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# Analysis On Hybrid Nanofluid Over a Shrinking Sheet with Transpiration and Uniform Shear Flow

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**Abstract**: This study is conducted to analyse the effects of the uniform shear flow and transpiration over the shrinking sheet on hybrid nanofluid. A similarity transformation is used to convert the governing equation from partial differential equations to ordinary differential equations using a set of similarity variables, and then the solutions are obtained using the shooting technique with RKF45 method in Maple software. The effects of several governing parameters such as transpiration parameter, shrinking parameter and nanoparticles volume fraction for alumina oxide and copper on the flow and heat transfer characteristics are presented in tables and graphs. Results elucidate that the skin friction coefficient decreases while the local Nusselt number increases as the shrinking parameter increased in regular fluid in the absence of transpiration. It is also found that in the presence of transpiration, alumina nanoparticle volume fraction decreases the heat transfer rate at the surface while it increases the skin friction coefficient. However, for copper nanoparticle volume fractions, the opposite results are obtained.

**Keywords**: Hybrid Nanofluid, Transpiration, Uniform Shear Flow, Shrinking Sheet

## 1. Introduction

Knowledge of heat transfer has gained much attention due to its importance in engineering and industrial application. Most engineering equipment such as heat exchangers and electronic devices use heat transfer fluid such as water, ethylene glycol and oil. However, these base fluids have lower thermal conductivity, therefore limiting the heat transfer enhancement. To improve the deficiency, scientists added a single type of nanosized particles to the fluid to form a compound called nanofluid which was introduced by Choi and Eastman [1]. Choi and Eastman proved that nanoparticles become excellent potentials by boosting the heat transfer rate and the thermal conductivity of the base fluids. Over the past decade, nanofluid technology is important in nanoscience, nanotechnology, and thermal

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engineering. Nanofluid aims to reach the highest possible thermal properties at the smallest concentrations. Moreover, some sorts of nanofluid phrases called hybrid nanofluid are developed to enhance the thermophysical properties of the regular nanofluid. Hybrid nanofluid is an extension of nanofluid formed of two different nanoparticles dispersed in the base fluid.

In this regard, the study on the heat transfer enhancement of a hybrid nanofluid has been investigated as a new concept in the boundary layer flow problem. For example, Aladdin et al. [2] investigated the problem of Cu-$Al_2O_3$ hybrid nanofluid over the permeable moving surface. The authors concluded that hybrid nanofluid possess better increment of shear stress and low heat transfer rate compared to nanofluid. Further study has been done by Zainal et al. [3] to show that centralization of alumina nanoparticles expended, the skin friction, and the heat transfer rate likewise increment. The expanding heat transfer is because of the presence of attractions boundary which influences the heat transfer proficiency. Research by Zeghbid and Bessaih [4] shows expending the consideration of mixture of nanofluid in the base fluid, the heat transfer shows increment value. Moreover, study of dual solutions of stretching/shrinking parameter are consider in Waini et al. [5], Observation of this research occur for both stretching or shrinking cases in suction parameter and discovered that only one of the dual solutions is stable and physically reliable. Study by Wahid et al. [6] also found that the shrinking surface boundary assists with making dual solutions more noticeable. A unique feature is that the transpiration parameter appears both in the boundary conditions and in the governing equations was found by Weidman et al. [7].

Uniform shear flow is also considered in this study, where the term shear flow refers to a character of fluid flow that is caused by forces, rather than by the forces themselves. In a shearing flow, adjacent layers of fluid move parallel to each other at different rates. The boundary layer flow with uniform shear free stream has not obtained much consideration compared to other flow conditions. According to Magyari and Weidman [8], the thermal characteristics of the Blasius flow problem over a semi-infinite flat are driven by a uniform shear flow in the field. Then, the effect of stretching/shrinking surface on flow and heat transfer characteristics with a uniform shear free stream has been studied by Fang [9] to fill the gap on a stationary surface on the study before this.

