the initial radius in the coordinate system. The $r_f = r_0 + f^* + u^*$is the final radius after deformation occurred.

The equation of motion of elastic tube in the radial direction could be written as follow [4]:

$$-\frac{\mu}{\lambda_2} \frac{\partial \Sigma}{\partial \lambda_2} + \mu R_0 \times \frac{\partial}{\partial z^*} \left\{ \frac{\left(f^{*\prime} + \frac{\partial u^*}{\partial z^*} \right)}{\left[1 + \left(f^{*\prime} + \frac{\partial u^*}{\partial z^*} \right)^2 \right]^{\frac{1}{2}}} \frac{\partial \Sigma}{\partial \lambda_1} \right\} + \frac{P_r^*}{H} (r_0 + f^* + u^*)$$

$$\times \left[1 + \left(f^{*\prime} + \frac{\partial u^*}{\partial z^*} \right)^2 \right]^{\frac{1}{2}} = \rho_0 \frac{R_0}{\lambda_z} \frac{\partial^2 u^*}{\partial t^{*2}}, \qquad Eq.\ 3$$

where

$$P_r^* = \left[1 + (f^{*\prime} + \frac{\partial u^*}{\partial z^*})^2 \right]^{-1/2} \times \left[P^* + 4\mu_v \frac{(f^{*\prime} + \frac{\partial u^*}{\partial z^*})}{(r_0 + f^* + u^*)} w^* \right].$$

R_0 is the radius of the tube, Σ is the strain energy density function membrane, μ is the shear modulus of the material of the tube, λ_z represents the axial stretch of the tube, λ_2 is the circumference of curves, P_r^* is a force where it is developed from the reaction of the fluid, H is the thickness of the tube, and $\rho_0$is the tube's mass density. Both equations of tube and fluid using the function, u and depends on the same fast, and slow variables. Fast variables are t and z while slow variables are ξ and τ.

The following non-dimensional quantities are introduced at this stage [4]:

$$t^* = \left(\frac{R_0}{c_0} \right) t, \qquad z^* = R_0 z, \qquad u^* = R_0 u,$$

$$m = \frac{p_0 H}{p_f R_0}, \qquad w^* = c_0 w, \qquad f^* = R_0 f, \qquad Eq.\ 4$$
$$r_0 = R_0 \lambda_\theta, \qquad P^* = p_f c_0^2 p, \qquad c_0^2 = \frac{\mu H}{p_f R_0},$$

By applying Eq. (4) into Eq. (1), (2) and (3) yield

$$\frac{\partial w}{\partial t}\frac{c_0^2}{R_0} + \frac{c_0^2}{R_0} w \left(\frac{\partial w}{\partial z}\right) + \frac{c_0^2}{R_0}\frac{\partial P}{\partial z} = 0,$$

$$2\left(\frac{\partial w^*}{\partial w}\frac{\partial w}{\partial z}\frac{\partial z}{\partial z^*}\right) + 2w^*\left(f^{*\prime} + \frac{\partial u^*}{\partial u}\frac{\partial u}{\partial z}\frac{\partial z}{\partial z^*}\right) + (r_0 + f^* + u^*) \times \left(\frac{\partial w^*}{\partial w}\frac{\partial w}{\partial z}\frac{\partial z}{\partial z^*}\right) = 0,$$

$$p = \frac{m}{\lambda_z(\lambda_\theta + f + u)}\frac{\partial^2 u}{\partial t^2} + \frac{1}{\lambda_z(\lambda_\theta + f + u)}\frac{\partial \Sigma}{\partial \lambda_2} - \frac{1}{(\lambda_\theta + f + u)}\frac{\partial}{\partial z} \times \left\{\frac{f' + \frac{\partial u}{\partial z}}{\left[1 + \left(f' + \frac{\partial u}{\partial z}\right)^2\right]^{\frac{1}{2}}}\frac{\partial \Sigma}{\partial \lambda_1}\right\} - 4\hat{v}\frac{f' + \frac{\partial u}{\partial z}}{(\lambda_\theta + f + u)} w, \qquad Eq.\ 5$$

where λ_θ is the stretch ratio in the circumferential direction.

3. Nonlinear Wave Modulation

In this section, the amplitude modulation of weakly non-linear waves in a fluid-filled thin elastic with a stenosis whose non-dimensional governing equations are given in equation (5) will be studied. Considering the dispersion relation of the linearized field equations and the nature of the problem of concern, which is a boundary-value problem, the following stretched coordinates is introduced:

$$\xi = \varepsilon(z - \lambda t), \quad \tau = \varepsilon^2 z, \qquad Eq.\ 6$$

where ξ is the wave profile and τ is the space. ε indicates the nonlinearity measurer's weakness with a small value and λ is the scale constant to be determined from the solution.

