Enhanced Knowledge in Sciences and Technology Vol. 2 No. 1 (2022) 402-412

EKST

Homepage: http://publisher.uthm.edu.my/periodicals/index.php/ekst
e-ISSN : 2773-6385

Numerical Analysis of Mhd Dusty Fluid Flow Over Stretching Sheet in A Porous Medium

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DOI: https://doi.org/10.30880/ekst.2022.02.01.044
Received 02 January 2022; Accepted 02 March 2022; Available online 1 August 2022

Abstract: The purpose of this research is to study the effects of magnetohydrodynamic dusty fluid flow over stretching sheet in a porous medium. A mathematical model has been done to simulate a steady, incompressible, two-dimensional MHD flow of electrically conducting dusty fluid over a stretching sheet in a porous medium. The partial differential equations of the model are reduced to ordinary differential equations by similarity transformation. The system of reduced ordinary differential equations is solved by bvp4c method in MATLAB software. The effects of the governing parameters on the velocity distribution and temperature distribution are demonstrated graphically while the skin friction coefficient and temperature gradient are also calculated and displayed in tabular form. The governing parameters that have been investigated are permeability parameter, magnetic parameter, thermal conductivity variation parameter, and slip parameter. The results show that each parameter will affects the velocity and temperature distribution differently.

Keywords: Magnetohydrodynamic, Porous Medium, Stretching Sheet, Bvp4c

1. Introduction

The MHD dusty fluid flow over a stretching solid surface continuously can be considered as an important and inevitable process in different industrial and manufacturing applications. Salt water, plasmas, and liquid metals are some of the examples of such fluids. In the manufacturing and industrial processes, material production involves and occurs in sheets of metal and polymer [1]. For instance, the cooling of reactors, power generators, petroleum industry, plastic sheet aerodynamic extrusion, paper production, crystal growing, glass blowing, and so on.

The production of the sheets requires that the melt emits from a crack and is stretched to obtain the targeted thickness. To get the targeted outcomes, the fluid mechanical properties would depend on the rate of stretching and the cooling liquid used. According to [2], for the cooling liquid used, the liquids that can transmit electric current, or we can say the liquids have the non-Newtonian characteristic are

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chosen because the heat transfer and the flow can be adjusted through some external agency. Besides, the rate of stretching also plays a main role in the process. The higher the rate of stretching, the higher the chance of sudden solidification may happen, and this would destroy the properties of the expected outcomes. During the construction process of the polymer sheet, the material must in a molten phase when passing through the extrusion stamper. Before the polymer sheet can be collected, it should be taking away from the stamper and leaves it cools and solidifies. The material is found to stretch while cooling, and in between the region of the stamper and cooling mechanism.

Throughout the years, there are many authors such as [3] had developed the issue of steady flow over a stretching solid in a viscous fluid. For example, a study of three-dimensional MHD flow and heat transfer over a stretching or shrinking surface in a viscoelastic fluid with different physical effects found that to fulfill two algebraic equations, the physical exponential type solution is demonstrated. For the entire physical parameters considered, unique solutions are present, and it is proven by the simultaneous solution. To be specific, the stretching or shrinking effect of decreasing or increasing slip factor on the flow and temperature fields is straightforward to expect from the present results. Many articles and journals have shown that porous media play a significant role in technology while on the other hand, the different types of technologies also vary depend on porous media [4]. Hydrology and petroleum engineering are the two most important technologies that depend on the characteristic of porous media. Hydrology is the study of water movement in earth and sand structures like water flows from water-bearing formations to wells while petroleum engineering is mostly related to production and gas exploration of natural gas and petroleum. The quantities of fluid contained among the rocks, the permeability of fluids through the rocks, and other concerning properties are also related to petroleum engineering. These concerning properties rely on the rock and usually upon the distribution of character of the fluid happening among the rock. Since there are many applications involving the theory of fluid flow over stretching or shrinking sheet in porous medium, the effects of different governing parameters towards the process become very important. If we can investigate the effects of governing parameters, we can control the time and the cost of the process more efficient.

According to [5], the effects of magnetic parameter, velocity ratio parameter, temperature index parameter, and Prandtl number on the fluid flow and heat transfer characteristics of the MHD stagnation point flow towards a stretching sheet immersed in a viscous fluid is investigated theoretically using a finite difference scheme known as the Keller-box method. From the numerical solutions obtained, the results corresponded with the exact solutions for some cases of the previous study. It is found that the heat transfer rate at the surface increases with the magnetic parameter when the free stream velocity exceeds the stretching velocity.