Motivated by the above mentioned works, the aim of this present paper is to solve and analyse the hybrid nanofluid over a shrinking sheet with transpiration and uniform shear flow by using RKF45 method with shooting technique. The nanofluid equations model presented by Tiwari and Das [10] is used in this study. The present numerical results are compared with previous study for validation purpose. New finding has also been obtained besides the current results.

## 2. Mathematical Formulation

Consider a two-dimensional steady flow and heat transfer over a shrinking sheet in a hybrid nanofluid. The coordinate systems are shows in Figure 1 where *x* and *y* axes are measured along the surface of the sheet and normal to it, respectively. The velocity of the shrinking sheet is taken as $u_w(x) = \left(\nu_f / L^{4/3}\right) x^{1/3}$ as in Fang [9], where *L* is the characteristic length of the surface and $\nu_f$ is the kinematics viscosity of the base fluid. The problem can be rescaled the equation with $L = \left(\nu_f / \beta\right)^{1/2}$ where $\beta$ is the constant strain rate. The surface temperature, $T_w$ and the far field fluid temperature, $T_\infty$ are also assumed to be constant. While $u_e\left(y\right) = \beta y$ is the free stream velocity with uniform shear. Considering the hybrid nanofluid, supposed that the size of nanoparticles is uniform, and the effect of the collection of nanoparticles is neglected because the base fluid and the nanoparticles are synthesized as a stable mixture.

By referring to Weidman et al. [7], Fang [9], and Devi and Devi [11] in Waini *et al.* [5] the governing equations of the hybrid nanofluid for the continuity, momentum and energy equations using the usual boundary layer approximations are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \text{Eq. 1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_{hnf}}\frac{\partial p}{\partial x}+\frac{\mu_{hnf}}{\rho_{hnf}}\frac{\partial^2 u}{\partial y^2} \qquad \text{Eq. 2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{hnf}}{\left(\rho C_p\right)_{hnf}}\frac{\partial^2 T}{\partial y^2} \qquad \text{Eq. 3}$$

subject to the boundary conditions that can be expressed as:

$$v=v_w\left(x\right),\ u=u_w\left(x\right)\lambda,\ T=T_w \text{ at } y=0,$$

$$u\to u_e\left(y\right),\ T\to T_\infty \text{ as } y\to\infty, \qquad \text{Eq. 4}$$

where $u$ and $v$ are the velocity components of the hybrid nanofluid along $x$ and $y$ axes, $T$ is the hybrid nanofluid temperature, $p$ is the pressure and $\lambda$ is the shrinking parameter. Further, $\rho_{hnf}$, $\mu_{hnf}$, $\left(\rho C_p\right)_{hnf}$ and $k_{hnf}$ are the density, dynamic viscosity, heat capacity, and thermal conductivity of the hybrid nanofluid respectively.

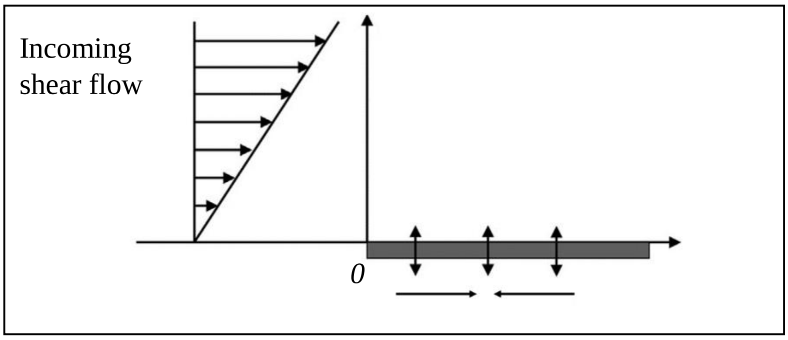


**Figure 1: Physical models and coordinate systems**

We employed the equations to evaluate the thermophysical properties for the nanofluid and the hybrid nanofluid as previous study which is provided in Table 1. Here, the volume fractions of $Al_2O_3$ and Cu nanoparticle as denoted as $\varphi_1$ and $\varphi_2$ respectively. While $\mu$, $\rho$, $C_p$, $(\rho C_p)$, and $k$ represent the dynamic viscosity, density, specific heat at constant pressure, heat capacity, and thermal conductivity respectively. Meanwhile, the subscripts $s1$, $s2$, $hnf$, $nf$, and $f$ represent $Al_2O_3$, Cu, hybrid nanofluid, nanofluid, and fluid respectively. The physical properties of the nanoparticles and the base fluid are shown in Table 2.