Since this study has a variable cross-section of tube, the order of $\hat{h}$ should be first-order, (ε), where $\hat{h}(\varepsilon, \tau) = \varepsilon h(\tau)$ [4]. The differential relations can be expressed as [4]:

$$\frac{\partial}{\partial z} \to \frac{\partial}{\partial z} + \varepsilon\frac{\partial}{\partial \xi} + \varepsilon^2\frac{\partial}{\partial \tau}, \qquad \frac{\partial}{\partial t} \to \frac{\partial}{\partial t} - \varepsilon\lambda\frac{\partial}{\partial \xi}. \qquad Eq.\ 7$$

The field quantities u, w and p are assumed can be expressed as asymptotic series in the following form [4]:

$$u = \varepsilon u_1 + \varepsilon^2 u_2 + \varepsilon^3 u_3 + \cdots$$
$$w = \varepsilon w_1 + \varepsilon^2 w_2 + \varepsilon^3 w_3 + \cdots$$
$$p = p_0 + \varepsilon p_1 + \varepsilon^2 p_2 + \varepsilon^3 p_3 + \cdots \qquad Eq.\ 8$$

where u, w, and p are the functions of fast variables and slow variables.

Introducing the expansions (6) – (8) into the equation (5), the following sets of differential equations are obtained.

$O(\varepsilon)$ order equation:

$$\frac{\partial w_1}{\partial t}+\frac{\partial p_1}{\partial z}=0, \qquad \frac{\partial u_1}{\partial t}+\frac{\lambda_\theta}{2}\frac{\partial \omega_1}{\partial z}=0, \qquad p_1=\frac{m}{\lambda_z\lambda_\theta}\frac{\partial^2 u_1}{\partial t^2}-a_0\frac{\partial^2 u_1}{\partial z^2}+\beta_1(u_1+h). \quad Eq.\ 9$$

$O(\varepsilon^2)$ order equation:

$$\frac{\partial w_2}{\partial t}+\frac{\partial p_2}{\partial z}-\lambda\frac{\partial w_1}{\partial \xi}+\frac{\partial p_1}{\partial \xi}+w_1\frac{\partial w_1}{\partial z}=0,$$

$$\frac{\partial u_2}{\partial t}+\frac{\lambda_\theta}{2}\frac{\partial \omega_2}{\partial z}-\lambda\frac{\partial u_1}{\partial \xi}+\frac{\lambda_\theta}{2}\frac{\partial \omega_1}{\partial \xi}+\omega_1\frac{\partial u_1}{\partial z}+\frac{u_1}{2}\frac{\partial \omega_1}{\partial z}=0,$$

$$p_2=\frac{m}{\lambda_z\lambda_\theta}\left(\frac{\partial^2 u_2}{\partial t^2}-2\lambda\frac{\partial^2 u_1}{\partial t\partial \xi}\right)-\frac{m}{\lambda_z\lambda_\theta^2}u_1\frac{\partial^2 u_1}{\partial t^2}+\beta_2u_1^2+\beta_1u_2-a_0\frac{\partial^2 u_2}{\partial z^2}-2a_0\frac{\partial^2 u_1}{\partial z\partial \xi}$$
$$+\left(\frac{\alpha_0}{\lambda_\theta}-2\alpha_1\right)u_1\frac{\partial^2 u_1}{\partial z^2}-\alpha_1\left(\frac{\partial u_1}{\partial z}\right)^2+\left[-\frac{m}{\lambda_z\lambda_\theta^2}\frac{\partial^2 u_1}{\partial t^2}+2\beta_2u_1+\left(\frac{\alpha_0}{\lambda_\theta}-2\alpha_1\right)\frac{\partial^2 u_1}{\partial z^2}\right]$$
$$\times h(\tau)+\beta_2(h)^2. \quad Eq.\ 10$$