The purpose of this research is to investigate the effects of governing parameters on magnetohydrodynamic dusty fluid flow over stretching sheet in a porous medium. The partial differential equations will first be transformed into ordinary differential equations. The equations are then solved by bvp4c method with boundary conditions in MATLAB software. The governing parameters that will be investigated are permeability parameter, magnetic parameter, thermal conductivity variation parameter, and slip parameter.

2. Materials and Methods

First, consider a steady, incompressible, two-dimensional MHD flow of electrically conducting dusty fluid over a stretching sheet in a porous medium, as in Figure 1. It is shown that the fluid flows in the x-direction which is normal to the y-axis. The magnetic field, $B(x)$ is applied, and it is perpendicular to the flow direction with a uniform strength B_0. As the magnetic Reynolds number is small enough, the magnetic field can be neglected.

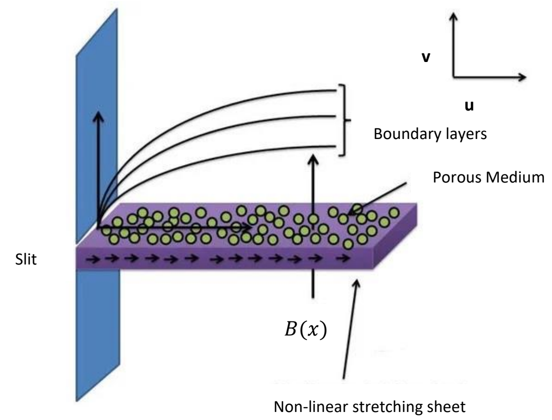

Figure 1: Physical diagram of the flow model

The basic two-dimensional boundary layer equations subject to the above consideration are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad Eq.1$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\upsilon}{k(x)}u-\frac{\sigma\, B(x)^2\, u}{\rho} \qquad Eq.2$$

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\,\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(\kappa\frac{\partial T}{\partial y}\right)+\sigma B(x)^2u^2 \qquad Eq.3$$

with the boundary conditions as below

$$y=0, u=U_w(x)=ce^{\frac{x}{L}}+\alpha_0\frac{\partial u}{\partial y}, v=V_w(x)=v_0e^{\frac{x}{2L}}, T=T_w(x)=T_\infty+T_0e^{\frac{x}{2L}} \qquad Eq.4$$

$$y\to\infty, u\to 0, T\to T_\infty \qquad Eq.5$$

where in $Eq.1$, u is the velocity components along with x-direction while v is the velocity components along with y-directions, $k(x)$ is the permeability of porous medium, C_p is the specific heat at constant pressure, υ is the kinematic viscosity, ρ is the density of the fluid, T is the temperature of the fluid, T_w is the variable temperature at the sheet, T_0 is the constant reference temperature, and T_∞ is the constant free stream temperature, $B(x)$ is the magnetic parameter, κ is the thermal conductivity of the fluid, α_0 is slip parameter. Further, L is the characteristic length, U_w is the stretching velocity of the sheet, V_w is the mass transfer velocity, where c is a nonnegative stretching constant and v_0 is a constant, where if v_0 smaller than 0, it corresponds to mass suction.

2.1 Similarity transformation

Next, we introduce the stream function $\Psi(x,y)$ and

$$u=\frac{\partial\Psi}{\partial y}, v=-\frac{\partial\Psi}{\partial x} \qquad Eq.6$$

So, referring to [6], the $Eq.1$ is similarly fulfilled and the similarity transformations are described as

$$\Psi = e^{\frac{x}{2L}}\sqrt{2vLc}\left(f(\eta)\right), \eta = ye^{\frac{x}{2L}}\sqrt{\frac{c}{2vL}}, \theta = \frac{T - T_\infty}{T_w - T_\infty} \quad Eq. 7$$

By using $Eq. 6$ and $Eq. 7$, we get the expressions of velocity components in non-dimensional form as

$$u = ce^{\frac{x}{L}}\left(f'(\eta)\right), v = -e^{\frac{x}{2L}}\sqrt{\frac{cv}{2L}}\left(\eta f'(\eta) + f(\eta)\right) \quad Eq. 8$$

We suppose that the permeability of the porous medium, k defined as below, to get the similarity solutions

$$k(x) = 2k_0 e^{-\frac{x}{L}} \quad Eq. 9$$

where k_0 is reference permeability.