Following Waini *et al.* [5], similarity solutions of Eq.1, Eq. 2, Eq. 3, and Eq. 4 using the following similarity variables in Eq. 5, where $\psi$ is the stream function which defined as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ that satisfies continuity Eq. 1.

$$\psi = v_f \left(\frac{x}{L}\right)^{2/3} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_W - T_\infty}, \quad \eta = \left(\frac{x}{L}\right)^{-1/3} \frac{y}{L} \qquad \text{Eq. 5}$$

the velocities can be expressed as:

$$u = \frac{v_f}{L^{4/3}} x^{1/3} f'(\eta), \quad v = -\frac{1}{3}\left(\frac{v_f}{L^{2/3}}\right) x^{-1/3}\left(2f(\eta) - \eta f'(\eta)\right) \qquad \text{Eq. 6}$$

and become:

$$v_x(x) = -\frac{2}{3}\frac{v_f}{L^{2/3}} x^{-1/3} S \qquad \text{Eq. 7}$$

The streamwise pressure gradient is necessary to maintain the flow and are given by Magyari and Weidman [8]:

$$-\frac{1}{\rho_{hnf}}\frac{\partial p}{\partial x} = -\frac{2}{3}\frac{v_f^{\,2}}{L^{8/3}} x^{-1/3} S \qquad \text{Eq. 8}$$

Here, $S$ is the transpiration parameter with $S > 0$ for suction parameter. The external shear flow is interacted with the boundary layer and transpiration parameter.

**Table 1: Thermophysical properties of nanofluid and hybrid nanofluid**

| Properties | Nanofluid | Hybrid nanofluid |
|---|---|---|
| Density | $\rho_{nf} = (1-\varphi_1)\rho_f + \varphi_1\rho_{s1}$ | $\rho_{hnf} = (1-\varphi_2)\left[(1-\varphi_1)\rho_f + \varphi_1\rho_{s1}\right] + \varphi_2\rho_{s2}$ |
| Heat capacity | $(\rho C_P)_{nf} = (1-\varphi_1)(\rho C_P)_f + \varphi_1(\rho C_P)_{s1}$ | $(\rho C_P)_{hnf} = (1-\varphi_2)\left[(1-\varphi_1)(\rho C_P)_f + \varphi_1(\rho C_P)_{s1}\right] + \varphi_2(\rho C_P)_{s2}$ |
| Dynamic viscosity | $\mu_{nf} = \frac{\mu_f}{(1-\varphi_1)^{2.5}}$ | $\mu_{hnf} = \frac{\mu_f}{(1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}}$ |
| Thermal conductivity | $k_{nf} = \frac{k_{s1}+2k_f-2\varphi_1(k_f-k_{s1})}{k_{s1}+2k_f+\varphi_1(k_f-k_{s1})}\times(k_f)$ | $k_{hnf} = \frac{k_{s2}+2k_{nf}-2\varphi_2(k_{nf}-k_{s2})}{k_{s2}+2k_{nf}+\varphi_2(k_{nf}-k_{s2})}\times(k_{nf})$ |

**Table 2: Thermophysical properties of the fluid and nanoparticles**

| Physical properties | Fluid phase (water) | $Al_2O_3$ | Cu |
|---|---|---|---|
| $\rho(kg/m^3)$ | 997.1 | 3970 | 8933 |
| $C_P(J/kgK)$ | 4179 | 765 | 385 |
| $k(W/mK)$ | 0.613 | 40 | 400 |

