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# Stagnation-Point Flow Over a Stretching Sheet in A Nanofluid

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**Abstract**: This research is regarding a study on a stagnation-point flow over a stretching sheet in a nanofluid. The governing equations are transferred from partial differential equations into a set of nonlinear ordinary differential equations by using similarity transformation. The similarity equation is solved for three types of nanoparticles which are Au-Metallic, Copper Oxide and Carbon Nanotubes. Then, the equation is used to determine the numerical solution for velocity for stagnation-point flow over a stretching sheet in a nanofluid by using Runge Kutta Fehlberg 45 method (RKF45 Method). The velocity profilesare presented graphically and discussed. It is found that the inclusion of nanoparticles into the base water fluid hasincreased heat transfer coefficients and skin friction.

**Keywords**: Stagnation Point Flow, Nanofluid, Shrinking Sheet

## 1. Introduction

This research is regarding a study on a stagnation-point flow over a stretching sheet in a nanofluid. In this research, we transform the governing equations from partial differential equations into a set of nonlinear ordinary differential equations using similarity transformation. Compare to the previous research which that they are using the equation by using dual solution method.

The flow over a stretching surface has a wide range of applications in engineering and several technological purposes. Mustafa [1] said that the extruded is drawn and stretched into a sheet, which is then solidified through the cooling process. This process is usually initiated by the use of a liquid electrolyte. This paper reports the flow of a nanofluid near a stagnation-point flow towards a stretching surface.

Sadiq [2] considered that stagnation point flow phenomenon has various functions in aerodynamic industries. These flows are primarily compact with fluid movement near the stagnated area of a solid surface flowing in the fluid material or maintained with dynamics. Many researchers have been studying about stagnation point because of its wide range of applications in engineering. In 1911, stagnation point flow was analyzed by Hiemenz. In 2012, Rameesh [3] studied the mechanical fluid properties

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required for the outcome of such a process 2 would depend primarily on two aspects, one is the cooling fluid used and the other is the stretching rate.

Nanofluid is a dispersion of tiny metal particles in the base fluid. Two phase and single phase are two ways for estimating the behavior of nanofluid. In first method, nanofluid is considered as homogenous fluid without any slip mechanism. But in the second method, slip velocities are included. Thermophoresis and Brownian motion impacts are taken into consideration for second approach.

Bachok [4] studied the steady two-dimensional stagnation-point flow over a shrinking or stretching sheet in its own plane. It is assumed that the stretching or shrinking velocity and the ambient fluid velocity are to vary linearly with the distance from stagnation point. The similarity equations are solved numerically for the three nanoparticles. This research concludes that the solution for shrinking sheet is non-unique. They also studied the effects of homogenous-heterogenous reactions on the steady boundary layer flow near the stagnation point on a stretching surface. The possible steady-states of this system are analyzed in the case when the diffusion coefficients of both reactant and auto catalyst are equal. The strength of this effect is represented by the dimensionless parameter. It is shown that for a fluid of small kinematic viscosity, a boundary layer is formed when the stretching velocity is less than the free stream velocity and an inverted boundary layer is formed when the stretching velocity exceeds the free stream velocity. The uniqueness of this problem lies on the fact that the solution is possible for all values of stretching surface, while for shrinking surface, solutions are possible only for limited range.

## 2. Mathematical Formulation

Consider the flow of an incompressible nanofluid in the region $y > 0$ driven by a stretching surface located at $y = 0$ with a fixed stagnation point at $x = 0$ as shown in Figure 2.1