We also know that the magnetic parameter takes the following form,

$$B(x) = B_0 e^{\frac{x}{2L}} \quad Eq. 10$$

We can also define the slip parameter as

$$\alpha = \alpha_0 e^{\frac{x}{2L}} \quad Eq. 11$$

Following the fluid's thermal conductivity is supposed to change with temperature in a linear form

$$\kappa = \kappa_\infty(1 + \varepsilon\theta) \quad Eq. 12$$

where ε is the thermal conductivity variation parameter. In general, ε is below 0 for fluids like lubrication oils, while ε is above 0 for fluids like water and air. It is further supposed that the temperature difference within the fluid is small enough so that T^4 can be expressed as a linear function of temperature T. This is done by expanding T^4 in a Taylor series about T_∞ and omitting higher-order term to yield

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad Eq. 13$$

By substitute $Eq. 8$ to $Eq. 13$ into $Eq. 2$ to $Eq. 5$, the momentum equation and energy equation will become ordinary differential equations as below

$$f''' + ff'' - 2f'^2 - \frac{f'}{K} - M^2 f' = 0 \quad Eq. 14$$

$$\theta'' + \varepsilon\theta\theta'' + \varepsilon\theta'^2 + Prf\theta' + M^2 Brf'^2 = 0 \quad Eq. 15$$

with boundary conditions

$$\eta \to 0: f'(\eta) = 1 + \alpha f'', f(\eta) = 0, \theta(\eta) = 1 \quad Eq. 16$$

$$\eta \to \infty: f''(\eta) \to 0, \theta(\eta) \to 0 \quad Eq. 17$$

where

$$K = \frac{ck_0}{Lv}, M^2 = \frac{2\sigma B_0^2 L}{\rho c}, Pr = \frac{\mu C_p}{\kappa_\infty}, Br = \frac{\mu C_p}{\kappa_\infty} \cdot \frac{U^2}{c_p(T_w - T_\infty)}$$

represent the permeability parameter, magnetic parameter, Prandtl number, and Brinkmann number, respectively.

2.2 Bvp4c

To find the numerical solutions of $Eq.14$ and $Eq.15$ which subject to the boundary conditions $Eq.16$ to $Eq.17$ by using the MATLAB built-in function bvp4c. The description and details of the method can refer to [7]. When using MATLAB to solve the functions, we define the value of infinity in boundary condition as equal to 5 and change the values of each governing parameters with different values to investigate the variation of each parameter. We then define the variables of each order of f and θ equations as

$$
\begin{aligned}
y_1 &= f, \\
y_2 &= f', \\
y_3 &= f'', \\
y_4 &= \theta, \\
y_5 &= \theta', \qquad\qquad Eq.18
\end{aligned}
$$

while the system of first-order equations is defined as:

$$
\begin{aligned}
y'_1 &= f' = y_2, \\
y'_2 &= f'' = y_3, \\
y'_3 &= f''' = 2(y_2)^2 - y_1 y_3 + \frac{y_2}{K} + M^2(y_2), \\
y'_4 &= \theta' = y_5, \\
y'_5 &= \theta'' = \frac{Pr(y_2 y_4 - y_1 y_5) - \varepsilon(y_5)^2 - M^2 Br(y_2)^2}{1+\varepsilon(y_4)}, \qquad Eq.19
\end{aligned}
$$

and boundary conditions are defined as:

$$
\begin{aligned}
y_0(1) &= 0, \\
y_0(2) - 1 - \alpha y_0(3) &= 0, \\
y_0(4) - 1 &= 0, \\
y_{Inf}(2) &= 0, \\
y_{Inf}(4) &= 0 \qquad\qquad Eq.20
\end{aligned}
$$

3. Results and Discussion

In this section, the focus is to analyze the effects of various governing parameters on the velocity and temperature. The results of the study are described and displayed in graphical as well as tabular form. With the help of MATLAB, the admissible values for different parameters are used to investigate the effects of each parameter to the velocity and temperature of the process of MHD dusty fluid flow over stretching sheet in porous medium.