Table 1 shows the thermophysical properties of nanofluid and hybrid nanofluid and Table 2 shows the thermophysical properties of the fluid and nanoparticles. The non-linear partial differential equations of continuity equation Eq. 1, momentum equation Eq. 2 and energy equation Eq. 3 are transformed to ordinary differential equation by using similarity transformation:

$$3\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f} f''' + 2ff'' - f'^2 = 2S, \quad \text{Eq. 9}$$

$$\frac{3}{\Pr}\frac{k_{hnf}/k_f}{\left(\rho C_p\right)_{hnf}/\left(\rho C_p\right)_f}\theta'' + 2f\theta' = 0, \quad \text{Eq. 10}$$

subject to the boundary conditions:

$$f(0) = S,\ f'(0) = \lambda,\ \theta(0) = 1$$

$$f''(\eta) \to 1,\ \theta(\eta) \to 0 \text{ as } y \to \infty \quad \text{Eq. 11}$$

where $\Pr = \mu_f (C_P)_f / k_f$ is the Prandtl number, and primes are denoted as differentiation respect to *η*. Eq. 9, Eq. 10, and Eq. 11 are reduced by Fang [9] in Waini et al. [5], and the present results are been compared for validation.

The skin friction coefficient, $C_f$ and the local Nusselt number, $Nu_x$, which are defined as follows, are the physical quantities of interest.

$$C_f = \frac{\tau_w}{\rho_f u_w^2},\quad Nu_x = \frac{xq_w}{k_f\left(T_w - T_\infty\right)} \quad \text{Eq. 12}$$

where $\tau_w$ is the shear stress along the surface and $q_w$ is the heat flux from the surface, which are given by:

$$\tau_w = \mu_{hnf}\left(\frac{\partial u}{\partial y}\right)_{y=0},\quad q_w = -k_{hnf}\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad \text{Eq. 13}$$

Using Eq. 5, Eq. 12, and Eq. 13, we obtain:

$$Re_x^{1/2}C_f = \frac{\mu_{hnf}}{\mu_f} f''(0),\quad Re_x^{1/2}Nu_x = -\frac{k_{hnf}}{k_f}\theta'(0) \quad \text{Eq. 14}$$

where the local Reynolds number is given as $Re_x = u_w(x)\,x/v_f$ .

## 3. Results and Discussion

The resulting of similarity Eq. 9 and Eq. 10 with subjected to boundary condition Eq. 11 are solved numerically by using shooting method with Runge-Kutta-Fehlberg (RFK45) in Maple and the analysis of the results are presented in tables and graphs. The 0.1 solid volume percentage of $Al_2O_3$ $(\text{i.e. } \varphi_1 = 0.1)$ is added to the base fluid as suggested by Devi and Devi [11] and added 0.1 solid volume fractions of Cu $(\text{i.e. } \varphi_2 = 0.1)$ to conduct this analysis. As a result, numerous solid volume fractions of Cu are added to the mixture to produce a Cu-$Al_2O_3$/water hybrid nanofluid. The Prandtl number of the base fluid is kept constant at Pr = 6.2 and will be utilised to create most of the results in this study. The skin friction coefficient, $f''(0)$ and the local Nusselt number, $\theta'(0)$ were successfully generated by using Maple. The parameter values of the Prandtl number, Pr, shrinking parameter, $\lambda$, nanoparticle volume fraction for alumina, $\varphi_1$, nanoparticle volume fraction for copper, $\varphi_2$, and the transpiration parameter, $S$ are the same as in the previous study.