$O(\varepsilon^3)$ order equation:

$$\frac{\partial w_3}{\partial t}+\frac{\partial p_3}{\partial z}-\lambda\frac{\partial w_2}{\partial \xi}+\frac{\partial p_2}{\partial \xi}+\frac{\partial p_1}{\partial \tau}+w_1\left(\frac{\partial w_2}{\partial z}+\frac{\partial w_1}{\partial \xi}\right)+w_2\frac{\partial w_1}{\partial z}=0,$$

$$\frac{\partial u_3}{\partial t}+\frac{\lambda_\theta}{2}\frac{\partial \omega_3}{\partial z}-\lambda\frac{\partial u_2}{\partial \xi}+\frac{\lambda_\theta}{2}\frac{\partial \omega_2}{\partial \xi}+\omega_1\left(\frac{\partial u_2}{\partial z}+\frac{\partial u_1}{\partial \xi}\right)+\frac{\partial u_1}{\partial z}\omega_2+\frac{u_1}{2}\left(\frac{\partial \omega_2}{\partial z}+\frac{\partial \omega_1}{\partial \xi}\right)$$
$$+\frac{u_2}{2}\frac{\partial \omega_1}{\partial z}+\frac{h(\tau)}{2}\left(\frac{\partial \omega_2}{\partial z}+\frac{\partial \omega_1}{\partial \xi}\right)=0,$$

$$p_3=\frac{m}{\lambda_z\lambda_\theta}\frac{\partial^2 u_3}{\partial t^2}-a_0\frac{\partial^2 u_3}{\partial z^2}+\beta_1u_3-2\lambda\frac{m}{\lambda_z\lambda_\theta}\frac{\partial^2 u_2}{\partial t\partial \xi}-2a_0\frac{\partial^2 u_2}{\partial z\partial \xi}$$
$$+\left(\lambda^2\frac{\alpha_0}{\lambda_\theta\lambda_z}-\alpha_0\right)\frac{\partial^2 u_1}{\partial \xi^2}+\frac{m}{\lambda_z\lambda_\theta^2}\left(2\lambda u_1\frac{\partial^2 u_1}{\partial t\partial \xi}-u_1\frac{\partial^2 u_2}{\partial t^2}-u_2\frac{\partial^2 u_1}{\partial t^2}\right)$$
$$+\frac{m}{\lambda_z\lambda_\theta^3}u_1^2\frac{\partial^2 u_1}{\partial t^2}+\beta_3u_1^3+2\beta_2u_1u_2-2a_0\frac{\partial^2 u_1}{\partial z\partial \tau}$$
$$+\left(\frac{3}{2}\alpha_0-3\gamma_1\right)\left(\frac{\partial u_1}{\partial z}\right)^2\frac{\partial^2 u_1}{\partial z^2}+\left(\frac{1}{\lambda_\theta}\alpha_0-2\alpha_1\right)\left(u_1\frac{\partial^2 u_2}{\partial z^2}+u_2\frac{\partial^2 u_1}{\partial z^2}\right)$$
$$-2\alpha_1\frac{\partial u_1}{\partial z}\left(\frac{\partial u_2}{\partial z}+\frac{\partial u_1}{\partial \xi}\right)+2\left(-\alpha_2+2\frac{\alpha_1}{\lambda_\theta}-\frac{\alpha_0}{\lambda_\theta^2}\right)u_1^2\frac{\partial^2 u_1}{\partial z^2}$$
$$+\left[\frac{m}{\lambda_z\lambda_\theta^2}\left(2\lambda\frac{\partial^2 u_1}{\partial t\partial \xi}-\frac{\partial^2 u_2}{\partial t^2}\right)+2u_1\frac{m}{\lambda_z\lambda_\theta^3}\frac{\partial^2 u_1}{\partial t^2}+3\beta_3u_1^2+2\beta_2u_2\right.$$
$$+\left(\frac{1}{\lambda_\theta}\alpha_0-2\alpha_1\right)\frac{\partial^2 u_2}{\partial z^2}+2\left(\frac{1}{\lambda_\theta}\alpha_0-2\alpha_1\right)\frac{\partial^2 u_1}{\partial z\partial \xi}+\left(-\alpha_2+\frac{\alpha_1}{\lambda_\theta}\right)\left(\frac{\partial u_1}{\partial z}\right)^2$$
$$\left.+2u_1\left(-\alpha_2+2\frac{\alpha_1}{\lambda_\theta}-\frac{\alpha_0}{\lambda_\theta^2}\right)\frac{\partial^2 u_1}{\partial z^2}\right]h(\tau)+\left(-\alpha_2+\frac{\alpha_1}{\lambda_\theta}\right)\left(\frac{\partial u_1}{\partial z}\right)^2u_1$$

$$+\left[\frac{m}{\lambda_z\lambda_\theta^3}\frac{\partial^2 u_1}{\partial t^2}+3\beta_3 u_1+\left(-\alpha_2+2\frac{\alpha_1}{\lambda_\theta}-\frac{\alpha_0}{\lambda_\theta^2}\right)\frac{\partial^2 u_1}{\partial z^2}\right]h^2(\tau)+h^3(\tau)\beta_3. \qquad Eq.\ 11$$

where $\alpha_0, \alpha_1, \alpha_2, \beta_0, \dots, \beta_3$ and γ_1 are defined by