In Table 1, when the values of the permeability parameter, K increase resultantly, the skin friction coefficient will be decreased. However, increase in the temperature gradient is observed. Besides, one can observe that the skin friction coefficient increases and the temperature gradient decreases with the increasing value of magnetic parameter, M^2. Besides, the skin friction coefficient increases with respect to the increase of thermal conductivity variation parameter, ε whereas the effect on temperature gradient is the opposite.

Table 1: Results for $f(0), -f''(0)$ and $-\theta'(0)$ obtained by different values of parameter K, M^2 and ε with $Pr = 6, Br = 1$, and $\alpha = 1$

K	M^2	ε	$f(0)$	$-f''(0)$	$-\theta'(0)$
1	2	0.1	0	0.6541	0.8089
2	–	–	0	0.6379	0.8327
10	–	–	0	0.6228	0.8500
100	–	–	0	0.6192	0.8536
10	2	0.1	0	0.6228	0.8500
–	4	–	0	0.6827	0.5527
–	6	–	0	0.7193	0.3842
–	8	–	0	0.7449	0.2781
10	2	0.1	0	0.622838643526222	0.8500
–	–	0.3	0	0.622838643526293	0.6037
–	–	0.5	0	0.622838643526299	0.4781
–	–	0.9	0	0.622838643526369	0.3530