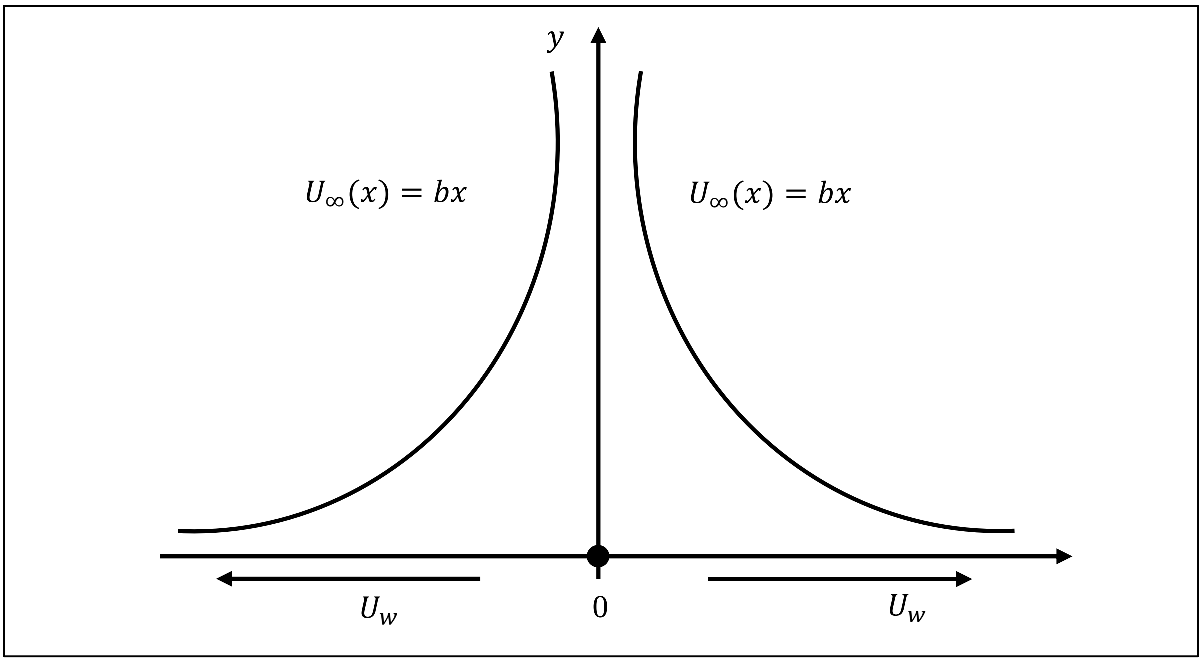


**Figure 2.1: Physical model and coordinate system**

The sheet is stretched with a velocity $U_w = cx$ and the velocity of the external flow is $U_e = ax$. The stretching velocity $U_w(x)$ and the ambient fluid velocity $U_\infty(x)$ are to vary linearly from the stagnation point, i.e., $U_w(x) = ax$ and $U =_\infty (x) = bx$, where $a$ and $b$ are constant with $b > 0$. We note that $a > 0$ correspond to stretching sheets. Based on these assumptions, the simplified two-dimensional equations governing the flow in the boundary layer of a steady, laminar, and incompressible nanofluid are (Mustafa [1] and Bachok [4])

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}= U_\infty\frac{dU_\infty}{dx}+\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2}, \tag{2.2}$$

where $u$ and $v$ are the velocity components along the $x$- and $y$- axes, respectively. $\mu_{nf}$ is the viscosity of the nanofluid and $\rho_{nf}$ is the density of the nanofluid, which are given by Oztop and Abu Nada [5]

$$\rho_{nf}=(1-\varphi)\rho_f+\varphi\rho_s, \tag{2.3}$$

$$\mu_{nf}=\frac{\mu_f}{(1-\varphi)^{2.5}}, \tag{2.4}$$

$\rho_f$ and $\rho_s$ are the densities of the fluid and of the solid fractions, respectively. The viscosity of the nanofluid $\mu_{nf}$ has been approximated by Brinkman as viscosity of a base fluid $\mu_f$ containing dilute suspension of fine spherical particles.

This equation is obtained from (2.4) subject to the boundary conditions

$$u=U_w(x),\ v=0$$
$$\text{at } \gamma=0,\ u\to U_\infty(x)$$
$$\text{as } \gamma\to\infty, \tag{2.5}$$

where $u$ and $v$ are the velocity components along the $x$- and $y$- axes, respectively.

The governing equation can be obtained by introducing the following dimensionless coordinates in term of similarity variable and similarity function as below

$$\eta=\left(\frac{b}{v_f}\right)^{1/2}\gamma,$$
$$\psi=(v_f b)^{1/2}xf(\eta), \tag{2.6}$$

where $\eta$ is the similarity variable and $\psi$ is the stream function defined as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$, which identically satisfy equation (2.1). Employing the similarity variables (2.6), equation (2.2) is reduce to the following ordinary differential equation:

$$\frac{1}{(1-\varphi)^{2.5}(1-\varphi+\frac{\varphi\rho_s}{\rho_f})}f'''+ff''-f'^2+1=0 \tag{2.7}$$

by using

$$u=bxf'(\eta),$$
$$v=-\sqrt{v_f b f(\eta)},$$
$$\eta=\sqrt{\frac{b}{v_f}}y,$$
$$U_\infty(x)=bx \tag{2.8}$$

Equation (2.7) is subjected to these boundary conditions:

$$f(0)=0, f'(0)=\varepsilon, \theta(0)=1$$
$$f'(\eta)\to 1, \theta(\eta)\to 0 \text{ as } \eta\to\infty \tag{2.9}$$

## 3. Results and Discussion

Numerical solutions were obtained in Maple2016 by using a Runge Kutta Fehlberg fourth-fifth order method (RKF45 Method). Table 3.1 displays the thermophysical properties of fluid and nanoparticles by Bachok [4].