$$\alpha_0=\frac{1}{\lambda_\theta}\frac{\partial\Sigma}{\partial\lambda_z},\qquad \alpha_1=\frac{1}{2\lambda_\theta}\frac{\partial^2\Sigma}{\partial\lambda_\theta\partial\lambda_z},\qquad \alpha_2=\frac{1}{2\lambda_\theta}\frac{\partial^2\Sigma}{\partial\lambda_\theta\partial\lambda_z}\qquad \gamma_1=\frac{\lambda_z}{2\lambda_\theta}\frac{\partial^2\Sigma}{\partial\lambda_z^2}$$

$$\beta_0=\frac{1}{\lambda_z\lambda_\theta}\frac{\partial\Sigma}{\partial\lambda_\theta},\qquad \beta_1=\frac{1}{\lambda_z\lambda_\theta}\frac{\partial^2\Sigma}{\partial\lambda_\theta^2}-\frac{\beta_0}{\lambda_\theta},\qquad \beta_2=\frac{1}{2\lambda_z\lambda_\theta}\frac{\partial^3\Sigma}{\partial\lambda_\theta^3}-\frac{\beta_1}{\lambda_\theta},\qquad \beta_3=\frac{1}{6}\frac{\partial^4\Sigma}{\partial\lambda_\theta^4}-\frac{\beta_2}{\lambda_\theta}.$$

Solving the Eq. (9), (10) and (11) give the following partial differential equation (PDE), which is the nonlinear Schrodinger (NLS) equation with variable coefficient,

$$i\frac{\partial U}{\partial\tau}+\mu_1\frac{\partial^2 U}{\partial\xi^2}+\mu_2|U|^2U-\mu_3 h_1(\tau)U=0, \qquad Eq.\ 12$$

where U is unknown function, and μ_1, μ_2, μ_3 are the variable coefficients shown as the following:

$$\mu=\frac{2\omega^2}{k}+3\alpha_0\lambda_\theta k^3-\frac{mk\omega^2}{\lambda_z}+\lambda_\theta\beta_1 k$$

$$\mu_1=\mu^{(-1)}\left[-\frac{4\lambda\omega}{k}+\frac{2\omega^2}{k^2}+2\lambda^2+\frac{m\lambda^2k^2}{\lambda_z}-3\alpha_0\lambda_\theta k^2+\frac{2m\omega\lambda k}{\lambda_z}\right]$$

$$\mu_2=\mu^{(-1)}\left\{\left[\frac{10\omega^2}{\lambda_\theta}+\lambda_\theta k^2\left(\frac{5m\omega^2}{\lambda_\theta^2\lambda_z}+6\alpha_1k^2-\frac{5\alpha_0k^2}{\lambda_\theta}+2\beta_2\right)\right]\Phi_0\right.$$
$$+\left[\frac{8\omega\lambda k}{\lambda_\theta}+\frac{2\omega^2}{\lambda_\theta}+\lambda_\theta k^2\left(\frac{m\omega^2}{\lambda_\theta^2\lambda_z}+2\alpha_1k^2-\frac{\alpha_0k^2}{\lambda_\theta}+2\beta_2\right)\right]\Phi_1$$
$$\left.-\frac{30\omega^2}{\lambda_\theta{}^2}+\lambda_\theta k^2\left[-\frac{3m\omega^2}{\lambda_\theta^3\lambda_z}+2\alpha_2k^2-\frac{5\alpha_1k^2}{\lambda_\theta}+\frac{3\alpha_0k^2}{\lambda_\theta^2}+3\left(\gamma_1-\frac{\alpha_0}{2}\right)k^4+3\beta_3\right]\right\},$$

$$\mu_3=\mu^{(-1)}\left\{\left[\frac{2\omega^2}{\lambda_\theta}+\frac{8\omega\lambda k}{\lambda_\theta}+k^2\lambda_\theta\left(\frac{m\omega^2}{\lambda_\theta^2\lambda_z}+2\alpha_1k^2-\frac{\alpha_0k^2}{\lambda_\theta}+2\beta_2\right)\right]\Phi_2\right.$$
$$\left.+\left[\frac{2\omega^2}{\lambda_\theta}+\lambda_\theta k^2\left(\frac{m\omega^2}{\lambda_\theta^2\lambda_z}+2\alpha_1k^2-\frac{\alpha_0k^2}{\lambda_\theta}+2\beta_2\right)\right]\right\}.$$