3.1 Effects of permeability parameter

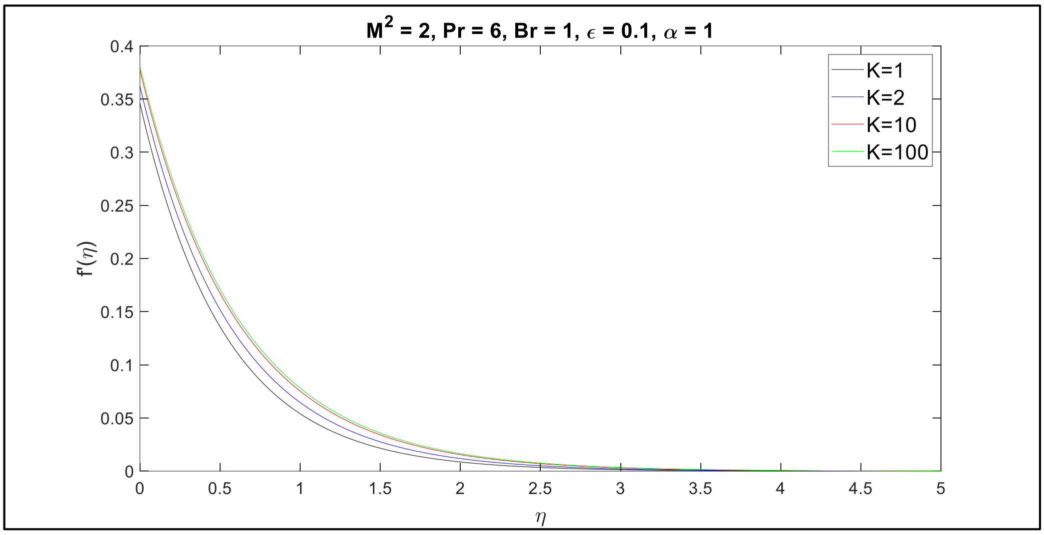

Figure 2: Velocity profile for different K

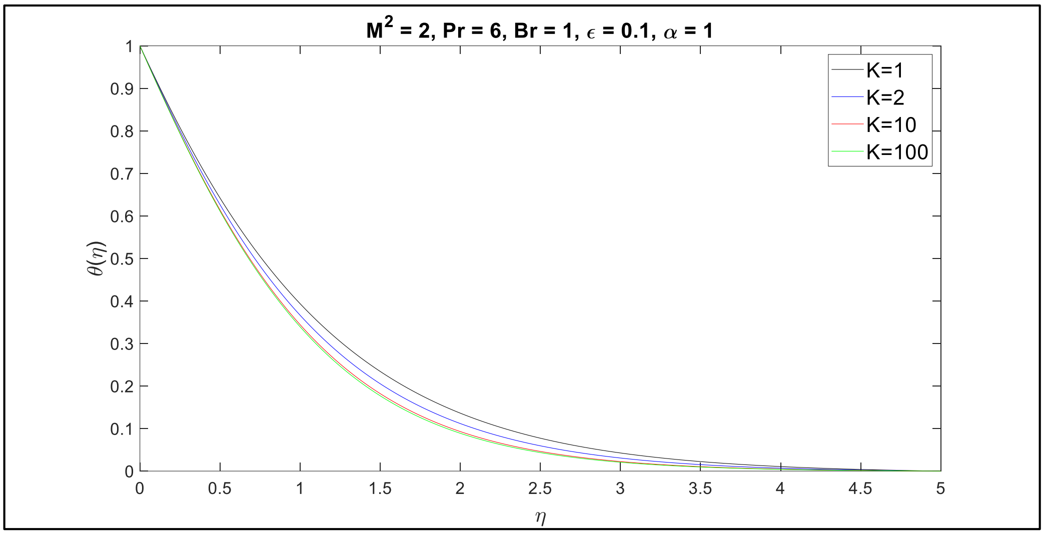

Figure 3: Temperature profile for different K

The variation of velocity and temperature for different values of K can be observed by fixing the other parameters, where $M^2 = 2, Pr = 6, Br = 1, \varepsilon = 0.1$, and $\alpha = 1$ in $Eq. 14$ to $Eq. 17$. The results are shown in Figure 2 and Figure 3, respectively. From Figure 2, it is observed that when the value of K increases $(1 \leq K \leq 100)$, the velocity profile also increases. However, the decreasing in temperature profile is observed in Figure 3. Permeability parameter characterises the strength of permeability of the porous medium. Physically, in a porous medium, we can ignore the resistance of the porosity medium when the holes are large enough. So, the resistance to the fluid is increased with the presence of the permeable surface will cause the increasing in velocity profile.