### 3.1 Comparison in regular fluid

Table 3 is provided to compare the numerical values of the skin friction coefficient with previous study by Waini *et al.* [5] in regular fluid in absence of transpiration ($S = 0$). It is observed that the comparison values of skin friction coefficient for the shrinking case ($\lambda < 0$) are in an excellent agreement with those obtained by Waini et al. [5]. It can also be seen that when the sheet shrinks in the absence of transpiration, the shear stress reduces. According to Aladdin et al. [2], as the value of alumina increases, the skin friction also increases. In comparison, hybrid nanofluid poses better increment of shear stress compared to nanofluid. On the other hand, the variation of the Nusselt number, $-\theta'(0)$ is also obtained for regular fluid ($\varphi_1 = \varphi_2 = 0$) when $S = 0$ as shown in Table 4. This table is displayed for future reference purpose. It can be observed that the heat transfer rate at the surface increases in absence of transpiration as the surface shrinking.

**Table 3: Values of $f''(0)$ for $\lambda < 0$ when $\varphi_1 = \varphi_2 = 0$ and $S = 0$.**

| $\lambda$ | Waini et al. [5] | Present results |
|---|---|---|
| -0.1 | 0.993440 | 0.993440 |
| -0.2 | 0.971925 | 0.971925 |
| -0.3 | 0.931424 | 0.931424 |
| -0.4 | 0.864453 | 0.864453 |
| -0.5 | 0.752585 | 0.752585 |

**Table 4: Values of $-\theta'(0)$ for $\lambda < 0$ when $\varphi_1 = \varphi_2 = 0$ and $S = 0$.**

| $\lambda$ | $-\theta'(0)$ |
|---|---|
| -0.1 | -0.887161 |
| -0.2 | -0.775748 |
| -0.3 | -0.651725 |
| -0.4 | -0.509834 |
| -0.5 | -0.338781 |

### 3.2 Effect on velocity profiles, shear stress profile, and temperature profiles

The profiles of the velocity, $f'(\eta)$, shear stress, $f''(\eta)$, and temperature, $\theta(\eta)$ for various values of $\varphi_1$ are shown in Figure 2, Figure 3, and Figure 4, respectively. In these figures, the volume fractions of

$Al_2O_3$ nanoparticles are set as $\varphi_1$ = 0.01, 0.05 and 0.1 with fixed values of shrinking parameter $\lambda = -0.5$, volume fractions of Cu nanoparticles $\varphi_2 = 0.1$, transpiration parameter $S = 0.1$ and Prandtl number $\Pr = 6.2$. It can be seen that the increase in $\varphi_1$ contributes to the increment of $f'(\eta)$ and $f''(\eta)$ while decrement in $\theta(\eta)$. Therefore, the effect of nanoparticle volume fractions of $Al_2O_3$ is to reduce the heat transfer rate and increase the skin friction coefficient of the hybrid nanofluid.

Meanwhile, the results obtained as in Figure 5, Figure 6, and Figure 7 shows the effect on velocity, $f'(\eta)$, shear stress, $f''(\eta)$, and temperature, $\theta(\eta)$ profiles for various values of volume fractions of Cu nanoparticles $\varphi_2$ with fixed values of $\lambda = -0.5$, $\varphi_1 = S = 0.1$ and $\Pr = 6.2$ when $\varphi_2$ = 0.01, 0.05 and 0.1. It can be observed that when the nanoparticle volume fractions of Cu increases, $\theta(\eta)$ increase, while $f'(\eta)$ and $f''(\eta)$ decrease. Therefore, the effect of nanoparticle volume fractions of Cu is to increase the heat transfer rate at the surface, but it decreases the skin friction coefficient of the hybrid nanofluid.