4. Results and Discussion

In this subsection, the progressive wave solution to the evolution equation given in (12) will be presented by introducing the following form:

$$V(\xi,\tau)=F(\zeta)\exp[i(K\xi-\Omega\tau)],\qquad \zeta=\xi-c\tau, \qquad Eq.\ 13$$

where Ω, K and c are some constants and $F(\zeta)$ is a real-valued unknown function to be determined from the solution. Introducing (13) into (12) yield

$$\mu_1\frac{\partial^2 F}{\partial\zeta^2}+i(2\mu_1K-c)\frac{\partial F}{\partial\zeta}+(\Omega-\mu_1K^2)F+\mu_2F^3=0. \qquad Eq.\ 14$$

By letting $c=2\mu_1K$, the $\frac{\partial F}{\partial\zeta}$ term can be eliminated and choosing $\Omega=\mu_1K^2-\mu_2a^2$, where a is the amplitude of the wave, it gives

$$\mu_1 \frac{\partial^2 F}{\partial \zeta^2} + \left(\Omega - \mu_1 K^2\right) F + \mu_2 F^3 = 0. \qquad \textit{Eq. } 15$$

By solving Eq. (15), it gives

$$U(\xi,\tau) = a\,\text{sech}(\xi)\exp\left[i\left(K\xi - \Omega\tau - \mu_3 \int_0^\tau h(s)\,ds\right)\right]. \qquad \textit{Eq. } 16$$

The graphical output for the solution of the NLS equation with variable coefficient (16) is illustrated using MATLAB. In this research, the numerical value of α is 1.948 [5]. Other than that, the axial stretch, λ_z and λ_θ are assumed as 0.8 and 1.2, respectively.

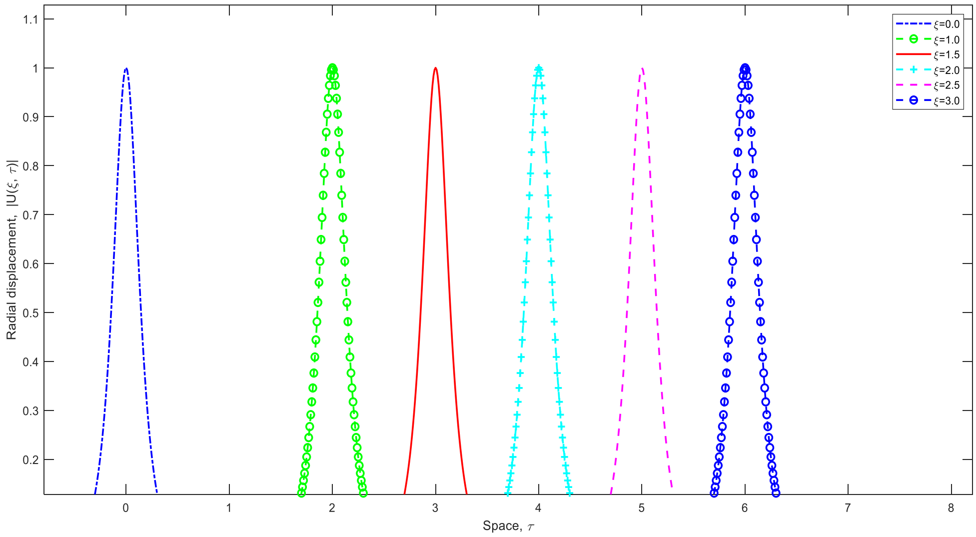

Figure 1: The solution of NLS equation with variable coefficient (16) versus space, τ for the different wave profile, ξ

From the Figure 1, the bell-shaped form and amplitude of wave remained the same along the space.The y-axis shows the radial displacement for $0 \leq \tau \leq 6$ and $0 \leq \xi \leq 1$. The wave propagates by preserving its bell-shaped form at the space points $\tau = 2$, $\tau = 3$, $\tau = 4$, $\tau = 5$ and $\tau = 6$.

5. Conclusion

In this study, it is focused on the solitary wave modulation in an inviscid fluid filled in an elastic tube with variable radius. The artery is considered as a thin-walled, and pre-stressed thin elastic tube with variable radius and the blood is treated as an inviscid fluid. There are one equation of tube and two equations of fluids used in this study. These dimensional equations of tube and fluid are converted to non-dimensional equations by introducing the non-dimensional quantities. Later, reductive perturbation method is used by introducing the stretched coordinates, asymptotic series and differential relations to obtain the various orders of differential equations.Next, the various orders of differential equations are solved and reduced to the nonlinear evolution equation which is the variable coefficient NLS equation. Then, a progressive wave type of solution is proposed to the NLS equation with variable equation. The graphical outputs of the progressive wave solution are studied and discussed. The results shown that the wave maintained its symmetrical bell-shaped curve propagates to the right as time going.

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