3.2 Effects of magnetic parameter

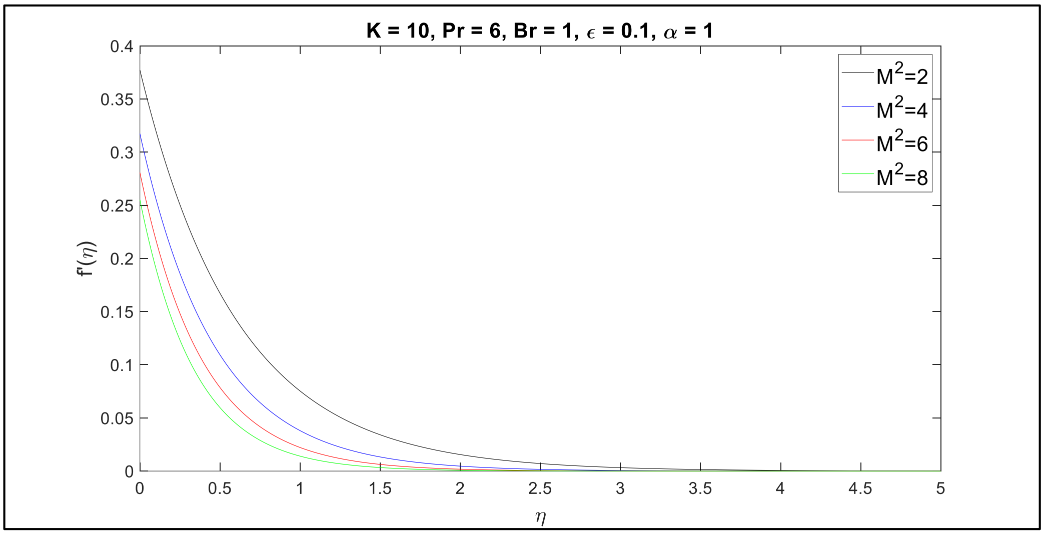

Figure 4: Velocity profile for different M^2

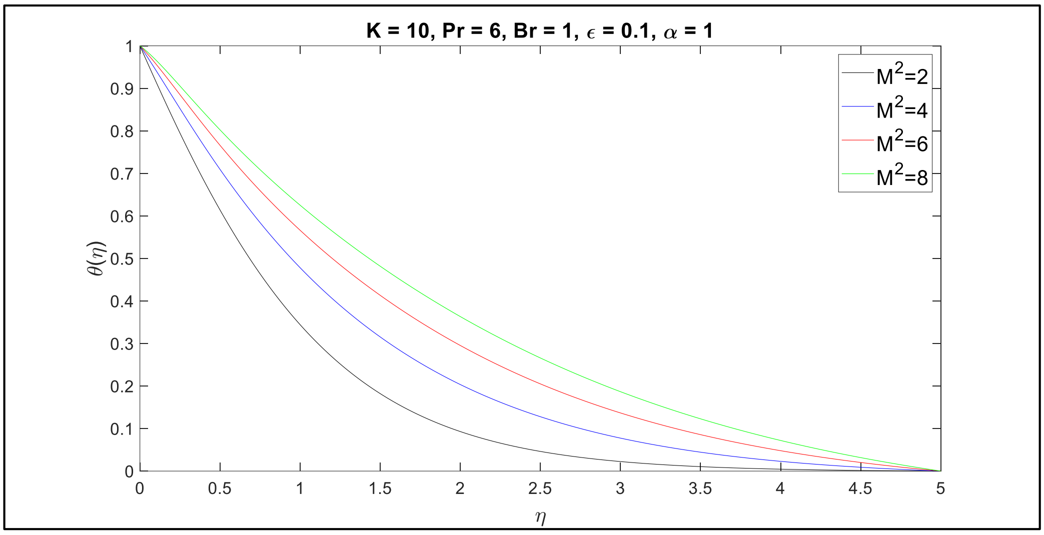

Figure 5: Temperature profile for different M^2

Figure 4 and Figure 5 show the trends of velocity and temperature when the magnetic parameter, M^2 increases. The other parameters are fixed as $K = 10, Pr = 6, Br = 1, \varepsilon = 0.1$, and $\alpha = 1$ in $Eq. 14$ to $Eq. 17$. In Figure 4, it shows that the velocity profile decreases due to the increasing of the value of M^2 $(2 \leq M^2 \leq 8)$ while the temperature profile increasing trend is shown in Figure 5. It is described that the presence of magnetic field in an electrically conducting fluid will produce Lorentz force. The Lorentz force is the force on a charged particle due to electric and magnetic fields. Lorentz force acts reversely in the direction of the flow if the magnetic field is enforced in the perpendicular direction which slows down the fluid velocity. This situation will produce the resistance to the flow, and it will produce heat that increase the temperature during the process. [8]