**Table 3.1: Thermophysical properties of fluid and nanoparticles [4]**

| Physical properties | Fluid phase (water) | Au-metallic | Copper Oxide, CuO | Carbon Nanotubes |
|---|---|---|---|---|
| $C_p$(J/kg K) | 4179 | 130.0 | 540 | 600 |
| $\rho$(kg/$m^3$) | 997.1 | 19.32 | 6500 | 1350 |

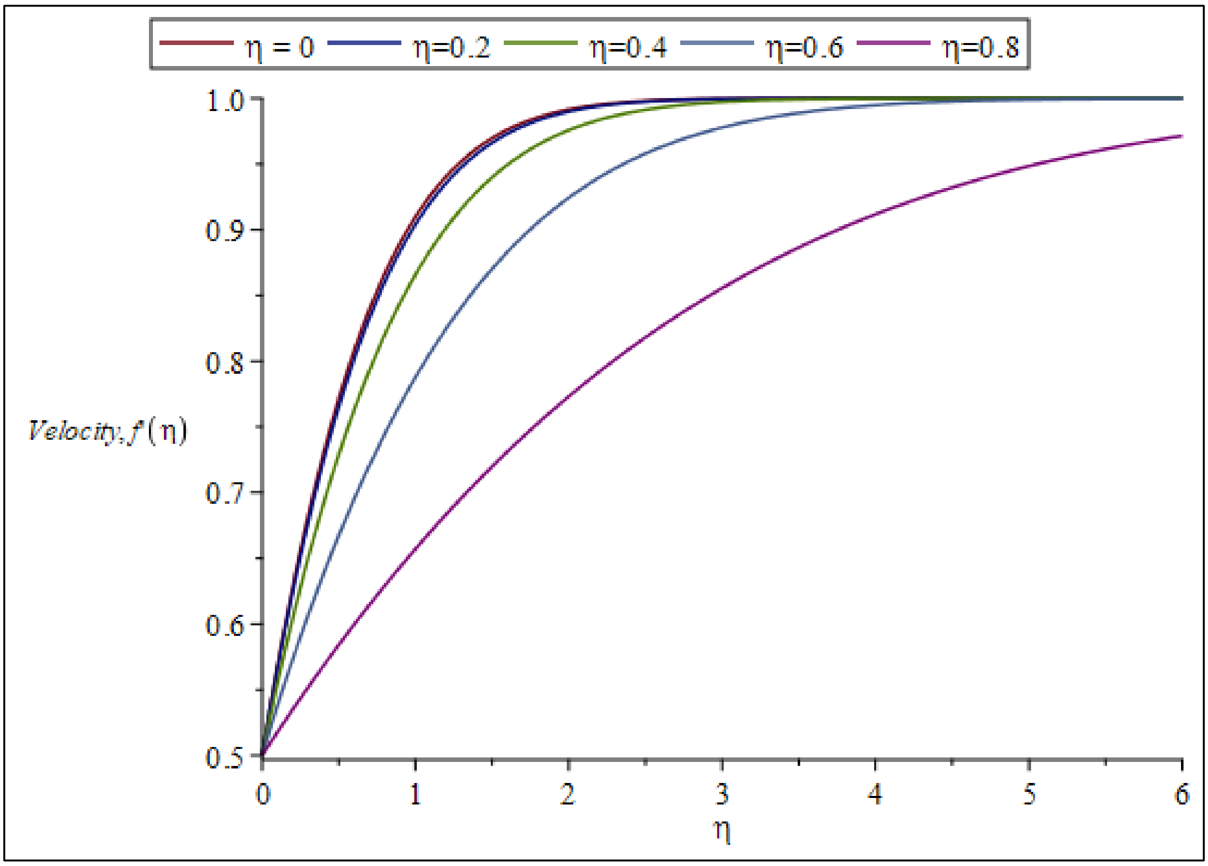


**Figure 3.1: Velocity profiles for water**

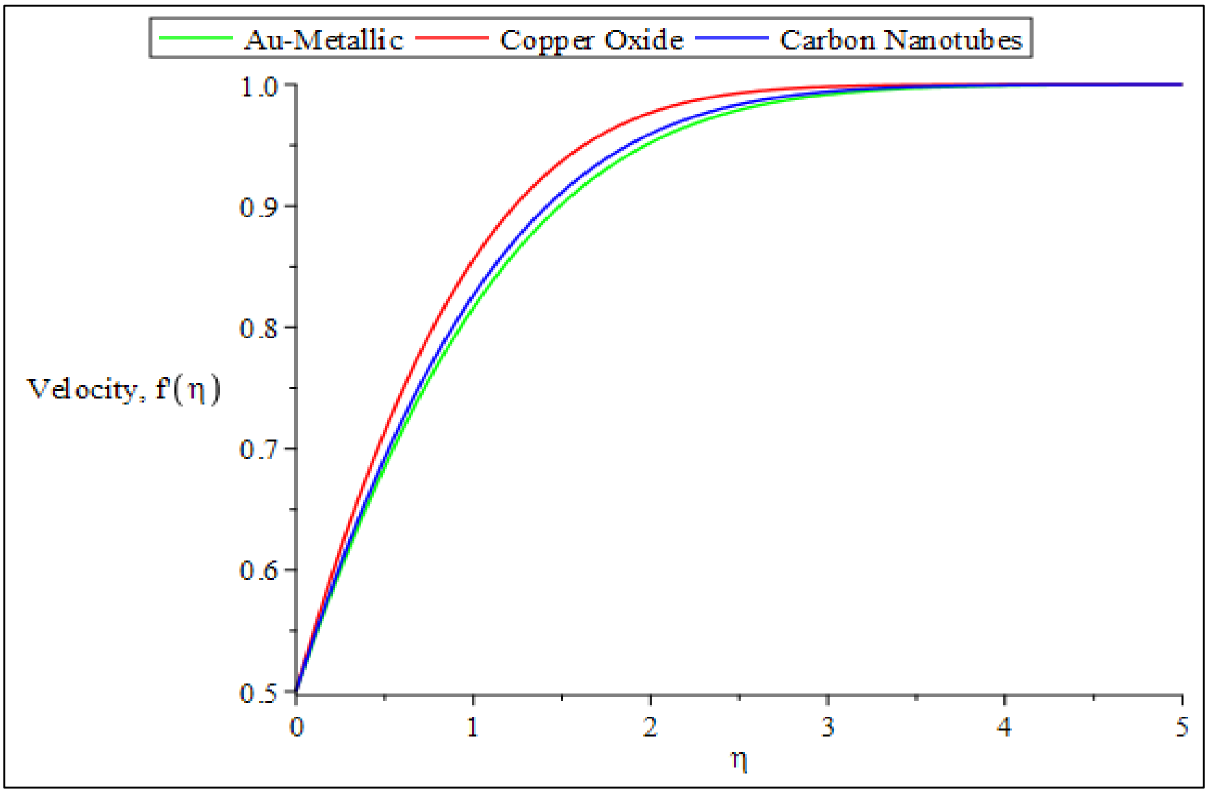


**Figure 3.2: Velocity profiles for nanoparticles**

Figures 3.1 and 3.2 show the velocity profiles for different nanoparticles and volume fraction used. These profiles have essentially the same form as in the case of regular field ($\varphi = 0.1$). Figures 3.1 – 3.2 shows that the far field boundary conditions (2.9) are satisfied asymptotically, thus supporting the validity of the numerical results.

Figure 3.1 shows the results obtained from Maple2016 by using Runge Kutta Fehlberg Forth-Fifth Method (RKF45). As nanoparticle volume fraction increases, the velocity increases. The volume fraction of nanoparticles used in this graph is from 0 – 0.8. This proves that the velocity of the nanofluid is influenced by the volume fraction. Furthermore, it is observed that the momentum boundary layer thickness decreases as nanoparticle volume fraction increases. This phenomenon implies that the skin friction coefficient increases with nanoparticle volume fraction which would result in higher probability of erosion.

By referring to Table 3.1, Figure 3.2 explains the velocity profiles for different nanoparticles which are Au-metallic, copper oxide (CuO) and carbon nanotubes. From the graph, copper oxide has higher velocity profiles as compared to Au-metallic and carbon nanotubes. This is because copper oxide is denser in fluid state compared to the au-metallic and carbon nanotubes.

## 4. Conclusion

In this study, stagnation-point flow over a stretching sheet in a nanofluid is studied numerically by using Runge Kutta Fehlberg Fourth-Fifth Order Method (RKF45). The main research objective is to transform the governing equations from partial differential equations into a set of nonlinear ordinary differential equations by using similarity transformation. This study analyzes the effects of nanoparticles and their volume fraction on velocity. The effects of volume fraction on the velocity of the nanoparticles are shown graphically and discussed briefly. It is concluded that the momentum boundary layer thickness decreases as nanoparticle volume fraction increases. The bigger the volume fraction of nanoparticles, the higher the velocity of the nanofluid. Furthermore, as the volume fraction of nanoparticles increases, the momentum boundary layer decreases.

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