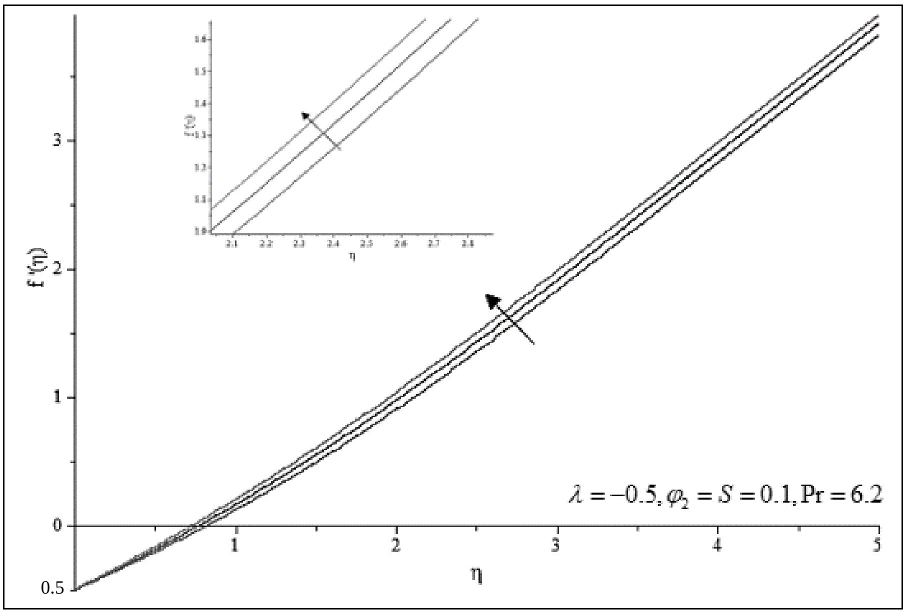


**Figure 2: Velocity profile, $f'(\eta)$ for various value of $\varphi_1$.**

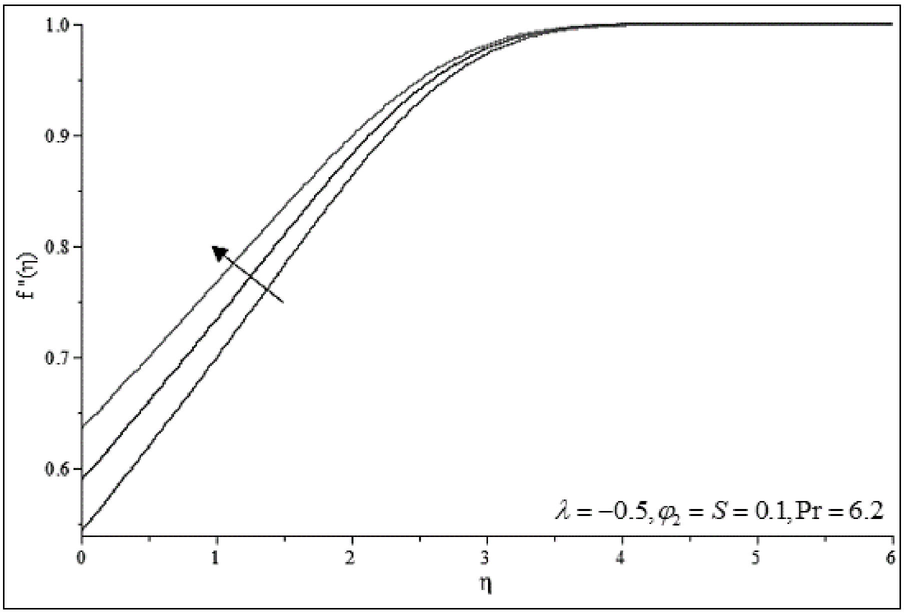


**Figure 3: Shear stress profile, $f''(\eta)$ for various value of $\varphi_1$.**

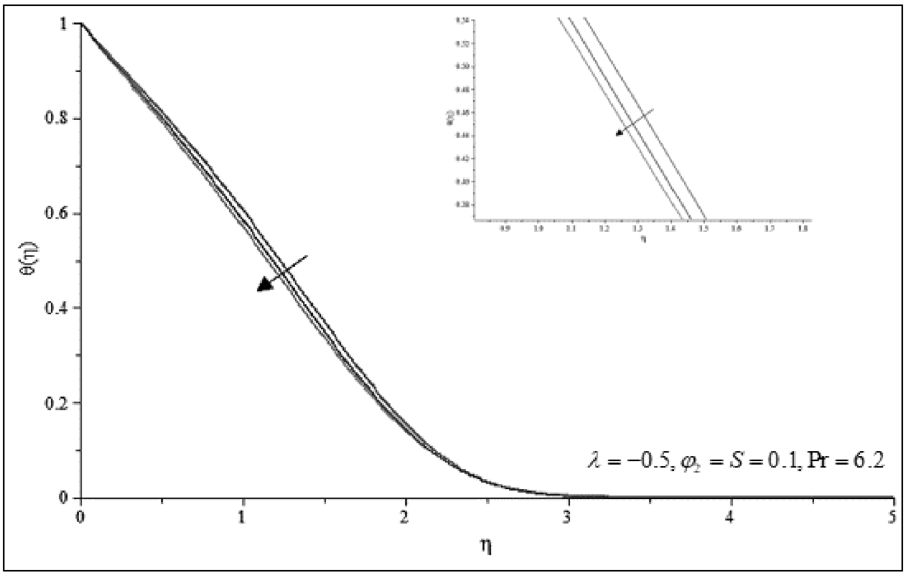


**Figure 4: Temperature profile, $\theta(\eta)$ for various value of $\varphi_1$.**

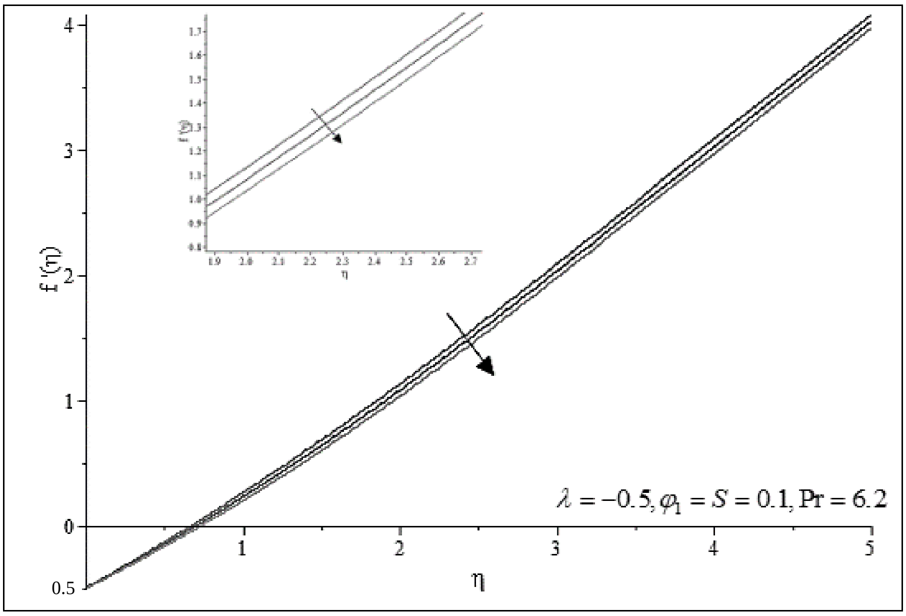


**Figure 5: Velocity profile, $f'(\eta)$ for various value of $\varphi_2$.**

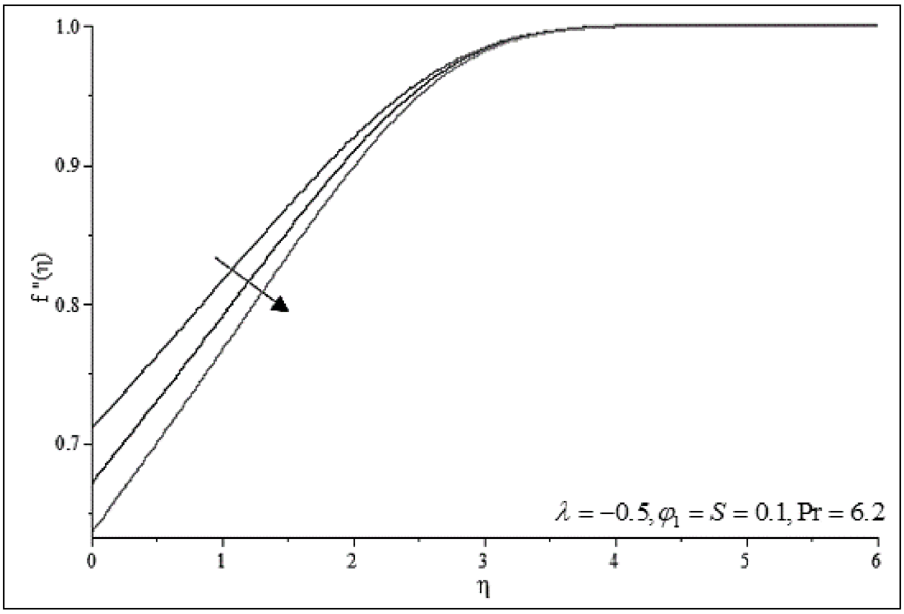


**Figure 6: Shear stress profile, $f''(\eta)$ for various value of $\varphi_2$.**

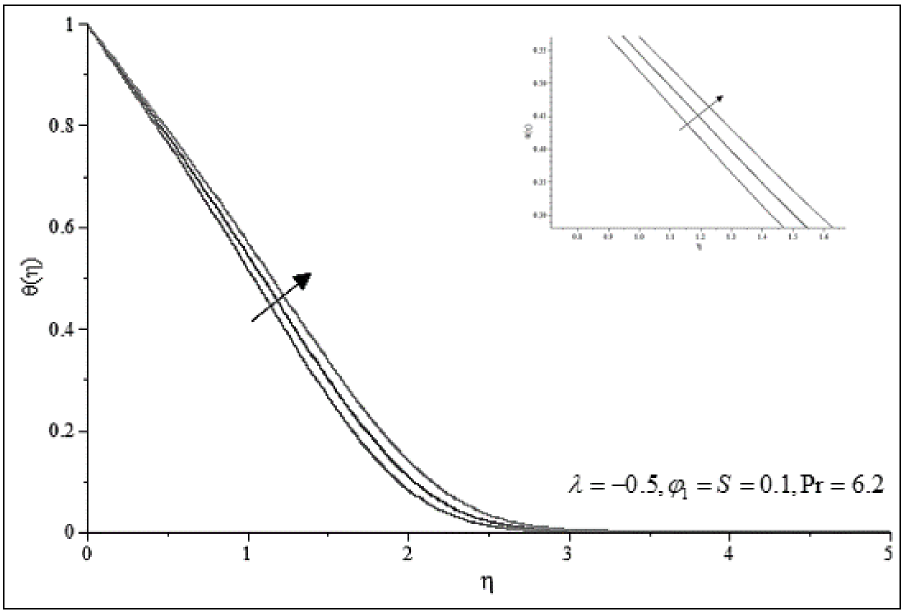


**Figure 7: Temperature profile, $\theta(\eta)$ for various value of $\varphi_2$.**

## 4. Conclusion

The effect of steady flow over a shrinking sheet with transpiration and uniform shear flow in hybrid nanofluid has been analysed in this study. The objectives of this study have been achieved. From this research, the initial objective that is to transform the governing equations of the hybrid nanofluid by employing the usual boundary layer, from partial differential equation (PDE) to ordinary differential equation (ODE) using similarity transformation technique has been reached. The second objective has also been attained since we have solved the transformed governing equations using shooting technique with Runge-Kutta-Fehlberg (RKF45) method in Maple software. New result has been obtained that is the effect of alumina oxide, $\varphi_1$ is to increase the velocity and shear stress, while it decreases the temperature, hence decreases the heat transfer rate, and increases the skin friction of the hybrid nanofluid for a fixed value of copper nanoparticles. Besides that, the effect of copper, $\varphi_2$ is to decrease the velocity and shear stress, but it increases the temperature thus increases the heat transfer rate at the surface and reduces the skin friction of the hybrid nanofluid.

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