3.3 Effects of thermal conductivity variation parameter

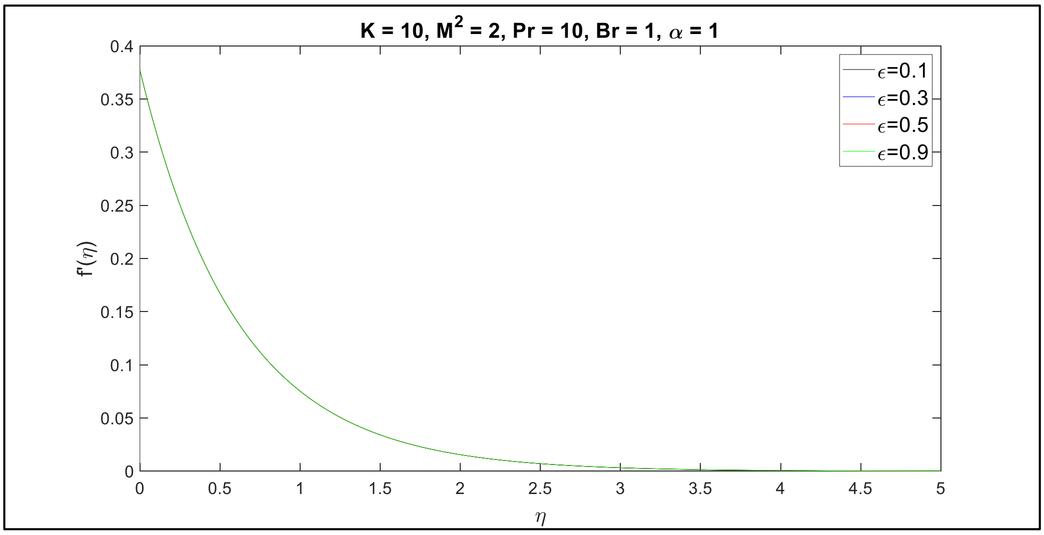

Figure 6: Velocity profile for different ε

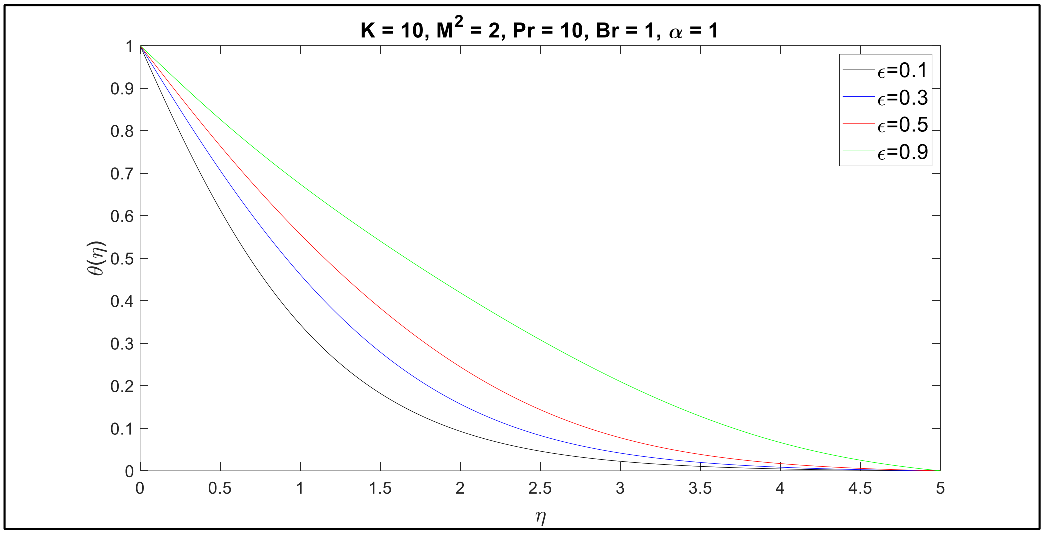

Figure 7: Temperature profile for different ε

By increasing the value of thermal conductivity variation parameter, ε, the effects of thermal conductivity to the velocity and temperature can be investigated and are shown in Figure 6 and Figure 7. The other parameters are fixed as $K = 10, M^2 = 2, Pr = 10, Br = 1$, and $\alpha = 1$ in $Eq. 14$ to $Eq. 17$. In Figure 6, the graph shows that when the value of ε is increasing $(0.1 \leq \varepsilon \leq 0.9)$, the decreases of the velocity profile are extremely small. This is because that there is no variable ε in $Eq. 14$ which means that the effect of thermal conductivity variation parameter on the velocity profile can be neglected. However, the effect on temperature profile is opposite in Figure 7. The materials of low thermal conductivity will have the lower rate of heat transfer compared to the material of high thermal conductivity which means that the higher the thermal conductivity, the higher the temperature.

3.4 Effects of slip parameter

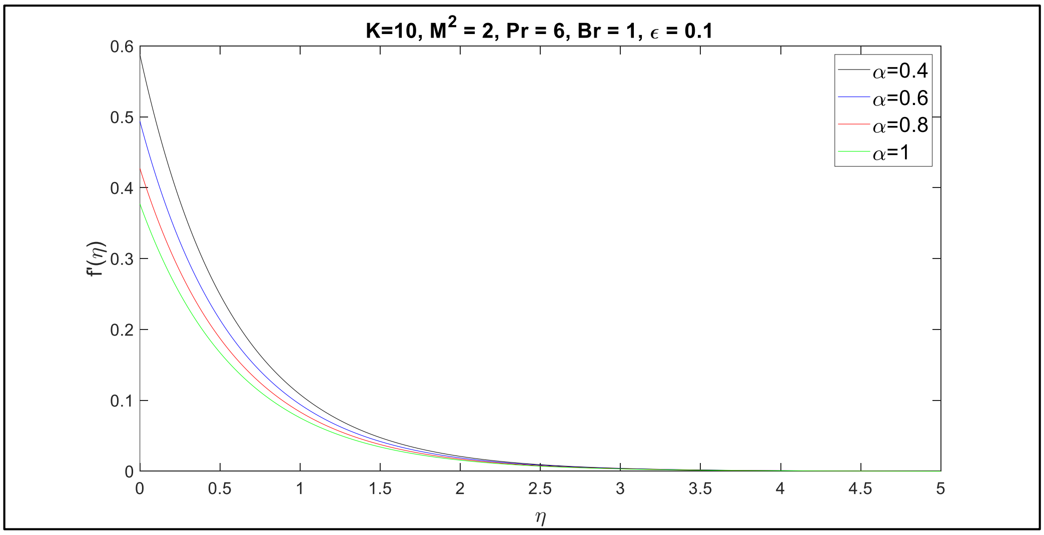

Figure 8: Velocity profile for different α

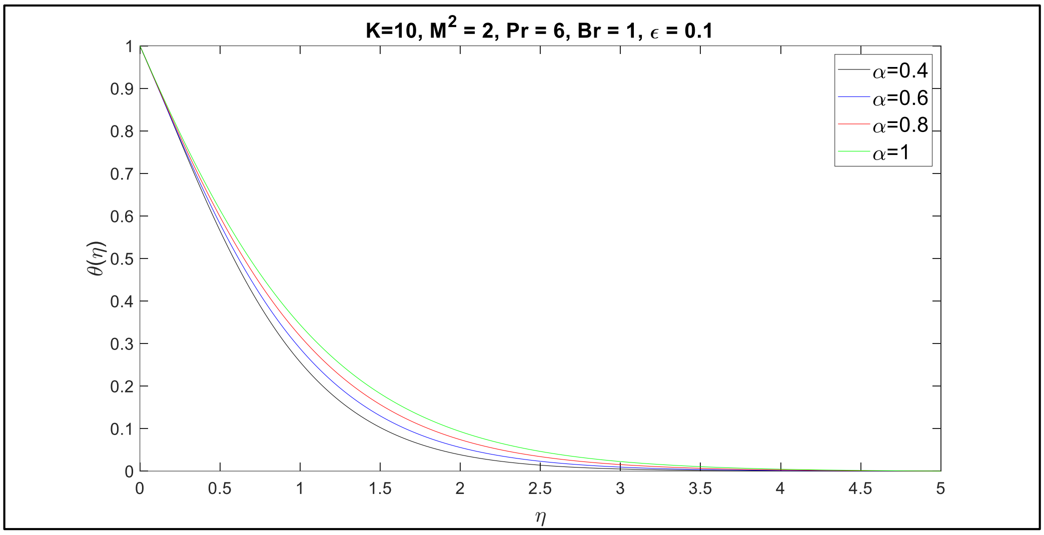

Figure 9: Temperature profile for different α

Lastly, the effects of slip parameter to the velocity and temperature are presented in Figure 8 and Figure 9 by fixing $K = 10, M^2 = 2, Pr = 6, Br = 1$, and $\varepsilon = 0.1$ in $Eq. 14$ to $Eq. 17$ while the value of slip parameter increases. In Figure 8, the trend of velocity profile is decreasing when the value of slip parameter is increasing ($0.4 \leq \alpha \leq 1$) while the trend of temperature profile is increasing shown in Figure 9. In the fluid dynamics, the fluid will have no velocity relative to the boundary when the no-slip boundary condition for viscous fluids is assumed at the solid boundary. However, this assumption is not realistic in the real situations. So, a partial slip condition will replace the no-slip boundary condition at the boundary layer and the effects are shown in this study. When the slip parameter increases, the thermal boundary layer thickness will become thinner due to the temperature field is suppressed. Therefore, the rate of heat transfer from the sheet increases.

4. Conclusion

The steady, incompressible, two-dimensional MHD flow of electrically conducting dusty fluid over a stretching sheet in a porous medium is analyzed under the velocity and temperature effects by using similarity transformation and corresponding boundary conditions. The system of nonlinear ODEs is processed by using bvp4c method with the help of MATLAB software. This method is used due to its simplicity to process and also is considered effective. The governing parameters is investigated and the results of this study are presented in both table and graph. The summarized of the observations are as follows:

(i) The increasing in the value of permeability parameter, K will cause the increases of momentum boundary layer and decreases of thermal boundary layer.
(ii) The increasing in the value of magnetic parameter, M^2 will cause the decreases of momentum boundary layer and increases of thermal boundary layer.
(iii) The increasing in the value of thermal conductivity variation parameter, ε will cause the decreases of momentum boundary layer and increases of thermal boundary layer.
(iv) The increasing in the value of slip parameter, α will cause the decreases of momentum boundary layer and the increases of thermal boundary layer.

Acknowledgement

The authors would also like to thank the Faculty of Applied Sciences and Technology, Universiti Tun Hussein Onn Malaysia for its support.

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